COST SAVING
VIA GRAPH COLORING APPROACH

Erika Fecková Škrabuľáková, Elena Grešová

Abstract: *The implementation of graph algorithms on diverse tasks of economic problems has reducing costs potential in different areas of business activities. In the present paper we deal with an identification of optimal allocation of guide signs in a large exhibition hall with given spatial organization, when the move of a visitor is allowed only along given corridors. In order to achieve our aim we use modeling by a graph and graph coloring algorithm approach. Although our implementation model concerns distribution of issues in a large exhibition hall, similar algorithm could be utilized in many other navigational situations. Solving these kinds of tasks is relevant from economic point of view, because it reflects in cost saving and business prosperity. The better is the organization of the event, the greater is the satisfaction of clients and exhibitors, the higher is the probability that they will come again and that the business will prosper for each of the parties involved.*

Keywords**:** *Cost Saving, Modeling via Graphs, Optimal Distribution of Issues, Palette Coloring.*

JEL Classification: C61, C65.

Introduction

The big benefit of usage of mathematical instruments in economy is known for several decades of years. Herstein (1953) claimed that modern mathematical instruments improved economic analysis in the same manner as new mathematical methods did earlier to physics. Newly also graph theory as a branch of discrete mathematics finds widespread use in economics. Besides the modeling via graphs and the application of decision trees in risk management and decision making in economics (see e. g. Chin et al., 2009; Drabiková and Fecková Škrabuľáková, 2017a, 2017b; Lockyer and Gordon, 2005; Trucco et al., 2008; Vitali, Glattfelder and Battiston, 2011; Weber et al., 2012) also the implementation of graph algorithms in order to reduce costs in different areas of business activities is known.

A number of empirical studies proved that graph theory is significantly valid in costs saving purposes. In this regards, for example, the supply chains topic was analyzed by Wagner and Neshat (2010). The riskiness of supply chains was considered via tools of graph theory by Faisal, Banwet and Shankar (2007), as well. Application of the graph theory accompanied by costs saving can be found also in works of Drabiková and Fecková Škrabuľáková (2018), Rao (2006), Torres et al. (2016) and elsewhere.

The above examples typify just a part of potential that the study of graphs offers for real issues implementation. Optimization of methods, algorithms and processes is new trend in every area of business. Dealing with a specific business, the subject can be also solution of some specific task and achievement of emerges. In this paper we describe the usage of graph algorithms for reducing costs by solving a partial business task which is intended for needs of the company organizing various events. An exposition in a large exhibition hall is the case examined in this work. Efforts lead, on the one hand, to optimizing organizational activities and, thereby, reducing costs of the organizer and, on the other hand, to the positive perception and feedback of visitors as well as exhibitors. From the long term strategic point of view, their repeated comeback is desired effect.

1 Problem formulation

In this paper we deal with a task how to identify an optimal allocation of guide signs in a large exhibition hall with given spatial organization when the move of a visitor is allowed only along given corridors.

We are solving this task by using graph coloring algorithms. According to our knowledge such an approach was never used before in order to find the allocation of guide signs in a large hall. Our graph coloring approach could be considered as a new possible way of solving similar problems on cost reducing. The issue that is solved here should be percepted as a particular task of a more complex work dealing with different tasks one has to solve while organizing an exhibition in a large exhibition hall.

Another particular task of this project dealing with an optimal distribution of information desks has been solved by Drabiková and Fecková Škrabuľáková (2018) by entirely different method than the one presented in this paper. In the paper of Drabiková and Fecková Škrabuľáková (2018) a method similar to k-means cluster method was used. While the original k-means cluster method uses the Euclidean distance as a metric and variance as a measure of cluster scatter, in that paper the square grid metric implemented on graph model of the exhibition hall was used and entirely different algorithm that lead to cost saving was introduced.

