Hierarchical Intuitionistic TSK Fuzzy System for Bitcoin Price Forecasting

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***Abstract*—There has been great interest in developing hierarchical structures of fuzzy rule-based systems due to their flexibility allowing to model complex problems. To cope with the high degree of uncertainty arising from the characteristics of cryptocurrency markets, this paper proposes a hierarchical intuitionistic TSK (Takagi–Sugeno–Kang) fuzzy system equipped with a feature selection and feature ranking component. The proposed system uses intuitionistic fuzzy sets, allowing to effectively model investor uncertainty in the decision-making on cryptocurrency markets. The hierarchical structure is a parallel tree-like fuzzy system that is based on relevant features while considering feature dependencies. Computational efficiency is achieved by using fuzzy c-means clustering to produce rule antecedents. The proposed system is validated using multivariate bitcoin data for the period 2018 to 2022, showing that the proposed system can accurately predict bitcoin prices while retaining an interpretable hierarchical structure.**

***Index Terms*—hierarchical structure, intuitionistic TSK fuzzy system, bitcoin, forecasting**

I. INTRODUCTION

A hierarchical fuzzy system (HFS) is a fuzzy rule-based system that is organized into multiple levels [1]. The hierarchical architecture allows the system to handle complex problems and make more accurate decisions by breaking them down into smaller, more manageable components (units). This not only overcomes the problem of the curse of dimensionality and rule explosion [2], but its flexibility also allows easier development and modification of the system, making it easier to scale the system to handle more complex problems, as new levels can be added as needed. Furthermore, the rule base in an HFS can provide a more human-like reasoning and decision-making process, which is often more intuitive and easier to understand than traditional rule-based systems. In fact, interpretability tends to be one of the primary considerations for justifying the use of HFSs [3]. In summary, HFSs offer universal approximation ability, while better handling of complexity and better modularity, making them a more suitable option for complex problems than single fuzzy rule-based systems [4]. Specifically, these remarkable features of HFSs have recently been exploited in a number of applications, including time-series forecasting [5] and human posture recognition [6].

To better cope with the uncertainty in the data and allow additional flexibility, a recent stream of research has focused on extending type-1 fuzzy rule-based systems [7]–[10]. Thus, the generalizations of fuzzy systems, such as interval type-2 and intuitionistic fuzzy systems, enable the expression of decision determinacy and hesitation when representing more complex and nuanced uncertainty than conventional type-1 fuzzy systems. The increased flexibility of such extensions can lead to more accurate decisions and better coping with dynamic environments, as they allow for the representation of evolving uncertainty over time [11]. In the current study, we opted for intuitionistic fuzzy systems on the basis of the specifics of the bitcoin market. Indeed, the bitcoin market is known for its high levels of uncertainty stemming from regulatory uncertainty, high volatility of bitcoin demand, lack of intrinsic value, and limited historical data, which makes it difficult to make informed investment decisions [12], [13]. Intuitionistic fuzzy sets allow us to express investor preferences and non-preferences. In cryptocurrency markets, it is convenient to represent the inherent uncertainty of investors in terms of positive and negative evidences, as investor preferences depend on their investment position and the related financial risk. The demand and supply factors of the bitcoin price can then be perceived to some extent positively and negatively, subject to the position (long/short) of the investors and their risk attitude and the related stop-loss or stop-limit orders, which is also reflected in the considerable impact of investor sentiment in cryptocurrency markets [14]. However, even intuitionistic fuzzy systems and other extensions of fuzzy rule-based systems struggle with the complexity of the bitcoin market. In particular, high volatility, lack of regulation, and technical complexity make it difficult for investors to predict the value of bitcoin. Therefore, to achieve a more accurate prediction, a number of determinants need to be considered, including blockchain fundamentals, transaction and mining features, and market forces [15]. To overcome the multidimensional complexity of bitcoin price forecasting, here we propose a hierarchical intuitionistic TSK fuzzy system equipped with a feature selection and feature ranking component. Unlike existing extensions based on interval-valued fuzzy sets [6], [16], the proposed system uses intuitionistic fuzzy sets, which allows us to model the intrinsic uncertainty in the cryptocurrency

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market.

