OPTIMAL CONTROL WITH DISTURBANCE ESTIMATION

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ABSTRACT

The paper deals with a very common situation in many control systems and this is the fact that, for zero control action, the controlled variable is nonzero. This is often caused by the existence of another process input which is uncontrolled. Classic controllers do not take into account the second input – instead, deviation variables are considered or some feedforward controller is used to compensate the variable. The authors propose a solution, that the process is considered as a system with two inputs and single output (TISO). Here, the uncontrolled input is estimated with the state observer and the controller is designed as the multivariable controller. A Linear-quadratic (LQ) state-feedback control and model predictive control (MPC) of simple thermal process simulations are provided to demonstrate the proposed control strategy.

INTRODUCTION

Control theory is frequently using models in the form of transfer functions, which from the definition, consider zero initial conditions (Åström and Murray 2010; Nise 2010; Ogata 1995; Skogestad and Postlethwaite 2005). This means practically that for zero control action the controlled variable will be zero as well. Unfortunately, this is not true for many practical applications. Even a P controller will not work very well and the situation is even worse for advanced controllers based on state space process models. These models are similar to P or PD controller formulations – without integral control action. One solution is subtracting working point variables and introducing deviation variables – "zero initial condition" will be met. Integral control action, to ensure offset-free reference tracking, is another interesting problem to solve (Maeder and Morari 2010; Dušek et al. 2015). But why not use the natural process model with disturbance variables and their dynamical effects, directly for the controller design? Then the control task can be solved as a multivariable control problem when only some of the process inputs are used as control variables, while the others are considered as disturbances. Disturbance modelling and state estimation for offset free reference tracking control problems, was published in (Muske and Badgwell 2002; Pannocchia and Rawlings 2003; Tatjewski 2014).

Authors propose to estimate the disturbance variable by the augmented state observer. Extended formulation of a standard LQ state-feedback controller and predictive controller, so that the disturbance information is an integral part of the controller, is presented in the paper. A simple thermal process with electrical heating, ambient temperature effect and temperature sensor is modeled analytically by the first principle approach. The model has two inputs and one output. One of the inputs is heating power, while the other is ambient temperature. The output is the temperature sensor measured temperature. A discrete time linear time invariant process model is used for LQ controller design with infinity horizon and asymptotic set point tracking and predictive controller with finite horizon and special formulation of the cost function. Deviations of future states from desired states, calculated from the future set point knowledge, are considered instead of the future control errors which are commonly used in the literature (Camacho and Bordons 2007; Kouvaritakis and Cannon 2015; Maciejowski 2002; Rawlings and Mayne 2009; Rossiter 2003).

PROCESS MODEL WITH OFFSETS

Let us consider the controlled process with variable u_m as the control variable (control action) and y_m as controlled variable. Disturbance (offset) variables u_0 and y_0 are considered as process input and additive disturbance on the process output – see block diagram in Fig. 1.

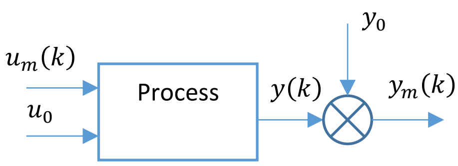

Figure 1: Process model

Discrete time state space process model can be written as,

$$\begin{aligned} \boldsymbol{x}(k+1) &= \mathbf{A}\boldsymbol{x}(k) + \mathbf{b}u_m(k) + \mathbf{b}_0 u_0 \\ y_m(k) &= \mathbf{c}\boldsymbol{x}(k) + y_0 \end{aligned} \qquad (1)$$

If we know the steady state input and both the offsets, the steady state output can be calculated as

$$y_m = \underbrace{\mathbf{c}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}}_{p}\, u_m + \underbrace{\mathbf{c}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{b}_0}_{p_0}\, u_0 + y_0 \quad (2)$$

and the steady state input (we will use this in controller design)

$$u_m(k) = \frac{1}{p}(y_m - y_0) - \frac{p_0}{p}u_0 \quad (3)$$

DISTURBANCE STATE ESTIMATION

The disturbances can be measured or estimated. In our case, we are using augmented state estimation for estimating the state vector and disturbance variable u_0, while y_0 must be known. It is not possible to estimate both offsets simultaneously. If u_0 is known, y_0 can be calculated from the steady state.