There are several reasons why to deal with these particular tasks coming from business expectations. The more optimal is the position of information desks detected, the lower are the organizational costs. The better is the allocation of guide signs, the easier is the orientation inside the exhibition hall. The better is the organization of the event, the greater is the satisfaction of clients and exhibitors. The greater is the satisfaction, the higher is the probability that clients and exhibitors will come again. The higher is the ratio of comeback people, the greater is the prosperity of the business for each of the parties involved.

There are many principles one has to follow when organizes exhibitions in a large exhibition hall given by minimal booth's size and their organization to larger blocks, local architectural conditions, business expectations, logical and other principles.

In our case the most determining facts were:

- the minimal booth size: 1 m × 2 m (the base building block of all booths used),
- the minimal block of region associated with one type of commodity: 4 m × 4 m,
- the minimal width of the corridor between booths: 2 m,
- not too large number of sets of commodities,
- logical principles,
- business expectations,

- local architectural conditions - fixed objects in the hall, e.g. restrooms, technical rooms, emergency exits, offices.

According to that all the booths and other movable objects were placed into the exhibition hall and the spatial organization of the hall was done.

2 Methods

Mathematics and graphs are used in different areas of everyday practice (see Ždímalová, 2016; Ždímalová and Vavríková, 2017). Beside architecture, construction and designing (see e.g. Rákay, Labant and Bartoš, 2018) they have widespread use in business activities as well, for instance for evaluating and managing supply chains sensibility (Wagner and Neshat, 2010), assessing the riskiness of clients (Faisal, Banwet and Shankar, 2007) and decision making (see e.g. Chitra and Subashini, 2013; Drabiková and Fecková Škrabuľáková, 2017a; Ince and Aktan, 2009).

The utilization is notable also in banking, finance, insurance and marketing. The special chapter in this sense is modeling by graphs - business networks, graphs of cooperation (see e.g. Vitali, Glattfelder and Battiston, 2011) and solving various economy-related problems via methods of graph theory, graph algorithms.

Graph theory is dealing with a special structures of discrete mathematics called graphs. Graphs are very worth for modeling objects - elements of some sets, and relations between them. Graph algorithms are then mathematical algorithms applied on these structures.

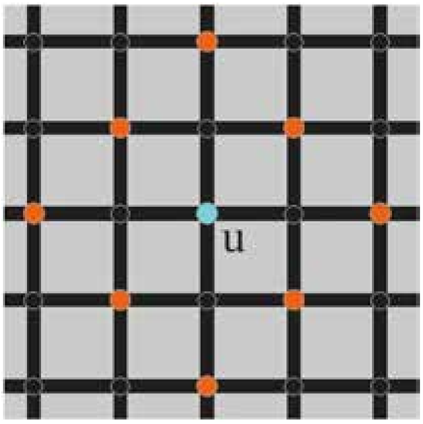
In this regard it should be mentioned that a graph H is an ordered pair $H=(V(H),E(H))$, where $V(H)$ is a finite set of elements called vertices, while $E(H)$ is a finite set of unordered pairs of vertices called edges. The coloring of the vertices of a graph H is a mapping from a set of colors to a set of vertices of H subject to certain constraints. As these constraints depend on the task that has to be solved, many different vertex-colorings of graphs have been defined and a large amount of different problems have been solved by using graph coloring algorithms.

A square grid graph is the graph $G_S(V(G_S), E(G_S))$ whose vertices correspond to the points in the plane with integer coordinates, and two vertices are connected by an edge whenever the corresponding points are at distance 1. A graph formed from a collection of points in the Euclidean plane by connecting two points by an edge whenever the distance between the two points is exactly one is called a unit distance graph. In this sense G_S is a unit distance graph.

In order to define a special type of neighborhood of vertices of G we used square metrics, which is known as postman metrics (see Fecková Škrabuľáková, 2016) too. The distance for a postman in a city where the roads are organized like a square grid is not the smallest distance between two points on the map of the city, but the length of the way he needs to walk using the streets.