The remainder of this paper is structured as follows. Section II outlines the proposed hierarchical intuitionistic TSK fuzzy system. Section III first presents the experimental setup, including the dataset used. Section III also gives experimental evidence for the effectiveness of the bitcoin price forecasting model based on the proposed hierarchical intuitionistic TSK fuzzy system. Finally, Section IV concludes.

II. HIERARCHICAL INTUITIONISTIC TSK FUZZY SYSTEM

In this section, the problem of developing hierarchical intuitionistic TSK fuzzy systems is concerned. This section first deals with the construction of the intuitionistic TSK fuzzy system, and then the process of building its hierarchical structure is presented.

A. Intuitionistic TSK Fuzzy System

In contrast to conventional fuzzy rule-based systems, the antecedent membership functions of an intuitionistic fuzzy rule-based system are represented by intuitionistic fuzzy sets. In an intuitionistic fuzzy set A, the degree of membership $\mu_A(x)$ is complemented by the degree of non-membership $\nu_A(x)$ for the element x as follows:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle x \in X\}, \tag{1}$$

where $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ and the degree of hesitation is given as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$. To obtain intuitionistic fuzzy sets of the inputs, we use the fuzzification method proposed in [11] while assuming that the fuzzy set $\mu(x)$ is given as a Gaussian membership function and, in agreement with [10], the variance of the last s values in the time series is reflected in the fuzzification parameter D:

$$\mu(x) = \mathrm{e}^{-(x-b)^2/2\sigma^2}, \tag{2}$$

$$\mu_A(x_i^t) = \mu(x_i^t) \times (1 - D), \tag{3}$$

$$\nu_A(x_i^t) = 1 - \mu(x_i^t) \times (1 - D) - D, \text{ where:} \tag{4}$$

$$\begin{aligned} D = (&\max(\mu(x_i^t), \mu(x_i^{t-1}), \dots, \mu(x_i^{t-s})) \\ &-\min(\mu(x_i^t), \mu(x_i^{t-1}), \dots, \mu(x_i^{t-s}))). \end{aligned} \tag{5}$$

Here we consider an intuitionistic TSK fuzzy rule-based system with n inputs (predictors) $x_1^t, x_2^t, \dots, x_n^t$ at time t and m rules $R_1, R_2, \dots, R_m$ defined as follows:

$$\begin{aligned} &R_j : \text{if } x_1^t \text{ is } A_{1,j} \text{ and } x_2^t \text{ is } A_{2,j} \text{ and } \dots \text{and } x_n^t \\ &\text{is } A_{n,j} \text{then } y_j^{t+k} = a_{0,j} + a_{1,j}x_1^t + \dots + a_{n,j}x_n^t, \end{aligned} \tag{6}$$

where $x_i^t \in \mathbb{R}$, $A_{i,j}$ denotes the intuitionistic fuzzy set of the antecedent for the j-th rule R_j and i-th input x_i^t, y_j^{t+k} is the forecast at time $t + k$, and $a_{i,j}$ represents the parameter of the consequent. For the j-th rule R_j, the firing weight w_j is defined as follows [17]:

$$w_j = w_j^{\mu} - w_j^{\nu}, \tag{7}$$

where the minimum t-norm and maximum t-conorm operators are used to obtain the membership and non-membership degrees of w_j, respectively, in the following way: $w_j^{\mu} = \min_{j=1,2,\dots,m}(\mu_A(x_1^t), \dots, \mu_A(x_n^t))$, and $w_j^{\nu} = \max_{j=1,2,\dots,m}(\nu_A(x_1^t), \dots, \nu_A(x_n^t))$. The firing weight w_j is normalized to w_j^{norm}. Then, the defuzzified output of the intuitionistic fuzzy system is calculated using the IFWA operator as follows:

$$y_{\mathrm{IFWA}}^{t+k} = \frac{\sum_{j=1}^{m} y_j^{t+k} w_j^{\mathrm{norm}}}{\sum_{j=1}^{m} w_j^{\mathrm{norm}}}. \tag{8}$$