We introduce the augmented state space model as

$$\underbrace{\begin{bmatrix}\mathbf{x}(k+1)\\ u_0\end{bmatrix}}_{\mathbf{x}_r(k+1)} = \underbrace{\begin{bmatrix}\mathbf{A} & \mathbf{b}_0\\ 0 & 1\end{bmatrix}}_{\mathbf{A}_r}\underbrace{\begin{bmatrix}\mathbf{x}(k)\\ u_0\end{bmatrix}}_{\mathbf{x}_r(k)} + \underbrace{\begin{bmatrix}\mathbf{b}\\ 0\end{bmatrix}}_{\mathbf{b}_r} u_m(k) \quad (4)$$
$$\underbrace{y_m(k) - y_0}_{y_r(k)} = \underbrace{\begin{bmatrix}\mathbf{c} & 0\end{bmatrix}}_{\mathbf{c}_r}\underbrace{\begin{bmatrix}\mathbf{x}(k)\\ u_0\end{bmatrix}}_{\mathbf{x}_r(k)}$$

State estimator with gain **K** has the form

$$\hat{\mathbf{x}}_r(k+1) = \mathbf{A}_r\hat{\mathbf{x}}_r(k) + \mathbf{b}_r u_m(k) + \\ + \mathbf{K}\big(y_m(k) - y_0 - \mathbf{c}_r\hat{\mathbf{x}}_r(k)\big) \quad (5)$$

We are estimating in vector $\hat{\mathbf{x}}_r(k)$, all the state variables and disturbance variable u_0, from variables $u_m(k)$, $y_m(k)$ and from the known output offset y_0.

CONTROLLER DESIGN

Two types of controllers based on the state space process model are modified so that the estimation of the disturbance variable u_0 can be used as an integral part of the controller design.

LQ controller

Linear-quadratic state-feedback controller with infinite horizon cost function is

$$J = \sum_{i=1}^{\infty}\begin{bmatrix}\mathbf{x}^T(k+i)\mathbf{Q}\mathbf{x}(k+i) + \\ u^T(k+i-1)\mathrm{R}u(k+i-1)\end{bmatrix} \quad (6)$$

Negative state feedback controller part is

$$u(k) = -\mathbf{L}\mathbf{x}(k) \quad (7)$$

To be able to follow the set point asymptotically we are introducing a feedforward path with control variable $u_f(k)$ – see Fig. 2.

Control action is

$$u_m(k) = u(k) + u_f(k) \quad (8)$$

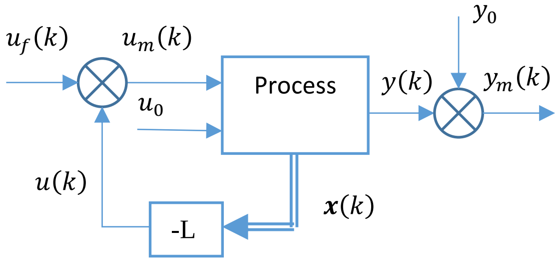

Figure 2: LQ Controller

Steady state output can be calculated as

$$y_m = \underbrace{\mathbf{c}(\mathbf{I}-\mathbf{A}+\mathbf{b}\mathbf{L})^{-1}\mathbf{b}}_{p_L}\, u_f + \\ + \underbrace{\mathbf{c}(\mathbf{I}-\mathbf{A}+\mathbf{b}\mathbf{L})^{-1}\mathbf{b}_0}_{p_{oL}}\, u_0 + y_0 \quad (9)$$

If $y_m = w(k)$ then the feedforward control variable is

$$u_f(k) = \frac{1}{p_L}(w(k) - y_0) - \frac{p_{oL}}{p_L}u_0 \quad (10)$$

and the control action is

$$u_m(k) = -\mathbf{L}\mathbf{x}(k) + u_f(k) \quad (11)$$

Model predictive controller

We consider the special matrix form cost function formulation for model predictive controller as

$$J(N) = (\mathbf{x}_N - \mathbf{x}_{Nw})^T\mathbf{Q}_N(\mathbf{x}_N - \mathbf{x}_{Nw}) + \mathbf{u}_N^T\mathbf{R}_N\mathbf{u}_N \quad (12)$$

where $\mathbf{u}_N$ is the vector of future control actions deviations from previous control action for a prediction horizon of N, which is given by,

$$\mathbf{u}_N = \underbrace{\begin{bmatrix}u_m(k)\\ u_m(k+1)\\ \vdots\\ u_m(k+N-1)\end{bmatrix}}_{\mathbf{u}_{Nm}} - \underbrace{\begin{bmatrix}u_m(k-1)\\ u_m(k-1)\\ \vdots\\ u_m(k-1)\end{bmatrix}}_{\mathbf{u}_{Nm0}}$$

