DECISION-MAKING ON IPO IMPLEMENTATION UNDER CONDITIONS OF UNCERTAINTY

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Abstract: *The theory of corporate financing considers the decision-making on the implementation of the IPO as one of the most important decision-making problems in the life-cycle of the undertaking. In addressing this decision-making task, however, are often not known all the necessary information – the likelihood of each condition. In a situation where we know only the fuzzy probabilities of certain states, it is appropriate to use quantitative methods, for example fuzzy linear programming. The aim of the contribution is to show on a case study use of the decision tree for deciding on the implementation of the IPO. Case study is divided into two parts. In the first part are known all the probabilities of each state, in the second part is known the likelihood of only some of the states, moreover, these probabilities are entered quite freely.*

Keywords: *IPO, Decision-making, Uncertainty, Water probability, Fuzzy logic.*

JEL Classification: *G32, C35.*

Introduction

The theory of corporate financing is considering the decision on execution or non-execution of IPO[4] for one of the most important decision-making problems during the life cycle of the company. There are offered many attitudes which aim to find a solution to this decision-making situation. The traditional approach is, regardless of the more detailed theoretical explanation, based on preparation of an overview of the possible advantages and drawbacks from implementation of the IPO. Similar reports are usually the result of surveys in enterprises making an IPO, or they reflect the views of the subjects who participate in the IPO. Lists are usually extensive, but not all in them mentioned advantages and disadvantages are necessary as a result of implementation of the IPO. Another possibility is usage of quantitative methods, such as fuzzy linear programming. The aim of the contribution is to show on a case study use of the decision tree for deciding on the implementation of the IPO. Case study is divided into two parts. In the first part are known all the probabilities of each state, in the second part is known the likelihood of only some of the states. It is therefore a task with partial ignorance.

1 Statement of a problem

Decision tasks are often represented by single root trees and sets of available III (input information items) probabilities, penalties etc. The full III set is either not available or some of its elements are prohibitively vague under realistic conditions, see e.g. [1]. A methodology / heuristic(s) is therefore needed to quantify the missing set of III.

[4] IPO is an abbreviation for „*Initial Public Offering*“. That is marked by the fact, that the company offers its shares to public for the first time, and at the same time it enters on the public organised stock market, represented the most frequently by the exchange stock as peak institution.

Decision makers who solve tasks are not willing to invest too much time into study of complex formal theories. They require such decisions which can be (re) checked by simple common sense reasoning to eliminate potential mistakes. Real-life decision making analysis must take into consideration the fact that precise information is missing, see e.g. [2]. The mentioned decision making theories have been very rarely studied from the company perspective see e.g. [3].

The following heuristic, see e.g. [4]:

The longer the path the less probable the path is (1)

is relatively easily accepted by those who routinely solve real life decision tasks. We strongly believe that there are several reasons for that: the heuristic is simple, based on common sense and its formalisation does not require sophisticated formal tools.

The algorithm studied in this paper is based on a strong analogy between a water flow through a one root tree system of pipes and the decision tree of the same topology. To make the common sense rechecks the following simplifications of the piping system are used:

- Each branch of the decision tree is considered to be a pipe of the same diameter and length. (2)
- All pipes are horizontally situated.

1.1 Water probability

Let us suppose that one liter of water is pumped into the root node of the decision tree and there is no accumulation of water in the tree. The consequence is that one liter of water must leave the tree through its terminal nodes. A reinterpretation of the heuristic (1) is:

The flowrate of water through a node is equal to its probability. (3)

The following algorithm is based on a simplified linear model to make the interpretation of the results more transparent for general audience.

The heuristic (1) is based on the assumption that the longer a decision (sub) branch is the less.