B. Hierarchical Structure of Intuitionistic TSK Fuzzy Systems

A hierarchical intuitionistic fuzzy system (HIFS) can be constructed, consisting of intuitionistic TSK fuzzy systems. To minimize the number of rules and decompose the problem into as simple subsystems as possible, this paper considers a parallel tree-like HIFS. At each level l, two inputs are used to produce a set of TSK rules. The j-th rule $R_j^{(l)}$ for the first level $l = 1$ is defined as follows:

$$\begin{aligned} &R_j^{(1)} : \text{if } x_1^t \text{ is } A_{1,j}^{(1)} \text{ and } x_2^t \text{ is } A_{2,j}^{(1)} \\ &\text{then } y_j^{(1)} = a_{0,j}^{(1)} + a_{1,j}^{(1)}x_1^t + a_{2,j}^{(1)}x_2^t, \end{aligned} \tag{9}$$

and the outputs from the previous level are taken as the inputs of the rules in the next level:

$$\begin{aligned} &R_j^{(l)} : \text{if } y_1^{l-1} \text{ is } A_{1,j}^{(l)} \text{ and } y_2^{l-1} \text{ is } A_{2,j}^{(l)} \\ &\text{then } y_j^{l} = a_{0,j}^{(l)} + a_{1,j}^{(l)}y_1^{l-1} + a_{2,j}^{(l)}y_2^{l-1}. \end{aligned} \tag{10}$$

The output of the final level L is obtained as follows:

$$\begin{aligned} &R_j^{(L)} : \text{if } y_1^{L-1} \text{ is } A_{1,j}^{(L)} \text{ and } y_2^{L-1} \text{ is } A_{2,j}^{(L)} \\ &\text{then } y_j^{L,t+k} = a_{0,j}^{(L)} + a_{1,j}^{(L)}y_1^{L-1} + a_{2,j}^{(L)}y_2^{L-1}. \end{aligned} \tag{11}$$

To achieve stable convergence, the gradient descent algorithm was used to adapt the consequent parameters $a_{i,j}^{(l)}$ [18]:

$$w_{i+1} = w_i - \eta \nabla_{\theta} J\left(w_i; x^{(t)}; y^{(t+k)}\right), \tag{12}$$

where w_i is the updated consequent parameter at iteration i, $w_i = \{a_{0,j}^{(l)}, a_{1,j}^{(l)}, \dots, a_{n,j}^{(l)}\}$, η is the learning rate and J is the objective function (RMSE).

There are two main problems to be addressed when building the structure of an HFS (or HIFS), namely the selection of input variables and the determination of their order of entry into the system [16]. Feature selection can improve system accuracy and greatly reduce the computational complexity of the system. In this study, consistent with the prevailing literature [19], the correlation-based feature selection was used to eliminate irrelevant or redundant inputs. More specifically, the correlation-based filter based on Pearson's correlation coefficient was used to consider individual predictive feature ability and its redundancy. For the heuristic search strategy, in accordance with [15], the flower pollination algorithm was employed and, then, to rank the selected features, the traditional ReliefF algorithm was used. Thus, redundant features were discarded and feature dependencies were detected, which allowed us to design an interpretable hierarchical structure.

The next critical step in generating HFSs is to determine the number of rules and antecedents of fuzzy sets. To determine the values of these parameters, we followed [15] and performed the FCM (fuzzy c-means) clustering for different values of c while evaluating the cluster validity using the Xie-Beni index.

III. MODEL VALIDATION FOR BITCOIN PRICE FORECASTING

A. Experimental Setup

To validate the effectivity of the proposed HIFS, its predictive performance was examined on a multivariate bitcoin dataset. Specifically, the aim of the experimental validation was to train the HIFS to predict one-day-ahead closing bitcoin prices. The dataset was gathered from the Refinitiv database and covered the period March 2018 to May 2022. In accordance with the current literature [20], [21], we included a wide range of features spanning the major determinants of the bitcoin price, namely (1) network usage and inter-exchange transactions; (2) mining variables, such as bitcoin hashrate; (3) bitcoin supply and demand; (4) prices of other cryptocurrencies; (5) investor sentiment (bitcoin misery index); (6) oil price; and (7) technical indicators (volatility indicators). In total, 55 features were used, a detailed description of which is given in [15].

