RCDue - experimental identification of continuous- and discrete-time models

František Dušek
Department of Process Control
Faculty of Electrical Engineering and Informatics, University of Pardubice
Pardubice, Czech Republic
orcid.org/0000-0002-6549-3112

Daniel Honc
Department of Process Control
Faculty of Electrical Engineering and Informatics, University of Pardubice
Pardubice, Czech Republic
orcid.org/0000-0001-8440-5272

Aleš Novotný
Department of Process Control
Faculty of Electrical Engineering and Informatics, University of Pardubice
Pardubice, Czech Republic
ales.novotny1@student.upce.cz

***Abstract*— The paper is devoted to the education and teaching of process control and automation. Various laboratory equipment is used to explain and better understand the theory and to gain practical experience. The authors have designed and developed a simple electrical dynamical system RCDue (dynamic model with passive RC components and Arduino Due as measurement and communication unit) that allows students to perform various laboratory experiments – e.g. static and dynamic characteristics measurements, modeling, experimental identification, control design and application of from the simplest strategies to advanced methods. Specifically, in this paper, the authors focus on experimental identification.**

Keywords—RCDue, process control, automation, education, experimental identification

I. Introduction

Laboratories and experiments are an integral part of the of education and teaching process in a large number of disciplines. Students, especially in technical fields, have to study a large number of theoretical subjects before they can finally try to apply the knowledge. It often happens that they do not see the goal of the endeavour and lose the concentration and motivation to study. Appropriately chosen experiments should help them not only to grasp theoretical knowledge but also to gain insight and practical experience. There are many companies that sell laboratory equipment for teaching process control and automation [1-3]. Most of the equipment is quite complex and therefore costly. Teachers around the world are trying to make practical demonstrations on low-cost hardware so that the students can work individually or even at home. Single-chip microcomputers are one way to reduce the cost of the instrumentation. An example of such a solution can be found e.g. in [4-7].

Presented paper is the second one of article series dealing with RCDue laboratory process for control education. The initial article [8] contained the motivation, basic concept, hardware, mathematical model, software and support for MATLAB environment, including the specific case of amplitude-phase frequency characteristic measurement.

This paper deals with Experimental Identification (EI), i.e., the unknown parameters determination or estimation of the mathematical description of the dynamic behaviour by using measured input and output signals. The determination of the parameter corrections of the continuous-time state-space first-principle model obtained by Mathematical Physical Analysis (MFA) will be demonstrated. Also, the direct determination of the parameters of the discrete-time input-output model (differential equation) of the appropriate order and the determination of the third-order discrete-time state-space approximation model will be shown. The procedures will be demonstrated on the example of R5C4 electrical system (containing 5 resistors and 4 capacitors). Paper is structured as follows. Chapter I. is an introduction. First-principle model with nominal parameters and model with corrected parameters, discrete-time input-output and state-space models are dealt in chapter II. Results are discussed in chapter III.

II. Experimental Identification

The term experimental identification means obtaining a description of the behaviour of the investigated system in the form of a mathematical model based on experimental data. Determining the structure of the model from the experimental data only is not a trivial problem. Therefore, additional information about the identified system obtained in other ways is usually used to determine the structure of the model. The concept of EI is usually reduced to the determination or estimation of the parameters of a model of known structure. Such parameters are sought that lead to the best approximation of the measured output response of the real system to the known input signals, i.e., usually formulated as the optimization problem.

In the case of describing the dynamic behaviour of linear systems, the structure of the model is given by a linear differential/difference equation. The structure of the model is then influenced only by the choice of order and possibly the inclusion of dependence on the derivatives of the input signal in the case of a continuous-time model or the given number of considered samples of the output and input signal in the case of a discrete-time model.