Let A be an event, and Ω is a set of all possible events, i.e. event space. An event probability p(A) must satisfy three following axioms [5]:

for all $A \in \Omega,\ p(A) \geq 0$, (4)

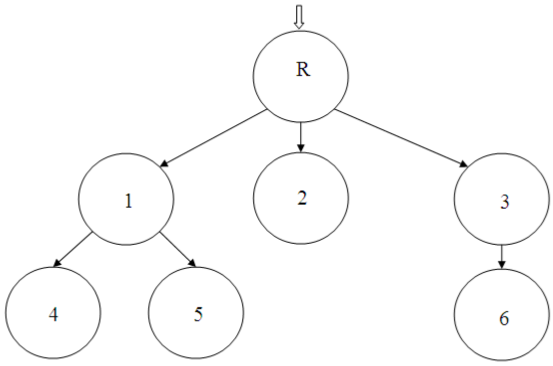
$p(\Omega) = 1$, (5)

$p(A_1 \cup A_2 \cup \ldots \cup A_i) = \sum_i p(A_i)$, (6) where A_i is a set of disjunctive events. The relevant water flow through the node N satisfies the axioms (4, 5, 6), see [4].

1.2 Topological resistance

A decision tree has one root node *R*, see e.g. Fig. 1:

Fig. 1: A Decision Tree



Source: own processing

The following definitions are used:

T be a set of terminals, see nodes 4, 5 and 6, Fig. 1.
N be a set of all nodes.
w_i is a number of ingoing and outgoing edges of *i*th node.
s_i is a number of edges of the subtree where *i* is the subroot.

$$s_i = \sum_j w_j \ . \tag{7}$$

where *j* represents nearest nodes of the subtree behind *i*th node. $a_{i,j}$ is a water/probability splitting ratio form *i*th node to *j*th node:

$$a_{i,j} = {w_j}/{s_i}, \text{ for all } j \in N - T \ . \tag{8}$$

K is a cardinality of N, the number of nodes. P_j of *j*th terminal for $j \in N$ is a flowrate of water through *j*th node. The value P_R of a root node is always equal one.

$$P_R = 1 \ . \tag{9}$$

For other nodes can be calculated by following (balance) equations

$$P_j = \sum_{i=1}^{k} \left(P_i \cdot a_{i,j}\right), \ j = 1,2,\ldots,K \ . \tag{10}$$

The set of K linear equations (10) where the set *P* is a vector of unknown variables and the splitting ratios *a* (8) are numerical constants can be easily solved.

Example The heuristic (3) can be used for evaluation of resistances (7), see Fig. 1:

(*i*th node)	Subtree	w_i	s_i	
1	R →1→ 4 → 5	3	2	
2	R →2	1	-(Node 2 is a terminal)	
3	R →3→ 6	2	1	(11)
4	1 → 4	1	-	
5	1 → 5	1	-	
6	3 → 6	1	-	

Where symbol "-" in fourth column indicates the *i*th node is a terminal.
The following splitting ratios (8) are:

$$
\begin{aligned}
&a_{r,1} = 1/6 = 0.167,\ a_{r,2} = 3/6 = 0.5,\ a_{r,3} = 2/6 = 0.333,\\
&a_{r,1} + a_{r,2} + a_{r,3} = 1,\\
&a_{1,4} = 1/2 = 0.5,\ a_{1,5} = 1/2 = 0.5,\\
&a_{1,4} + a_{1,5} = 1,\\
&a_{3,6} = 1.
\end{aligned}
\tag{12}
$$

The system of linear equations (9, 10) is as follows, see Fig. 1:

$$
\begin{aligned}
&1 + 0\cdot P_1 + 0\cdot P_2 + 0\cdot P_3 + 0\cdot P_4 + 0\cdot P_5 + 0\cdot P_6 = P_R,\\
&0.167\cdot P_R = P_1,\\
&0.500\cdot P_R = P_2,\\
&0.333\cdot P_R = P_3,\\
&0.500\cdot P_1 = P_4,\\
&0.500\cdot P_1 = P_5,\\
&1\cdot P_3 = P_6.
\end{aligned}
\tag{13}
$$

Solving system of equations (13) gives the probabilities of the terminal hypothesis.