First, consistent with previous studies [22], sequential validation was used to partition the bitcoin time-series data into the training set (1230 trading days) followed immediately by the testing set (307 trading days) in a 4:1 ratio. It is worth noting that the testing data covered a highly volatile and challenging period when bitcoin prices hit their highs (in November 2021) only to experience a historic decline (from the end of 2021).

To determine the hierarchical structure of the HFIS, feature selection and feature ranking was first conducted, using the correlation-based filter (the feature space was explored using the flower pollination algorithm) and the ReliefF algorithm, respectively. For setting the parameters of the flower pollination algorithm, we followed the recommendation of [23], that is, the size of the population was set to 20, pollination = 0.33, probability of mutation = 0.01, chaotic coefficient = 4.0, and no. of iterations = 20. Note that, among the swarm search methods tested in [23], the flower pollination algorithm performed best as the search strategy for the correlation-based filter. The ranking of features using the ReliefF algorithm was performed with 10 nearest neighbors for feature estimation and the influence parameter sigma = 2.

To construct the rules and rule antecedents, the fuzzification parameter D was first set by using the last seven values (trading days), s = 7, which is considered a reasonable value for the cryptocurrency markets (traded without time limits) [24]. Then, the value of c of the FCM algorithm was examined for each subsystem, as its choice is crucial to avoid overfitting [19]. To retain the interpretability of the rule base, the c parameter (determining the number of rules and rule antecedents) was examined in the range $c = \{2, 3, 5, 7\}$. To adapt the rule consequent parameters, the gradient descent algorithm was employed with $\eta = 0.01$ and 100 iterations [10]. The experiments were conducted in the Fuzzy Logic Toolbox in MATLAB R2022a.

To illustrate the prediction effectiveness of the HFIS, its performance was compared with the following models used in earlier research for bitcoin price forecasting:

- SVR (support vector regression) [25] – a univariate prediction model with five lagged bitcoin price values as feature values. Consistent with [25], radial basis kernel with a width of 4.0 and a complexity parameter $C = 4$ was used.
- ARIMA [26] – a multivariate prediction model automatically designed using the pmdarima Python library with foreign exchange rates as input features.
- MLP (multilayer perceptron) neural network with technical indicators used as predictors and a single hidden layer with 12 units [27].
- ES (exponential smoothing) [27] – a triple Holt-Winters model using the smoothing factor of 0.2.

Furthermore, the following fuzzy rule-based models were used for comparative purposes:

- HFS (hierarchical TSK fuzzy system) to examine the effect of using intuitionistic fuzzy sets when constructing the HIFS. The settting for HFS was the same as for HIFS, except that fuzzification was performed using eq. (2).
- ANFIS-GA (ANFIS adapted using the genetic algorithm) [28] with the GA parameters recommended in [28].
- INFN-PSO (intuitionistic neuro-fuzzy network tuned with the particle swarm optimization algorithm) [29] adopting the setting of the PSO algorithm from [29].
- IT2AIFLS (interval type-2 Atanassov-intuitionistic fuzzy logic system) [30] adapted using the gradient descent algorithm.

For the compared methods, different numbers of rules (and rule antecedents) $m = \{2, 3, 5, 7\}$ were tested while using the FCM algorithm to construct antecedent fuzzy sets for HFIS and ANFIS-GA, while the PSO and gradient descent algorithm were used to tune the antecedent intuitionistic fuzzy sets and interval type-2 intuitionistic fuzzy sets, respectively, in INFN-PSO and IT2AIFLS.

The performance of the models was evaluated using the following metrics: RMSE (root mean square error), MAE (mean absolute error), and MAPE (mean absolute percentage error).

B. Experimental Results

To reduce computational complexity and reveal dependencies between features, feature selection was carried out using the training data and the selected features were ranked. From the original set of 55 features, only 8 were selected and ranked as follows: (1) previous bitcoin (BTC) price, (2) 180-day volatility of bitcoin returns (Vlt180d), (3) transaction value (TrVal), (4) adjusted transaction value (AdjTrVal), (5) market capitalization (CapMrkt), (6) total issuance (IssTot), (7) bitcoin misery index (BMI), and (8) sentiment direction (from fear to greed) (SentDir). Basic descriptive statistics of the selected features are presented in Table I.