In the case of identification of dynamic systems, the choice of an appropriate input signal schape is also important. Since the description of the system properties is determined from experimentally acquired data, they must contain information about the system behaviour. An ideal input signal should act on the identified system at all frequencies with the same energy, i.e. it should contain all frequencies with the same power. White noise has these properties but is physically unfeasible. For practical applications, we are usually interested in describing the behaviour of the system in certain frequency region. Frequently input signal in the form of one or more step changes with sufficiently long constant sections is used. The advantage of such a signal is that the structure of the model can be estimated from the response of the system. The amplitude of the input signal changes should be large enough that the system response is not lost in measurement noise. At the same time, constant (preferably zero) values of

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any disturbance variables acting on the identified system should be ensured.

It is advisable to ensure that the first change of the input signal is starting from the steady state which can be considered as initial condition. This is suitable for verifying the reproducibility of experiments.

A. First-principle model of R5C4 system

The aim of the modelling and experimental identification is to obtain the best possible mathematical description of the dynamic behaviour of the R5C4 system consisting of five resistors and four capacitors according to Fig. 1.

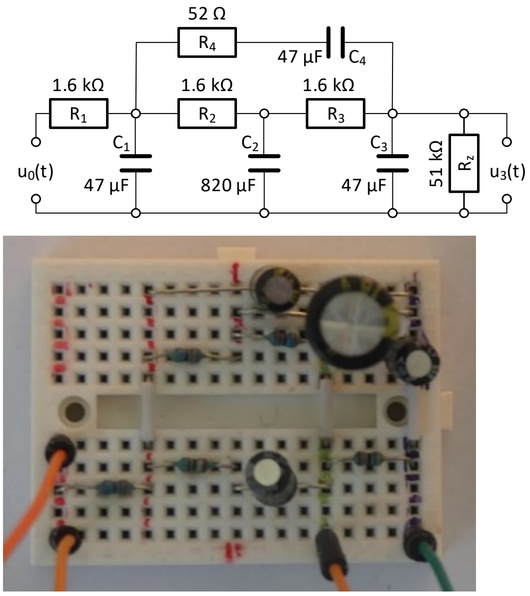

Fig. 1. Wiring diagram and implementation of the R5C4 system

The first-principle model of the R5C4 system, i.e. the dependence of the currents i and voltages of capacitors C_1 to C_4 on the input voltage u_0, was obtained by MFA. These dependencies expressed in a matrix form are described by equation (1a)

$$\mathbf{Ii} = \mathbf{Uu} + \mathbf{m}u_0, \quad \mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{1a}$$

where

$$\mathbf{I} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & R_2 & R_2 + R_3 & -R_2 - R_3 - R_4 \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} -\frac{1}{R_1} & 0 & -\frac{1}{R_z} & 0 \\ \frac{1}{R_2} & -\frac{1}{R_2} & -\frac{1}{R_z} & 0 \\ 0 & \frac{1}{R_3} & -\frac{1}{R_3} - \frac{1}{R_z} & 0 \\ 0 & 0 & -\frac{R_2 + R_3}{R_z} & 1 \end{bmatrix}, \mathbf{m} = \begin{bmatrix} \frac{1}{R_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Equation (1b) expresses the relationship of currents and voltages on capacitors

$$\mathbf{K}\frac{d\mathbf{u}}{dt} = \mathbf{i} \tag{1b}$$

where

$$\mathbf{K} = \begin{bmatrix} C_1 & 0 & 0 & 0 \\ 0 & C_2 & 0 & 0 \\ 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & C_4 \end{bmatrix}.$$

Equations (1a) and (1b) can be easily converted to the standard form of the state-space model (2), where the voltage u_3 on the capacitor C_3 is also the output voltage of the R5C4 system

$$\begin{aligned} \frac{d\mathbf{u}}{dt} &= \mathbf{Au} + \mathbf{b}u_0 \\ u_3 &= \mathbf{cu} \end{aligned} \tag{2}$$

where

$$\mathbf{A} = \mathbf{K}^{-1}\mathbf{I}^{-1}\mathbf{U}, \quad \mathbf{b} = \mathbf{K}^{-1}\mathbf{I}^{-1}\mathbf{m}, \quad \mathbf{c} = [0 \quad 0 \quad 1 \quad 0].$$

These equations describe the dynamical behaviour of the system under the assumption of ideal behaviour of the electrical components and conformity of the actual parameters with the nominal ones – given in Table 1.