and $\mathbf{x}_N$ is the vector of future predicted states deviations from future desired states $\mathbf{x}_{Nw}$

$$\mathbf{x}_N - \mathbf{x}_{Nw} = \underbrace{\mathbf{S}_{xx}\mathbf{x}(k) + \mathbf{S}_{xu}\mathbf{u}_{Nm} + \mathbf{S}_{xu0}\mathbf{u}_{N0}}_{\mathbf{x}_N} - \mathbf{x}_{Nw} = \\ = \mathbf{S}_{xx}\mathbf{x}(k) + \mathbf{S}_{xu}\mathbf{u}_N + \underbrace{\mathbf{S}_{xu}\mathbf{u}_{Nm0} + \mathbf{S}_{xu0}\mathbf{u}_{N0} - \mathbf{x}_{Nw}}_{\mathbf{o}}$$

$$\mathbf{x}_N = \begin{bmatrix}\mathbf{x}(k+1)\\ \mathbf{x}(k+2)\\ \vdots\\ \mathbf{x}(k+N)\end{bmatrix}, \mathbf{S}_{xx} = \begin{bmatrix}\mathbf{A}\\ \mathbf{A}^2\\ \vdots\\ \mathbf{A}^N\end{bmatrix}, \mathbf{u}_{N0} = \begin{bmatrix}u_0\\ u_0\\ \vdots\\ u_0\end{bmatrix},$$

$$\mathbf{S}_{xu} = \begin{bmatrix} \mathbf{b} & 0 & \cdots & 0 \\ \mathbf{Ab} & \mathbf{b} & \ddots & \vdots \\ \vdots & & & \\ \mathbf{A}^{N-2}\mathbf{b} & \mathbf{A}^{N-3}\mathbf{b} & \cdots & \mathbf{b} \quad 0 \\ \mathbf{A}^{N-1}\mathbf{b} & \mathbf{A}^{N-2}\mathbf{b} & & \mathbf{Ab} \quad \mathbf{b} \end{bmatrix}$$

$$\mathbf{S}_{xu0} = \begin{bmatrix} \mathbf{b}_0 & 0 & \cdots & 0 \\ \mathbf{Ab}_0 & \mathbf{b}_0 & \ddots & \vdots \\ \vdots & & & \\ \mathbf{A}^{N-2}\mathbf{b}_0 & \mathbf{A}^{N-3}\mathbf{b}_0 & \cdots & \mathbf{b}_0 \quad 0 \\ \mathbf{A}^{N-1}\mathbf{b}_0 & \mathbf{A}^{N-2}\mathbf{b}_0 & & \mathbf{Ab}_0 \quad \mathbf{b}_0 \end{bmatrix}.$$

Cost function (12) can be transformed to a form,

$$\begin{aligned} J(N) = \; & \mathbf{u}_N^T \underbrace{(\mathbf{R}_N + \mathbf{S}_{xu}^T \mathbf{Q}_N \mathbf{S}_{xu})}_{\mathbf{M}} \mathbf{u}_N + \\ & \mathbf{u}_N^T \underbrace{\mathbf{S}_{xu}^T \mathbf{Q}_N [\mathbf{S}_{xx}\mathbf{x}(k) + \mathbf{o}]}_{\mathbf{m}} + \underbrace{[\mathbf{S}_{xx}\mathbf{x}(k) + \mathbf{o}]^T \mathbf{Q}_N \mathbf{S}_{xu}}_{\mathbf{m}^T} \mathbf{u}_N + \\ & \underbrace{\mathbf{x}^T(k)\mathbf{S}_{xx}^T \mathbf{Q}_N \mathbf{S}_{xx}\mathbf{x}(k) + \mathbf{x}^T(k)\mathbf{S}_{xx}^T \mathbf{Q}_N \mathbf{o} + \mathbf{o}^T \mathbf{Q}_N \mathbf{S}_{xx}\mathbf{x}(k) + \mathbf{o}^T \mathbf{Q}_N \mathbf{o}}_{c} \end{aligned} \tag{13}$$

Solution for the unconstrained case to this quadratic form can be calculated analytically as

$$\mathbf{u}_N = -\mathbf{M}^{-1}\mathbf{m} \tag{14}$$

and the actual control action is

$$u_m(k) = u_m(k-1) + \mathbf{u}_N(1) \tag{15}$$

where $\mathbf{u}_N(1)$ is first element of vector of optimal future control actions deviation from previous control action.