1.3 Partial ignorance

A typical feature of all realistic decision tasks is a shortage of information. Isolated information items, e.g. probabilities of certain events are known. The concept of the total ignorance represented by e.g. the meta heuristic (1) helps to incorporate a set of isolated specific information items within a general framework of metaheuristics.

An incomplete set of probabilities

$$R \equiv (R_1, R_2, \ldots, R_h) \tag{14}$$

has h elements. However, the set R is not complete and therefore it is not possible to use well known algorithms of quantitative decision making.
For example the probability of node 2, see Fig. 1 is *small*. And this is the only known / given quantitative value:

$$R_2 = small. \tag{15}$$

It means that h = 2, see (14). The quantification is verbal. It is possible to discover if the additional quantitative items R (14) are consistent with the heuristic (1).

If the differences

$$|R_i - P_i|, i = 1,2,\ldots,N \tag{16}$$

are too significant, and this is nearly always the case, a reconciliation is inevitable.

1.4 Reconciliation

There are different reconciliation procedures. The problems of reconciliation are very important and have been studied for more than 30 years [6].

A very flexible and productive idea is a fuzzy interpretation of the numerical values of the set (14):

$$P = R_i, \tag{17}$$

where R_i is the fuzzy set which is given as a part of the problem specification. The over specified set of fuzzy set of linear equation is (10, 17):

$$\mathbf{A} \cdot \mathbf{P} = \mathbf{B} \ \cup \ \mathbf{P} = \mathbf{R}, \tag{18}$$

has n + h equations and n variables **P** (see (14)). The reconciliation can be solved by a fuzzy linear programming, see e.g. [7, 8].

1.5 Fuzzy reconciliation

The set of equations (17) is over specified. Different equations are differently reliable. Rather often certain equations cannot be violated at all. For example probability of a terminal event, see e.g. node 4 in Fig. 1, is known very accurately. Therefore the corresponding additional probability P must be reconciliation algorithm which takes into consideration different violation of different equations (18).
A simple example is used to demonstrate the basic idea of fuzzy reconciliation. However, A level /extent of violation is specified by a fuzzy interpretation of the set R_i. Our

experience indicates that the fuzzy set R_i is mean fully characterised by triangular grades of memberships, see e.g. Fig. 3

$$a \quad b = c \quad d. \tag{19}$$

$$\mathbf{P} = \mathbf{R}. \tag{20}$$

The vector of right hand side coefficients R_P is quantified using fuzzy numbers [9]. To simplify the problem let us suppose that the triangular numbers are used (19).
The *j*-th equation (20) can be transformed into four linear inequalities [10] by introducing two slack variables S_u and S_l.

$$\sum_{i=1}^{h} \left(a_{ji} \cdot P_i\right) + S_{uj} \geq b_j \;, \tag{21}$$

$$\sum_{i=1}^{h} \left(a_{ji} \cdot P_i\right) - S_{lj} \leq b_j \;, \tag{22}$$

$$0 \leq S_{uj} \leq b_j - a_j \,, \tag{23}$$

$$0 \leq S_{lj} \leq d_j - b_j \,, \tag{24}$$

where

b_j, a_j, d_j - see (19),
S_{uj} - is the *j*-th upper slack variable,
S_{lj} - is the j-th lower slack variable.

The set of inequalities (21–24) represents fuzzy description of *j*-th linear equation (10).