In this sense 0-neighborhood of a vertex u of square grid G_S equals u itself, *1*-neighborhood is represented by a set of neighboring vertices of u, further *2*-neighborhood by a set of vertices in the distance two from u - see the highlighted vertices at Fig. 1, and so on. Clearly, if G is a subgraph of the square grid G_S, i-neighborhood of some vertex of G, is defined as a set of those vertices of G which distance from the very vertex equals i.

Fig. 1: 2-neighborhood of a vertex u



Source: (Authors)

All the graph theory terms used, but not defined here, can be found in Bondy and Murty (1976).

For achieving the aims and solving our practical problem, an adequate graph model (in the graph theory sense) of the exhibition hall was prepared.

This was done in several steps:

- drawing a computer-made scheme of the exhibition hall with all the booths in an appropriate scale,
- coloring the zones according to similar commodities presented,
- drawing a graph G_S,
- defining the appropriate coloring of the vertices of G_S coloring of these vertices
- defining a graph G of possible clients' moves,
- palette-coloring the vertices of the graph G.

The computer-made plan of the exhibition hall, see Fig. 2, was made in a scale with a help of a square grid G_S. The areas rented by exhibitors were colored according to the presented set of commodities, diverse set of commodities obtained different colors. No set of commodities was colored full-white, or full-black. The same colors were assigned to the vertices of G_S covered by rented areas. All the remaining vertices of G_S obtained full-white color.

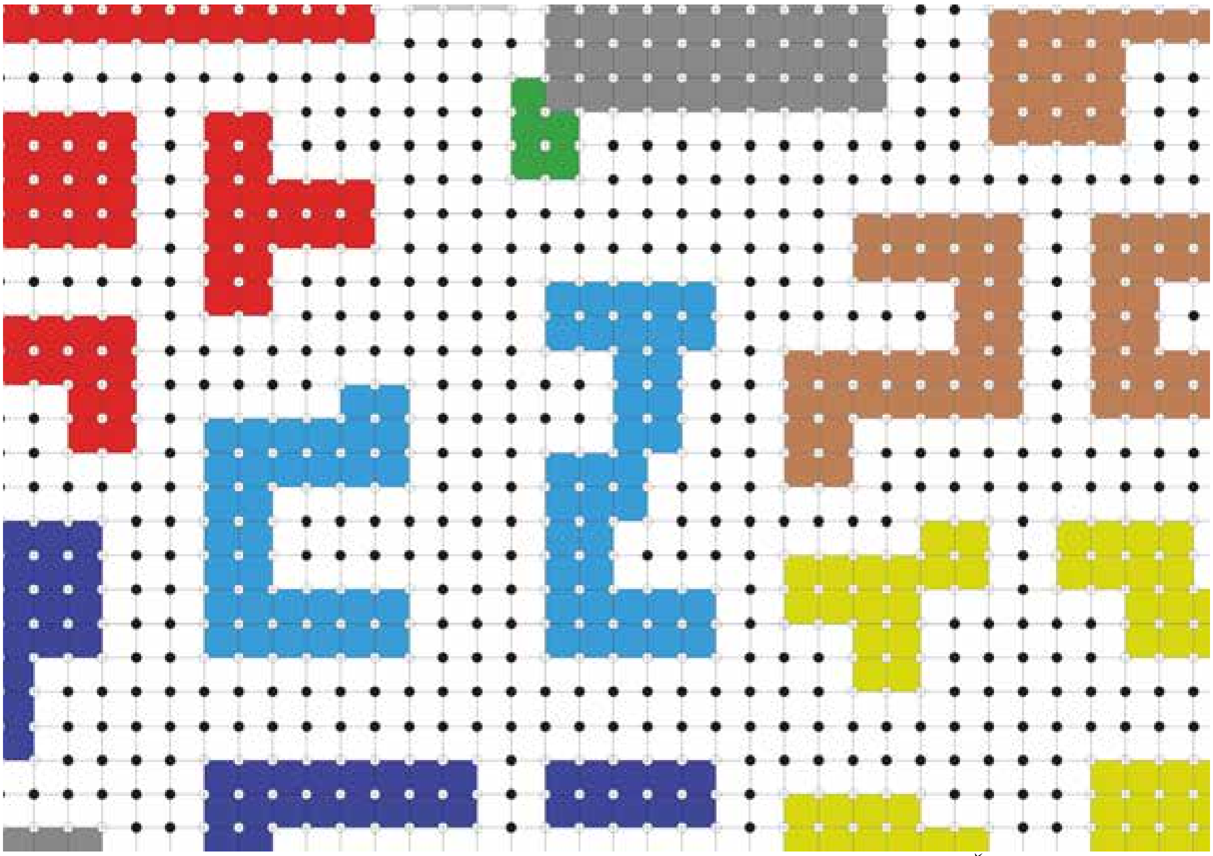
A graph G of slightly simplified possible movements of clients was defined as an induced subgraph of G_S on the set of vertices $V(G) \subset V(G_S)$ colored full-black at Fig. 2. Clearly, $G(V(G),E(G))$ is an unit distance and planar graph.