TABLE I
DESCRIPTIVE STATISTICS OF THE INPUT FEATURES USED FOR FORECASTING BITCOIN PRICES.

Feature	**Mean**	**StDev**	**Min**	**Max**
BTC price	20,611	18,362	3,183	67,555
Vlt180d	0.040	0.008	0.025	0.061
TrVal (mean) [USD]	21,365	32,833	2,716	506,174
AdjTrVal [billion USD]	5.96	6.81	0.55	36.47
CapMrkt [billion USD]	382.9	348.7	55.5	1,270.0
IssTot [million USD]	22.38	15.07	4.59	71.70
BMI	44.73	22.64	5.00	95.00
SentDir	2.591	1.362	1.000	5.000

It should be noted that interpretable intermediate features of the hierarchical structure are required to achieve an interpretable HFS [31]. Therefore, the semantic hierarchy of features should be followed. To this end, the structure of an HIFS was defined for the forecasting of bitcoin prices (Fig. 1). Market demand-based determinants are given by bitcoin market data (price and volatility) and market transactions, while market supply is the outcome of market capitalization and total bitcoin issuance. At the last level, market valuation (the result of bitcoin market demand and supply) is combined with market sentiment to predict the next day bitcoin price.

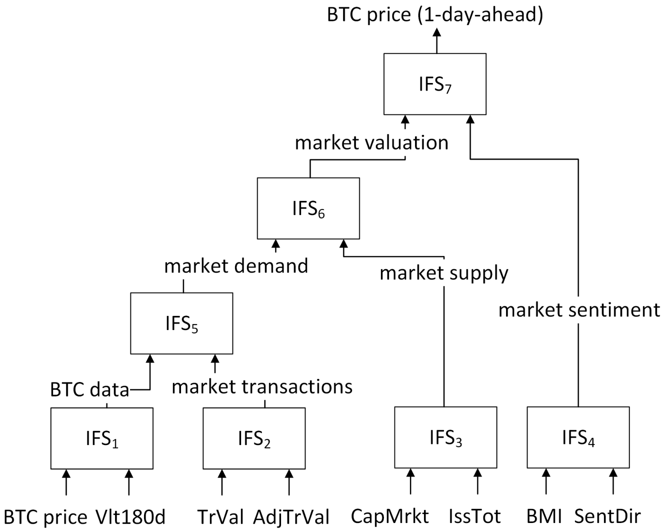

Fig. 1. A hierarchical structure of HIFS for bitcoin price forecasting.

To illustrate the fuzzification process,

Next, we examined different numbers of rules and rule antecedents based on the values of the Xie-Beni cluster validation index. It turned out that the HIFS is easily susceptible to overfitting and so for all subsystems at the first level except market sentiment (with c=3), two clusters, c=2, were sufficient. At subsequent levels, c=3 rules and rule antecedents were generated, resulting in a total of 18 rules. To illustrate the fuzzification process, leading to the construction of intuitionistic fuzzy sets, Fig. 2 shows the membership and non-membership degrees on the example of the BMI feature, indicating a rise in the sentiment index until November 2021 and a decline from the end of 2021.

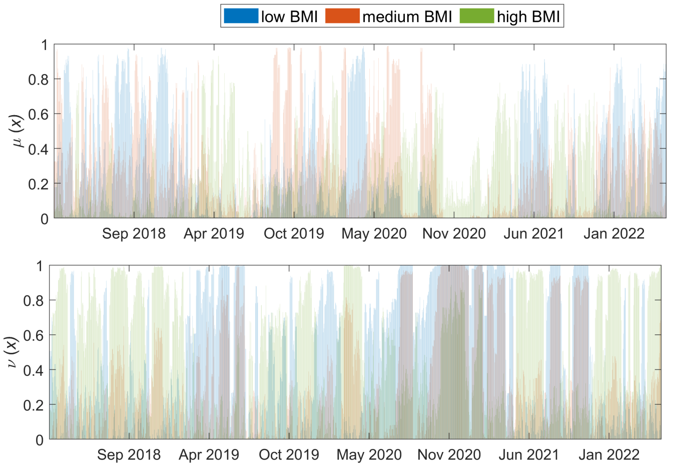

Fig. 2. Illustration of membership and non-membership degrees for bitcoin misery index (BMI).