Vector of future desired states $\mathbf{x}_{Nw}$ is calculated from the future set points as

$$\mathbf{x}_{Nw} = \begin{bmatrix} \mathbf{x}_w(k+1) \\ \mathbf{x}_w(k+2) \\ \vdots \\ \mathbf{x}_w(k+N) \end{bmatrix} \tag{16}$$

where

$$\mathbf{x}_w(k+i) = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}u_w(k+i) + (\mathbf{I} - \mathbf{A})^{-1}\mathbf{b}_0 u_0$$

and

$$u_w(k+i) = \frac{1}{p}[w(k+i) - y_0] - \frac{p_0}{p}u_0$$

THERMAL PROCESS

We consider the simple thermal process, where E is a heating power, T_o is ambient temperature, T_E, T and T_C are temperatures of the heating element, body of the system and the temperature sensor respectively. The system has two inputs and one output – see Fig. 3.

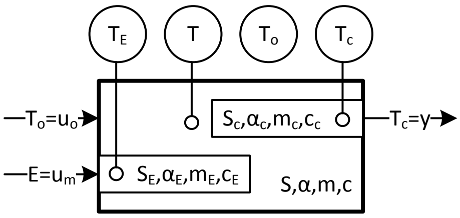

Figure 3: Thermal process

We are modeling the process analytically with first principle and we consider individual subsystems as systems with lumped parameters for the sake of simplicity.

Energy balance of the heating element is

$$E = \underbrace{\alpha_E S_E}_{s_1}(T_E - T) + \underbrace{m_E c_E}_{m_1}\frac{dT_E}{dt} \tag{17}$$

Energy balance of body of the system is

$$\alpha_E S_E(T_E - T) = \alpha_c S_c(T - T_c) + \underbrace{\alpha S}_{s_2}(T - T_o) + \\ + \underbrace{mc}_{m_2}\frac{dT}{dt} \tag{18}$$

Energy balance of the temperature sensor is

$$\underbrace{\alpha_c S_c}_{s_3}(T - T_c) = \underbrace{m_c c_c}_{m_3}\frac{dT_c}{dt} \tag{19}$$

State space model of the whole process is

$$\begin{bmatrix} \frac{dT_E}{dt} \\ \frac{dT}{dt} \\ \frac{dT_c}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{s_1}{m_1} & \frac{s_1}{m_1} & 0 \\ \frac{s_1}{m_2} & -\frac{s_1 + s_2 + s_3}{m_2} & \frac{s_3}{m_2} \\ 0 & \frac{s_3}{m_3} & -\frac{s_3}{m_3} \end{bmatrix} \begin{bmatrix} T_E \\ T \\ T_c \end{bmatrix} + \\ + \begin{bmatrix} \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} E + \begin{bmatrix} 0 \\ \frac{s_2}{m_2} \\ 0 \end{bmatrix} T_o \tag{20}$$

For the following simulations, we consider the parameters of the process as given in Table 1.

Table 1: Process parameters

		$J.s^{-1}.K^{-1}$		$J.K^{-1}$
Heating	s_1	0.5	m_1	1
Body	s_2	2.5	m_2	25
Sensor	s_3	0.1	m_3	0.5

SIMULATION RESULTS

The gain of the state observer is calculated as a solution of dual problem to a linear-quadratic state-feedback controller for discrete-time state-space system calculated in MATLAB as with command

$$[\mathbf{K}^{\mathrm{T}},\sim,\sim] = \mathrm{dlqr}(\mathbf{A}_{\mathrm{r}}^{\mathrm{T}},\mathbf{c}_{\mathrm{r}}^{\mathrm{T}},\mathbf{Q}_{\mathbf{e}},\mathrm{R}_{\mathrm{e}})$$

The penalization matrices are selected as $\mathbf{Q}_{\mathbf{e}} = \mathrm{eye}(4)$ and $\mathrm{R}_{\mathrm{e}} = 0.1$, and the sample time $Ts = 2.5$ s. State and disturbance estimation are demonstrated in Fig. 4. After a few seconds the state estimation errors drop to zero and the disturbance variable T_{o} is correctly estimated.