A palette coloring $\pi(G)$ of the graph G in which one or more color is assigned to each vertex was defined by the procedure that is summarized in the particular points below.

Let v be a vertex of a graph G. A color palette $P(v)$ (see Fig. 3) is assigned to the vertex v according to the following conditions:

- full-black color b is in $P(v)$,
- color c is in $P(v)$ whenever the Euclidean distance of the vertex v from the rented area colored c is lesser or equal to 1.

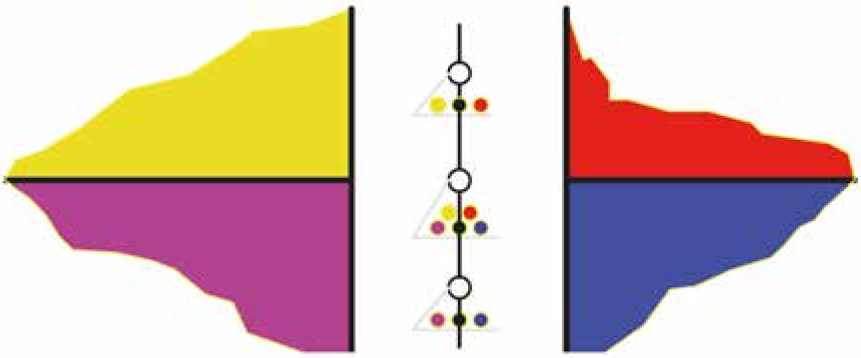
Fig. 2: Graphs G_S and G



Source: (Authors, based on Drabiková and Fecková Škrabuľáková, 2018)

Now denote $S_{P(v)}$ the size of a palette *P(v)*, i.e. the number of colors in *P(v)*. Clearly, $S_{P(v)} \leq 5$ for each $v \in V(G)$, see Fig. 3, as *G* is an unit distance graph and for every $v \in V(G)$; $b \in P(v)$. Finally, denote $N_iS_{P(v)}$ the number of different colors in palettes of the *i*-neighborhood of *v*.

Fig. 3: The situation in which there is a vertex v with $S_{P(v)} = 5$



Source: (Authors)