In Fig. 2, the input u_0 and output u_3 voltage signals are measured with a sampling period of 50 ms. The output voltage signal calculated from the model for the same input voltage signal and nominal values of resistors and capacitors is plotted. The model error signal including standard deviation is also given.

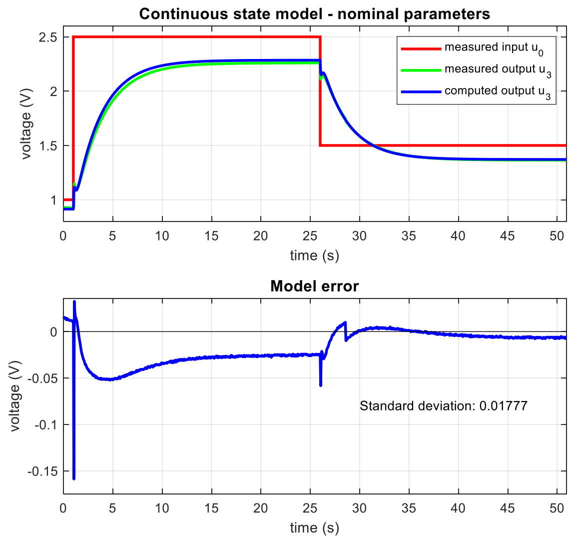

Fig. 2. Measured and calculated voltage signals - continuous-time state-space model with nominal parameters of the components

B. Continous-time state-space model of R5C4 system - with corrected parameters

The identification of time-invariant parameters of a continuous-time linear dynamic model is based on a straightforward optimization. It minimizes the difference between the measured and calculated output signal at the same sampling time instants. Due to the application of the numerical optimization method, the objective function can be expressed not only as the commonly used sum of the squares of the differences between the measured and calculated values at the given time instants but also, e.g., as the sum of the absolute errors.

EI is used to correct the parameters of the electrical components of model (1) obtained by MFA. This model is converted into the form of a continuous-time state-space model (2). This is a very favorable situation, because we know both the structure of the model (2) and the dependence of its parameters on the parameters of the electrical components used. It is convenient to search for the values of the components instead of the model parameters. The fact that the searched parameters have physical meaning allows a basic verification of the correctness of the result, i.e. whether the found solution does not contradict the reality. Unrealistic values (e.g. negative values of resistances) may lead to a lower value of the objective function, but they indicate that either the model structure does not match the real device or the necessary information is not present in the measured data.

The minimization of the objective function of the form (3) was used to determine the corrections of the electrical component parameters

$$\mathbf{p}_{opt} = \arg\min_{\mathbf{p}} J(\mathbf{p})$$
$$J(\mathbf{p}) = \sum_{k=1}^{N} [y_m(t_k) - y(t_k, \mathbf{p})]^2 \quad (3)$$

where

$t_k = t_0 + kT_s$ is sampling time with constant sampling period T_s,

$\mathbf{p} \in R^{9\times1}$ is vector of electrical component parameters,

$y_m(t_k)$ are values of output voltage u_3 at time t_k – measured and

$y(t_k, \mathbf{p})$ are values of the output voltage u_3 at time t_k - calculated from model (3) for the corrected parameters **p** of the nominal electrical components and the measured value of the input signal $u_m(t_k)$. We assume that the input voltage is constant between sampling intervals.

This fundamentally simple approach has two practical problems:

(a) the problem of determining the global extremum - the dependence of the objective function on the parameter values is nonlinear, i.e. the problem of determining the global extremum of a general nonlinear function of several variables is not generally solvable

b) the problem of computational complexity, including the choice of robust algorithms for the calculation of subproblems - each step of numerical optimization requires numerical solution of the differential equation.