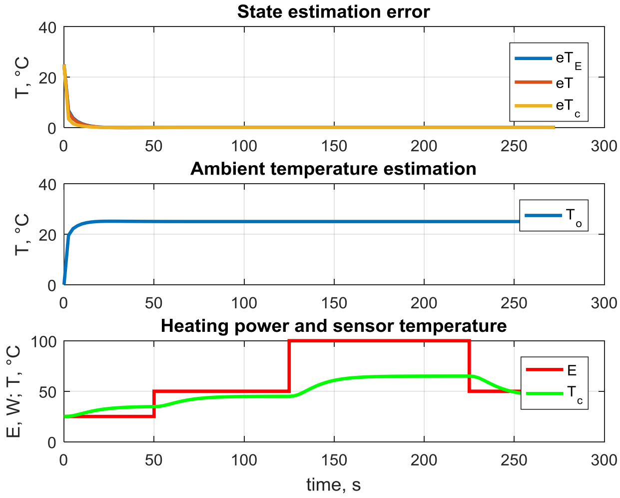

Figure 4: State and disturbance estimation

The gain of the LQ controller is calculated in the same way as the observer gain with MATLAB command, but only with modified penalization matrix **Q**

$$[\mathbf{L},\sim,\sim] = \mathrm{dlqr}(\mathbf{A},\mathbf{b},\mathbf{Q},\mathrm{R})$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

The control experiment can be seen in Fig. 5. The set point w is followed by the output T_{c} by controlling the heating power E (we do not consider constrains).

The predictive controller has identical parameters. The prediction horizon $N = 15$. Fig. 6 shows the control response with the predictive controller. It can be seen that, the predictive controller starts in advance before the set point change and the quality of the control is slightly higher – standard deviation (SD) of control error is 8.9 °C compared to 12.7 °C for LQ controller.

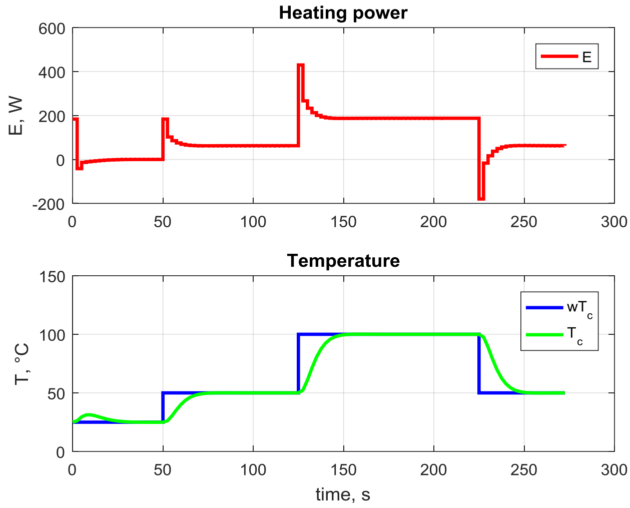

Figure 5: Control with LQ controller

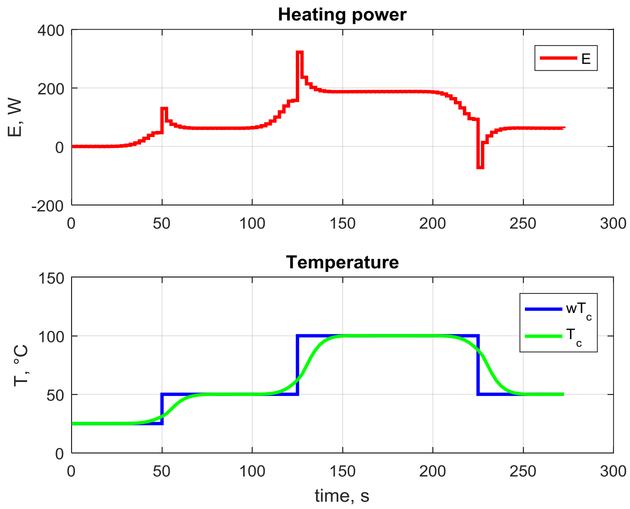

Figure 6: Control with predictive controller

CONCLUSION

The paper deals with a practical control issue, where for the steady state zero control input, the controlled variable is nonzero because of the offset or disturbance. Classic control methods are dealing with this problem by introducing deviations from a working point, integral actions, or feedforward parts of the standard single input single output (SISO) controllers. Authors propose, to work with the processes as with multivariable systems, and to design the controllers as a multivariable system. The disturbance (uncontrolled input variable) estimation method is presented in the paper. Subsequently, LQ and predictive controller design methods are modified that the estimated disturbance can be used as an integral part of the controllers. The set point is followed asymptotically with the LQ controller – feedforward controller path uses the offset information. Similarly, offset is used in the model predictive controller in free response calculation and for future desired states calculation as well. We are controlling sensor temperature but, it is also possible (without any problems) to control temperature of the body of the system; only by changing vector **c** of the process model for the controller design.

The paper is a nice example of the strength and elegance of state space methods for modelling, estimation and control. According to authors' opinion these methods will become acutely relevant in the future.

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