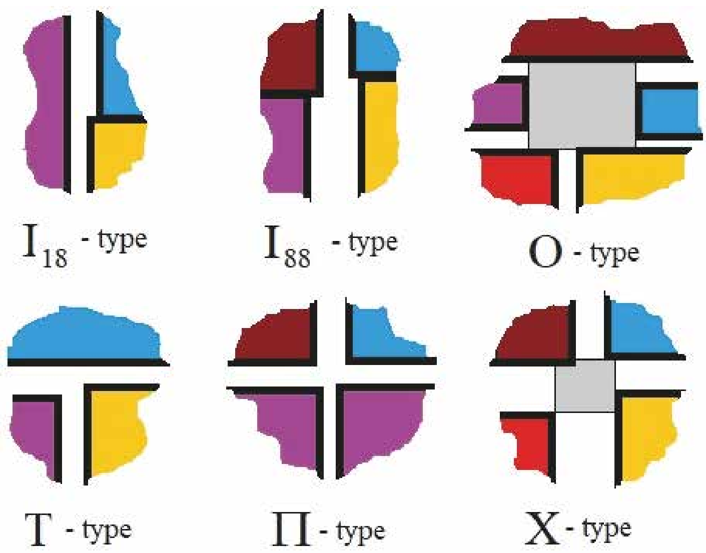
The preparation of an adequate and useful graph model was the crucial task of the research. Defining the appropriate coloring of the vertices of G_S and a graph *G* of possible clients' moves were the major objectives. Although the palette-coloring of graphs is known term in the discrete mathematics, its conjunction with the solution of economic task is the novelty of authors. According to our knowledge, it is one of the first, if not the first at all, application of palette-coloring of graphs in order to find a solution of an optimal allocation problem.

3 Solving the problem

In general, expositions in exhibition halls are organized functionally, that means that similar commodities are grouped together in some area of the hall although these areas might not be unique. The reasons are pure logical: duplicitous customers field of interest, price comparison of individual products and services, targeting on a specific group of clients, achievement of the competition, link-up new partners and many more. The better is the organization of the event, the greater is the satisfaction of clients and exhibitors, the higher is the probability that they will come again and that the business will prosper for each of the parties involved. The satisfaction depends on many factors, not only on big issues, but from details, as well. Therefore, good organizer of an event has to take care about the identification of optimal allocation of guide signs too. With the help of the graph *G* described in the previous section this task is easy solvable.

As the move of the visitor is allowed only along given paths, the guide signs should be placed at some points of these corridors. Their position should help the visitors of an event find one's way in the exhibition hall. The number of the guide signs should not be too large, because it lifts the expenses of the organizers, acts disturbingly and causes bewilderment. The guide signs have to be easy readable, hence, their shape, position and distance between two consecutive guide signs should respect the visual function of healthy human eye. The strategy of finding an optimal allocation of guide signs in the hall should respect all these natural conditions of the organizers of an event in the large exhibition hall.

Fig. 4: Meet of three or more color zones



Source: (Authors)

Remark, that a place, where 3 or more color zones of Fig. 2 meet, could be one of the following types: I_{18} I_{88}, T, π, X, or O, where O groups together all the remaining situations. Out of these types only T, π, X and O are called crossroads and grey area in the middle of X-type and O-type crossroad, see Fig. 4, is called crossroad-square here.

The orientation at T-type and π-type crossroads is intuitive and easy, therefore, putting a guide sign here would not be economical. Hence, only X-type and O-type crossroads should be associated with a sign. Naturally, guide signs should be placed at crossroad-squares of the graph G. Cost saving and practical reasons do not allow to associate all such vertices with a sign. The optimal allocation of the guide signs can be found in two steps:

1. Let v be a vertex of crossroad-square and let $i > 0$ be as small as possible such that $5 \leq N_i S_{P(v)}$. Then we place a guide sign at a position of v except that it would be necessary to apply some moves defined by condition 2.
2. Check, whether the distance (in postman metrics) between two consecutive guide signs of the same crossroad-square is greater or equal to 5. If not, replace these two signs by one (containing the join information from both) placed in the middle-point of the path between the points of two signs and repeat this procedure until the distance condition is fulfilled.

4 Reasoning

The only case when $N_i S_{P(v)} = S_{P(v)} = 5$ is for $i = 0$ and it is the one when v has distance 1 from four differently colored commodity sets. But in this case v is not lying at a crossroad-square - see Fig. 3, its degree is 2 and there is no reason to put a guide sign at a place of v. Hence, $i > 0$.