Classical numerical methods of finding the extremum of a function of several variables guarantee finding the extremum only if the initial estimate of the variables is close enough to the sought extremum. Since in this case it is a matter of finding parameter values with physical meaning, then the nominal parameters of the components provide a good estimate of the initial solution.

The choice of numerically robust and computationally efficient algorithms is ensured by using MATLAB, which provides robust and computationally efficient numerical algorithms. The *lsim* function is used to calculate the response to the input signal (solution of the linear differential equation). Since the searched parameters have significantly different magnitudes (about 6 orders of magnitude), the parameter are not searched directly but we search the corrections to the nominal parameters. This improves the numerical robustness of the optimization algorithm because all the searched corrections are in the order of unity. The initial values of the searched corrections are set to 1 with a constraining to a range from 0.5 to 1.5. The constraints guarantee that parameter values correspond to the reality. To find the minimum of the objective function, the function *fminbnd* is used, allowing the inclusion of band constraints on the values of the search parameters.

The results of the identification, i.e., the determination of the corrections to the nominal electrical component parameters for signals sampled with two sampling periods, are summarized in Table 1. Fig. 3 compares the model output with the corrected component parameters with the measured data (T_s = 0.05 s). Corrections are dimensionless with an expected value of 1 (initial value for optimization).

TABLE I. CORRECTIONS OF CONTINUOUS-TIME MODEL PARAMETERS

Component	Nominal value	Correction	
		T_s=0.05 s	T_s=0.10 s
R_1	1.6 kΩ	1.0207	0.9168
R_2	1.6 kΩ	1.4459	1.4084
R_3	1.6 kΩ	0.7670	0.7278
R_4	52 Ω	0.7473	1.0064
R_z	51 kΩ	0.9766	0.9238
C_1	47 µF	1.1289	1.3192
C_2	820 µF	0.8665	0.9170
C_3	47 µF	1.0807	1.2542
C_4	47 µF	1.2226	1.2292

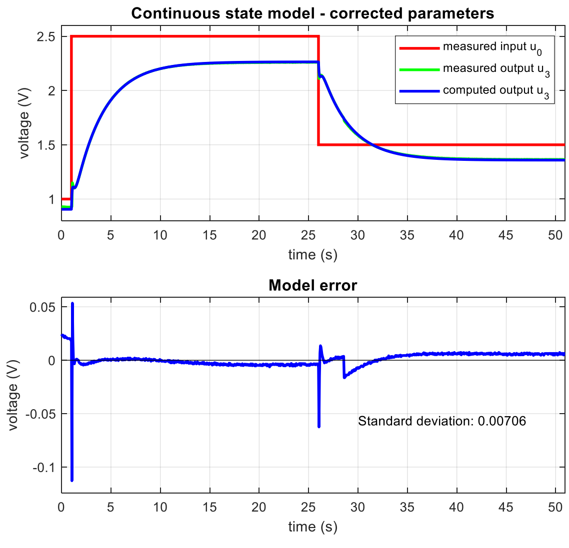

Fig. 3. Measured and calculated voltage signals - continuous-time state-space model with corrected parameters of the components

C. *Discrete-time input-output model of R5C4 system*

Experimental identification is purely mathematical procedure in the case of a discrete-time model in a form of the linear difference equations. Parameters of the model of known structure that provide the best fit (in the sense of the objective function) to the measured data are estimated. This procedure does not provide any information about mismatched structure or missing information in the data.

Knowing a sufficient number of samples of the input and output signals, the parameters of the discrete-time linear input-output model can be determined. We estimate the coefficients of the difference equation, i.e. na coefficients a_1 to a_{na}. and nb coefficients b_1 to b_{nb}

$$y(t_k) + a_1 y(t_{k-1}) + \cdots + a_{n_a} y(t_{k-n_a}) = b_1 u(t_{k-1}) + \cdots + b_{n_b} u(t_{k-n_b}) \quad (4)$$

where

$y(t_k)$ are output signal samples at times t_k and
$u(t_k)$ are input signal samples at times t_k.