The total ignorance is always dangerous. The more the system of linear equations is over specified the better. The set of additional information (15) allows us to cross check not only the metaheuristics but the additional information items as well. The following simple requirements must be satisfied

$$m < h. \tag{25}$$

One possible objective function which represents a meaningful trade-off is

$$Q = \min_{S_u S_l} \left\lfloor \sum_{j=1}^{m} \left(S_{uj} / \left(b_j - a_j\right) + S_{lj} / \left(d_j - b_j\right)\right) \right\rfloor. \tag{26}$$

It is possible to transfer the minimisation problem (26) to a fuzzy linear programming problem and this problem can be finally solved as a conventional linear programming task, see e g. [11, 12].
For example the probability of node 2, see e.g. Fig. 1, is defined as '*small*'. This verbal '*small*' can be expressed by a fuzzy set (19)

$$a = 0.1;\; b = c = 0.2;\; d = 0.25. \tag{27}$$

The equation $R_2 = (0.1;0.2;0.25)$ (e.g. balance) can be transformed by (21–24) into four linear inequalities by introducing two slack variables S_u and S_l.

$$\begin{aligned} &0.1 \cdot P_2 + S_u \geq 0.2, \\ &0.1 \cdot P_2 - S_l \leq 0.2, \\ &0 \leq S_u \leq 0.1, \\ &0 \leq S_l \leq 0.05. \end{aligned} \tag{28}$$

The inequalities (28) are added to the balance equations (13). This system of linear equations is supplemented by one possible objective function (29) and it is solved using linear programming methods.

$$Q = \min_{S_u S_l} \left[S_u / 0.1 + S_l / 0.05 \right]. \tag{29}$$

2 Methods

To achieve the stated objectives have been first selected three factors, which we consider to be a key for the decision-making on implementation of the IPO. Further we were considering about two situations. In the first one are known all the necessary information for decision-making, in the second are known fuzzy likelihood of only certain conditions. The first situation required a solution by normally used method of the decision-making tree. The second solution consisted in finding missing probabilities by use of the water heuristic, adding the specified fuzzy probabilities and solving of this problem using linear programming.

3 Problem solving

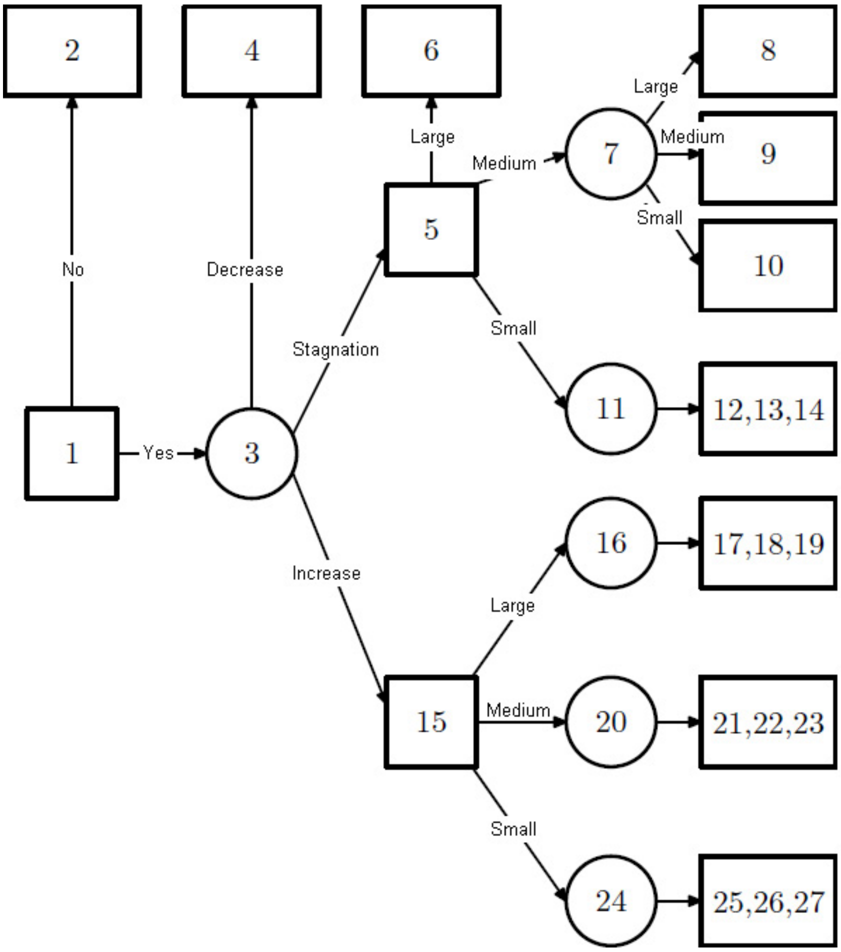
For the sake of keeping the scope of this article within acceptable limits there was selected a medium-sized decision-making tree. This decision-making tree addresses the entry of the company on the Stock Exchange trough IPO. When you build the decision-making tree there have been taken into account the following factors:

- macro-economic growth (increments of GDP),
- Investor's interest in the IPO,
- size of the issue (number of issued shares multiplied by their emission rate).

We consider these factors to be key for the decision-making on the implementation of the IPO, it is, however, only a selection from many others, which affect the success of the IPO. Modified version of this decision-making tree is seen on figure 3. In the decision-making tree are used two types of nodes: situations (rings) and decision-making (squares and boxes). The rings represent the "Lottery", where the decision-maker is not able to influence the future state (the state is decided by neighbourhood of the decision-maker). The squares represent decisions which are within the competence of decision-maker. Boxes represent the end nodes.

Case study is divided in two parts. In the first part are known all the probabilities of each state, in the second part are known fuzzy probabilities of only some of the states (partial ignorance).

Fig. 2: Decision-making tree IPO



Source: own processing

From the figure 2 it is apparent that in the framework of the space savings are the individual boxes 12-14, 17-19, 21-23, 25-27 unified into one. In fact, they are handled as three separated rectangles (as well as with the rectangles 8-10). In the following table is described the importance of the main knots of the chart.

Tab. 4: Importance of knots

The Knot no.	**Meaning**	**The Knot no.**	**Meaning**
1	Implement the IPO?	15	Size of the IPO
2	STOP	16	Investor's interest
3	GDP	20.	Investor's interest
4	STOP	24	Investor's interest
5	Size of the IPO		
6	STOP		
7	Investor's interest		
11	Investor's interest		

Source: own processing

3.1 Problem solving in case of knowledge of probabilities of all conditions

The following table shows the profits of the individual variants of the decision and the relevant known probabilities.

Tab. 5: Profit and the specified probabilities

Variant	Probability of the variant	Profit (mil. CZK)	Variant	Probability of the variant	Profit (mil. CZK)
3–4	0.15	0	16–17	0.50	100
3–5	0.30		16–18	0.30	50
3–6	0.55		16–19	0.20	10
7–8	0.20	30	20–21	0.30	40
7–9	0.50	10	20–22	0.50	20
7–10	0.30	–5	20–23	0.20	0
11–12	0.10	10	24–25	0.20	30
11–13	0.50	–5	24–26	0.50	20
11–14	0.40	–20	24–27	0.30	–10

Source: own processing

On the basis of these data were calculated average profits for individual variants (tab. 3).

Tab. 6: Average of the profit

Variant	5–7	5–11	15–16	15–20	15–24	1–3	1–2
Average profit (mil. CZK)	9.5	–9.5	67	22	13	34.7	0

Source: own processing

Evaluation:

From these calculations is obvious that in the present case, it is recommended:

1) Implement the IPO in case of stagnation, or of GDP growth,
2) in case of stagnation of GDP choose middle issue, in the case of GDP growth choose a big issue,
3) largest profit will be achieved in the case of great interest of investors in both variants.

Average value of profit is CZK 34.7 mil.

3.2 Problem solving in case of fuzzy knowledge of only certain conditions

In the following section is described the case of the decision tree, where are known only some probabilities. The following table shows the profits of the individual variants of the decision and the relevant entered probabilities.