To show the forecasts of the HIFS model with the tuned consequent parameters, Fig. 3 shows that the predicted one-day-ahead bitcoin prices are close to the actual prices for the testing data. However, it is obvious that the previous bitcoin price was crucial to the predictions, as the output of the HIFS responded to the price change with some delay.

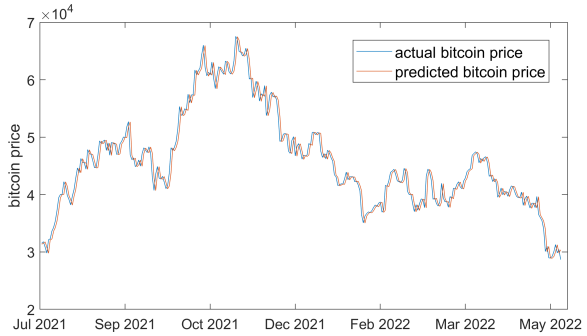

Fig. 3. Daily bitcoin prices predicted using HIFS.

To validate the performance of the proposed HIFS model, its prediction performance was compared with four methods previously used for bitcoin price forecasting and four fuzzy rule-based methods. Table II shows that among the benchmark methods previously used, SVR performed best, which is consistent with the findings of [15]. For the single fuzzy rule-based models (ANFIS-GA, IT2AIFLS, and INFN-PSO), only three rules (and rule antecedents) were created (higher values of c resulted in overfitting for all of these models), which proved inadequate for achieving an accurate prediction model. The higher accuracy achieved by the HFS and HIFS models can be attributed to both the higher number of rules (still acceptable in terms of interpretability) and a convenient hierarchical structure giving preference to key features with high predictive power while allowing greater flexibility in terms of the fuzzy set (intuitionistic fuzzy set) antecedents of individual features. Furthermore, the greater capacity of HIFS to handle uncertainty in the data led to further improvements over conventional HFS in terms of all evaluation metrics.

TABLE II
RESULTS OF BITCOIN PRICE FORECASTING.

Model	MAE	RMSE	MAPE [%]
SVR	1218.5	1631.8	2.68
MLP	1264.4	1676.8	3.03
ES	2541.5	3586.2	6.44
ARIMA	1241.1	1648.7	2.75
ANFIS-GA	1394.2	1906.4	3.13
IT2AIFLS	1429.5	1872.4	3.22
INFN-PSO	1257.8	1819.4	2.78
HFS	1180.1	1587.8	2.62
HIFS	**1171.3**	**1576.9**	**2.60**

Finally, to demonstrate the financial performance of the proposed model, the predicted bitcoin prices were used to generate trading signals ('buy' or 'hold' for the upward bitcoin price prediction, and 'sell' for the downward prediction of bitcoin price). As a result, an average return of 6.73% was obtained, which substantially outperformed the traditional benchmark buy-and-hold strategy (-10.03%), which supports the effectiveness of the proposed model and its underlying investment strategy.

IV. CONCLUSION

In this paper, we have outlined a hierarchical TSK fuzzy rule-based structure using intuitionistic fuzzy sets. The proposed architecture represents a parallel tree-like fuzzy system that exploits the feature selection and ranking component to achieve the computational effectiveness and interpretability of the intermediate features. We have obtained encouraging results demonstrating that the proposed HIFS can be used to forecast highly volatile bitcoin prices. However, it should be noted that the current study was limited in several ways. First, the correlation-based filter used in this study only considers linear relationships between features, which can be limiting for complex high-dimensional problems. The use of a recently proposed feature selection method based on independence tests [16] appears to be a viable alternative. Moreover, this study focused on point predictions of bitcoin prices, while interval forecasts are gaining attention [13]. To overcome this problem, our intention is to extend the proposed system to hierarchical inverval type-2 intuitionistic TSK fuzzy system. Our results should also be validated by a larger sample of cryptocurrencies.

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