Choosing an objective function of the form (3) is a linear regression problem. The solution can be obtained by a standard procedure using partial derivatives of the objective function according to the estimated parameters. Another possibility is to use matrix operations, which leads to the same result.

Equation (4) needs to be rewritten into a matrix expression involving N measured samples of the input and output signals

$$y(t_k) = \mathbf{d}_k \boldsymbol{\theta} \quad (5a)$$

where

$\mathbf{d}_k = [u(t_{k-1}) \quad \cdots \quad u(t_{k-n_b}) \quad -y(t_{k-1}) \quad \cdots \quad -y(t_{k-n_a})] \in R^{1\times(n_a+n_b)}$,

$\boldsymbol{\theta} = [b_1 \quad \cdots \quad b_{n_b} \quad a_1 \quad \cdots \quad a_{n_a}]^T \in R^{(n_a+n_b)\times 1}$.

$$\mathbf{Y}_N = \mathbf{D}_N \boldsymbol{\theta} \quad (5b)$$

where

$\mathbf{Y}_N = [y(t_{n_{ab}}) \quad \cdots \quad y(t_N)]^T \in R^{(N-n_{ab})\times 1}$,

$$\mathbf{D}_N = \begin{bmatrix} \mathbf{d}_{n_{ab}+1} \\ \vdots \\ \mathbf{d}_N \end{bmatrix} = \begin{bmatrix} u(t_{n_b}) & \cdots & u(t_1) & -y(t_{n_b}) & \cdots & -y(t_1) \\ & & & \vdots & & \\ u(t_{N-1}) & \cdots & u(t_{N-n_b}) & -y(t_{N-1}) & \cdots & -y(t_{N-n_a}) \end{bmatrix} \in R^{(N-n_{ab})\times(n_a+n_b)}.$$

The estimation of the unknown parameters is expresses as pseudoinversion (the rectangular matrix $\mathbf{D}_N$ cannot be inverted)

$$\boldsymbol{\theta} = (\mathbf{D}_N^T \mathbf{D}_N)^{-1} \mathbf{D}_N^T \mathbf{Y}_N \quad (6)$$

A continuous-time state-space model with four states (4th degree differential equation) corresponds to a 4th degree difference equation in the form (4) with eight unknown coefficients a_1 to a_4 and b_1 to b_4. The identification results, i.e., the values of the coefficients of the differential equation (4) calculated from equation (6), are summarized in Table 2 for the sampling period $T_s = 0.05$ s and $T_s = 0.1$ s.

TABLE II. PARAMETERS OF THE DISCRETE-TIME MODEL

$T_s = 0.05$ s	a_1	a_2	a_3	a_4
	-1.294	-0.1832	0.6475	-0.1644
	b_1	b_2	b_3	b_4
	0.1146	-0.1258	-0.0513	0.0677

$T_s = 0.1$ s	a_1	a_2	a_3	a_4
	-0.987	-0.4039	0.4601	-0.0486
	b_1	b_2	b_3	b_4
	0.1386	-0.1454	-0.0528	0.0783

Fig. 4 compares the model output in the form of a difference equation with the measured data at sampling period $T_s = 0.05$ s.

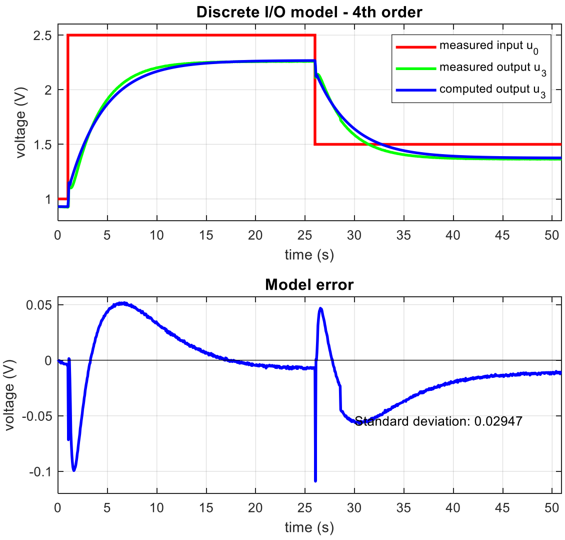

Fig. 4. Measured and calculated voltage signals - approximation by a model in the form of a 4th order difference equation