From the palette-coloring algorithm it is clear, that at crossroad-square of *X*-type or *O*-type crossroad there exist a vertex v for which $N_i S_{P(v)} \geq 5$ for some $i > 0$.

The second condition is natural, based on cost saving too. The condition for the minimal distance between two consecutive guide signs was not chosen randomly, but it rose from a visual function of healthy human eye.

For visual acuity testing are frequently used Snellen charts. Snellen's charts published in 1862 (Snellen, 1862) used alphabetic capitals and numbers in the 5×5 grid. Subsequent rows had increasing number of alphanumeric symbols decreasing in size. The patient red the symbols from the distance 6 m with one eye covered. This method is widely practiced also nowadays, most often with distance 5 - 6 m between the patient and the board, although the research of Ferris et al. (1982) showed that maximum visual acuity and minimum dispersion of visual acuity scores can be obtained at a test distance close to 4 m. Therefore, we consider minimal distance 5 m between two consecutive guide signs being optimal.

Conclusions

In the present paper we have shown an implementation of graph models and algorithms by solving a model cost saving task. We have described a fraction of more complex problem for a company organizing exhibition events. Namely, we have dealt with an identification of optimal allocation of guide signs via graph models and graph coloring.

We have found an adequate graph model of the given exhibition hall. We have defined its coloring thanks to which we were able to define the graph of possible clients' moves. The palette-coloring of this graph helped us to find an optimal allocation of guide signs in the large exhibition hall keeping in the mind all the conditions given by the organizers of an event in the hall. These natural conditions were defined with respect to visual function of healthy human eye, easy orientation in the hall, cost saving and satisfaction of both clients and organizers.

Although graphs were already used in many areas of economics in order to solve different problems, the usage of palette-coloring in this connection is a novel technique. According to our knowledge, it was never used before in this connection.

Solving the task presented is interesting both from economic and business-prosper point of view. The better organization of an event reflects in both clients and exhibitors satisfaction. Their comeback is welcomed for all parties involved. What is more, our proposed solution of the task has also a secondary effect reflected in cost-saving, as it is always a big advantage to know about the optimal quantity of information signs or optimal allocations of other items required.

Although our implementation model concerns distribution of issues in a large exhibition hall, similar algorithm could be utilized in many other navigational situations starting from orientation at a large campus, airport, bus or train station, shopping mall, city market, up to sport activities.

Acknowledgment

This work was supported by the Slovak Research and Development Agency under the contract No. APVV-14-0892 and by the grant VEGA 1/0908/15.

The authors would like to thank the anonymous referees for their valuable comments that improved the quality of this paper.

References

Bondy, J. A., Murty, U. S. R. (1976). *Graph theory with applications*. London: Macmillan.

Chin, K. S. et al. (2009). Assessing New Product Development Project Risk by Bayesian Network with a Systematic Probability Generation Methodology, *Expert Systems with Applications*, 36(6), pp. 9879-9890.

Chitra, K., Subashini, B. (2013). Data mining techniques and its applications in banking sector. *International Journal of Emerging Technology and Advanced Engineering*, 3(8), pp. 219-226.

Drabiková, E., Fecková Škrabuľáková, E. (2017a). Decision trees - a powerful tool in mathematical and economic modeling. In: *Proceedings of the 18th International Carpathian Control Conference (ICCC 2017)*. Danvers: IEEE, pp. 34-39.

Drabiková, E., Fecková Škrabuľáková, E. (2017b): Monitoring and Controlling of Economic Tasks through Tools of Graph Theory, *Economic Spectrum, 7(2),* pp. 1.-8.

Drabiková, E., Fecková Škrabuľáková, E. (2018). Reducing costs by graph algorithms. In: *Proceedings of the 19th International Carpathian Control Conference (ICCC 2018)*. Danvers: IEEE, pp. 113-117.