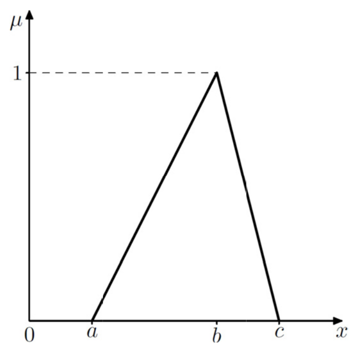
Tab. 7: Profit and the specified probabilities

Variant	Probability of the variant	Profit (mil. CZK)	Variant	Probability of the variant	Profit (mil. CZK)
3–4		0	16–17	about 0.05	100
3–5			16–18		50
3–6			16–19	about 0.80	10
7–8		30	20–21	about 0.90	40
7–9	about 0.20	10	20–22		20
7–10	about 0.70	–5	20–23		0
11–12		10	24–25		30
11–13		–5	24–26		20
11–14		–20	24–27		–10

Source: own processing

From the table is obvious, that there are known only the probabilities of some variants. In addition, these probabilities are entered very freely. It is thus a partial ignorance task which is described in the previous chapter. In the calculation of the remaining probabilities is used metaheuristics (3). Freely entered probabilities can be for further calculations converted to fuzzy values. The transcription may be executed with a transcript to a triangular fuzzy number (see fig. 3).

Fig. 3: Triangular fuzzy number



Source: own processing

Numeric conversion is seen in the following table.

Tab. 8: Conversion of probabilities to fuzzy

	a	b	c
$a_{7.9}$ =	0.1	0.2	0.3
$a_{7.10}$ =	0.6	0.7	0.85
$a_{16.17}$ =	0.03	0.05	0.07
$a_{16.19}$ =	0.7	0.8	0.85
$a_{20.21}$ =	0.8	0.9	0.95

Source: own processing

The remaining probabilities are calculated on the basis of topological resistance (7), balancing equations (10) and the partial ignorance (15) – see the following table.

Tab. 9: The dividing ratios

Variant	Probability of	Variant	Probability of
3–4	0.542	16–17	0.050
3–5	0.417	16–18	0.200
3–6	0.042	16–19	0.750
7–8	0.100	20–21	0.900
7–9	0.200	20–22	0.100
7–10	0.700	20–23	0.000
11–12	0.333	24–25	0.333
11–13	0.333	24–26	0.333
11–14	0.333	24–27	0.333

Source: own processing

On the basis of these data were calculated average profits for individual variants (tab. 7).

Tab. 10: Average profit

Variant	5–7	5–11	15–16	15–20	15–24	1–3	1–2
Average profit (mil. CZK)	1.5	–4.995	22.5	38	13.32	–2.79	0

Source: own processing

Evaluation:

In the present case, it is recommended not to realise the IPO, since the mean value of the profit results at CZK –2.79 mil.

Conclusion

The theory of corporate financing considers the decision-making on the implementation of the IPO as one of the most important decision-making problems in the life-cycle of the undertaking. In addressing this decision-making task, however, are often not known all the necessary information – the likelihood of each condition. In a situation where we know only the probability of certain states, and where these probabilities are entered freely, it is appropriate to use quantitative methods, for example fuzzy linear programming. In this method, it is first necessary to specify the probabilities for singe conditions by using a water metaheuristics. This heuristic is based on the principle of water flow through a piping where individual separating conditions in the monitored nodes correspond to the searched probabilities. On the basis of established separating ratios (probabilities) is created with support of balance equations a system of linear equations. To these equations are then added the linear equations, based on the entered fuzzy probabilities. Thus created system of the equations is further supplemented by a specific function. In this function we are trying to minimize the deviation of the found probabilities from the given fuzzy probabilities. The advantage of this approach lies in the fact that the specific function and the equations of the individual restrictions are linear, and therefore easily solvable using commonly known simplex method. In addition, this method is also applicable in other fields where it is needed to make decisions under conditions of uncertainty.

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