D. Discrete-time state-space model of R5C4 system

If the vector of the state variables $\mathbf{x}$ can also be measured, in addition to the vector of input $\mathbf{u}$ and output $\mathbf{y}$ signals, then the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ of the Multi-Input Multi-Output (MIMO) state-space model (7) can be determined by the same procedure by using matrix operations

$$\begin{aligned} \mathbf{x}(t_{k+1}) &= \mathbf{A}\mathbf{x}(t_k) + \mathbf{B}\mathbf{u}(t_k) \\ \mathbf{y}(t_k) &= \mathbf{C}\mathbf{x}(t_k) + \mathbf{D}\mathbf{u}(t_k) \end{aligned} \quad (7)$$

where

$$\mathbf{x}(t_k) = \begin{bmatrix} x_1(t_k) \\ \vdots \\ x_{n_x}(t_k) \end{bmatrix}, \mathbf{u}(t_k) = \begin{bmatrix} u_1(t_k) \\ \vdots \\ u_{n_u}(t_k) \end{bmatrix}, \mathbf{y}(k) = \begin{bmatrix} y_1(t_k) \\ \vdots \\ y_{n_y}(t_k) \end{bmatrix},$$

$\mathbf{A} \in R^{n_x\times n_x}$, $\mathbf{B} \in R^{n_x\times n_u}$, $\mathbf{C} \in R^{n_y\times n_x}$, $\mathbf{D} \in R^{n_y\times n_u}$.

We can write equation (7) in matrix form for N measured samples of input, output and state signals

$$\begin{aligned} \mathbf{X}_{N1} &= \boldsymbol{\theta}_{AB} \mathbf{X}_{Nu} \\ \mathbf{Y}_N &= \boldsymbol{\theta}_{CD} \mathbf{X}_{Nu} \end{aligned} \quad (8)$$

where

$\boldsymbol{\theta}_{AB} = [\mathbf{A} \quad \mathbf{B}] \in R^{n_x\times(n_x+n_u)}$,

$\boldsymbol{\theta}_{CD} = [\mathbf{C} \quad \mathbf{D}] \in R^{n_y\times(n_x+n_u)}$,

$\mathbf{X}_{N1} = [\mathbf{x}(t_N) \quad \cdots \quad \mathbf{x}(t_2)] \in R^{n_x\times(N-1)}$,

$\mathbf{Y}_N = [\mathbf{y}(t_{N-1}) \quad \cdots \quad \mathbf{y}(t_1)] \in R^{n_y\times(N-1)}$,

$$\mathbf{X}_{Nu} = \begin{bmatrix} \mathbf{x}(t_{N-1}) & \cdots & \mathbf{x}(t_1) \\ \mathbf{u}(t_{N-1}) & \cdots & \mathbf{u}(t_1) \end{bmatrix} \in R^{(n_x+n_u)\times(N-1)}.$$

The solution of equation (8) i.e. the matrix $\boldsymbol{\theta}_{AB}$ containing the searched matrices **A**, **B** and the matrix $\boldsymbol{\theta}_{CD}$ containing the searched matrices **C**, **D** is

$$\begin{aligned} \boldsymbol{\theta}_{AB} &= \mathbf{X}_{N1}\mathbf{X}_{Nu}^T(\mathbf{X}_{Nu}\mathbf{X}_{Nu}^T)^{-1} \\ \boldsymbol{\theta}_{CD} &= \mathbf{Y}_N\mathbf{X}_{Nu}^T(\mathbf{X}_{Nu}\mathbf{X}_{Nu}^T)^{-1} \end{aligned} \tag{9}$$

In the case of the R5C4 system it is possible to measure the voltage only on capacitors C_1 and C_3, i.e. only three from the four state variables are measurable. Therefore, the matrices of the third-order discrete-time state-space approximation model were identified. The identification results, i.e., the values of the discrete-time state-space model matrices in (7) calculated from the equation (9), are summarized in Table 3 for the sampling period $T_s = 0.05$ s and $T_s = 0.1$ s.