Faisal, M. N., Banwet, D. K., Shankar, R. (2007). Quantification of risk mitigation environment of supply chains using graph theory and matrix methods. *European Journal of Industrial Engineering*, 1(1), pp. 22-39.

Fecková Škrabuľáková, E. (2016). A game on exact mathematical verbalization. In: *Engineering Education in the 21st Century*. Košice: Technical University of Košice, pp. 19-22.

Ferris, F. L. et al. (1982). New visual acuity charts for clinical research. *American Journal of Ophthalmology*, 94(1), pp. 91-96.

Herstein, I. N. (1953). Some mathematical methods and techniques in economics, *Quarterly of Applied Mathematics,* 11(3), pp. 249-262.

Ince, H., Aktan, B. (2009). A comparison of data mining techniques for credit scoring in banking: A managerial perspective. *Journal of Business Economics and Management*, 10(3), pp. 233-240.

Lockyer, K. G., Gordon, J. (2005). *Project Management and Project Network Techniques*, Pearson Education.

Rákay, Š., Labant, S., Bartoš K. (2018). Verification of floor planarity by trigonometrical measurement of heights on a 5-storey monolithic building. *Geodesy and Cartography*, 44(1), pp. 14-21.

Rao, R. V. (2006). A material selection model using graph theory and matrix approach. *Materials Science and Engineering: A*, 431(1), pp. 248-255.

Snellen, H. (1862). *Optotypi ad visum determinandum (Letterproeven tot bepaling der Gezichtsscherpte; Probebuchstaben zur bestimmung der Sehschaerfe).* Utrecht: Weyers.

Torres, J. M. et al. (2016). Exploring topological effects on water distribution system performance using graph theory and statistical models. *Journal of Water Resources Planning and Management*, 143(1), 04016068.

Trucco, P. et al. (2008). A Bayesian Belief Network Modelling of Organisational Factors in Risk Analysis: A Case Study in Maritime Transportation, *Reliability Engineering & System Safety*, 93(6), pp. 845-856.

Vitali, S., Glattfelder, J. B., Battiston, S. (2011). The network of global corporate control. *PloS one*, *6*(10), e25995.

Wagner, S. M., Neshat, N. (2010). Assessing the vulnerability of supply chains using graph theory. *International Journal of Production Economics*, 126(1), pp. 121-129.

Weber, P. et al. (2012). Overview on Bayesian Networks Applications for Dependability, Risk Analysis and Maintenance Areas, *Engineering Applications of Artificial Intelligence*, 25(4), pp. 671-682.

Ždímalová, M. (2016). Mathematics in Architecture, Design and Structure Engineering. In: *Proceedings of Aplimat (Aplimat 2016)*. Bratislava: STU Bratislava, pp. 1154-1163.

Ždímalová, M., Vavríková, L. (2017). Mathematics, Garden and Landscape Architecture. In: *Proceedings of Aplimat (Aplimat 2017).* Bratislava: STU Bratislava, pp. 1746-1771.

Contact Address

RNDr. Erika Fecková Škrabuľáková, PhD.
Technical University of Košice, Faculty of Mining, Ecology, Process Control and Geotechnology, Institute of Control and Informatization of Production Processes
B. Němcovej 3, 042 00 Košice, Slovakia
Email: erika.feckova.skrabulakova@tuke.sk
Phone number: +421556025162

Ing. Elena Grešová, PhD.
Technical University of Košice, Faculty of Mining, Ecology, Process Control and Geotechnology, Institute of Control and Informatization of Production Processes
B. Němcovej 3, 042 00 Košice, Slovakia
Email: elena.gresova@tuke.sk
Phone number: +421556025162

Received: 24. 04. 2018, reviewed: 13. 09. 2018
Approved for publication: 08. 04. 2019