TABLE III. PARAMETERS OF THE DISCRETE-TIME MODEL

x	T_s	**A**			**B**	**u**
u_3	0.05 s	0.708	0.389	-0.222	0.115	u_0
	0.10 s	0.545	0.553	-0.253	0.140	
u_2	0.05 s	0.030	0.944	0.040	0.017	
	0.10 s	0.045	0.912	0.017	0.026	
u_1	0.05 s	-0.134	0.398	0.627	0.459	
	0.10 s	-0.142	0.503	0.503	0.248	
y		**C**			**D**	**u**
u_3	0.05 s	1.0	9.05×10^{-14}	-2.47×10^{-14}	0	u_0
	0.10 s	1.0	2.22×10^{-12}	-7.26×10^{-14}	0	

Fig. 5 compares the output of the third-order approximate discrete-time state-space model with the measured data at sampling period $T_s = 0.05$ s.

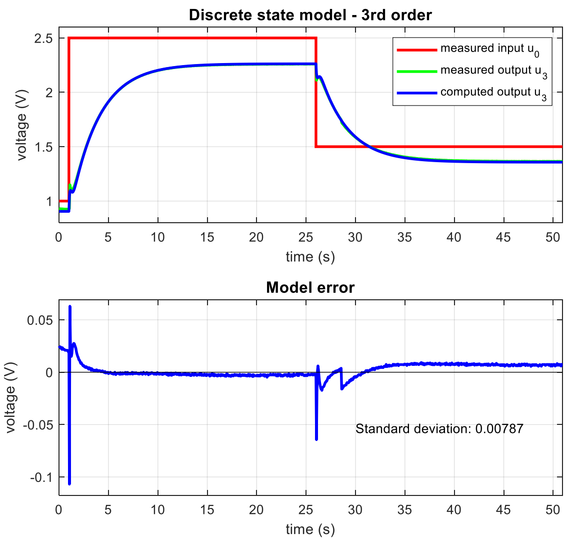

Fig. 5. Measured and calculated voltage signal - approximation by discrete-time state-space model of the 3rd order

The standard deviations of the error of the continuous-time model with the nominal electrical component parameters and the corrected parameters and the discrete-time models (difference equation and approximate state-space form) for both sampling periods are given in Table 4.

TABLE IV. COMPARISON OF STANDARD DEVIATIONS OF ERRORS FOR ALL MODELS

Sample period	Continuous-time model		Discrete-time model	
	nominal parameters	corrected parameters	differential equation	approximate state-space
T_s=0.05 s	0.01777	0.00706	0.02947	0.00787
	100.00 %	41.98 %	165.94 %	44.29 %
T_s=0.10 s	0.01729	0.00317	0.00429	0.00335
	97.30 %	17.84 %	24.14 %	18.85 %

III. CONCLUSION

The article is the second one of the article series devoted to the R5C4Due laboratory device designed and developed for the teaching process control and automation purposes by the authors. It is an electrical circuit composed of resistors and capacitors complemented by a single-chip Arduino microcomputer. The whole system is designed to be used together with MATLAB environment. This paper is devoted to the experimental identification of the RCDue laboratory device. Corrections to the parameters of the first-principle model based on the measured experimental data are estimated. Furthermore, the parameters of the input-output model and the state-space model in the discrete-time domain are determined. If the corrections are applied, the quality of the model is significantly improved. For the discrete-time input-output description, the sampling period plays a role. On the other hand, the state-space model describes the system well for different sampling periods. Measurement noise in a RCDue system with passive components is negligible. The next paper of the article series will be an article dealing with the control design starting from the classical up to the advanced control design methods.

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