Application of Model Predictive Controller to Magnetic Levitation

1st Aleš Novotný
Department of Process Control
University of Pardubice
Pardubice, Czech Republic
ales.novotny1@student.upce.cz

2nd Daniel Honc
Department of Process Control
University of Pardubice
Pardubice, Czech Republic

***Abstract*—Model Predictive Control (MPC) is an advanced process control method that is widely used for controlling both linear and under some modifications for non-linear systems. The aim of this work is to show a way how to apply MPC to a non-linear Magnetic Levitation System (MLS) and its capability of stabilization and closed-loop performance. This work is a continuation of the previous article where the laboratory plant CE 152 MLS was identified, and a non-linear model was designed. This paper proposes a control circuit consisting of linearized discretized non-linear MLS model, Extended Kalman Filter (EKF) algorithm for state estimation and linear MPC. The results are verified in simulation and real-world experiment.**

***Index Terms*—model predictive control, non-linear system, magnetic levitation system, extended Kalman filter, laboratory plant CE 152**

I. Introduction

In recent years, MPC is widely used strategy for controlling various systems (linear or non-linear). MPC algorithm is described in many books and publications in various forms, for example in [1–5].

Magnetic levitation is a physical phenomenon when an object is held in a stable position in the air only by using magnetic fields (without mechanical support). The levitating object can eliminate the gravitation field using its own (electro-magnetic) magnetic field, the (electro-magnetic) magnetic field of the base or both. This allows an object to move in the area of lower friction resistance. This is used in many practical cases in the industry. For example, it is used in commonly known magnetic levitation trains (maglev), contactless bearings [6] and another example of the magnetic levitation principle is used in metallurgy as a non-contact material melting [7].

The aim of this work is to follow up the previous article [8] where a non-linear model of the magnetic levitation laboratory Single-Input Single-Output (SISO) plant CE 152 produced by Humusoft (see Figure 1) was identified and apply a controller based on MPC in order to create a more sophisticated controller for this plant.

This model is well described in the manual [9], and it is a subject of many other publications such as [10–14], etc. Many control strategies for this plant are presented in available publications. In this contribution, a linear MPC-based approach is used for controlling CE 152.

Fig. 1. Laboratory plant CE152.[1]

In this paper, the physical model is summarized in Chapter II. Then the non-linear model linearization and discretization process are described in Chapter III. The usage of Extended Kalman filter (EKF) is briefly explained, and the enumeration process of EKF matrices is described in Chapter IV. The MPC is comprehensively described in Chapter V where the controller is designed. Results are discussed in Chapter VI.

II. Magnetic levitation plant

According to [8], the resulting force acting on the ball is given by the difference between the two main forces, gravitational F_g and magnetic F_m forces.

$$F(t) = F_m(t) - F_g. \tag{1}$$

This research was supported by SGS 2023 grant at Faculty of Electrical Engineering and Informatics, University of Pardubice.

[1] Source: https://www.humusoft.cz/data/img/models/ce152.jpg

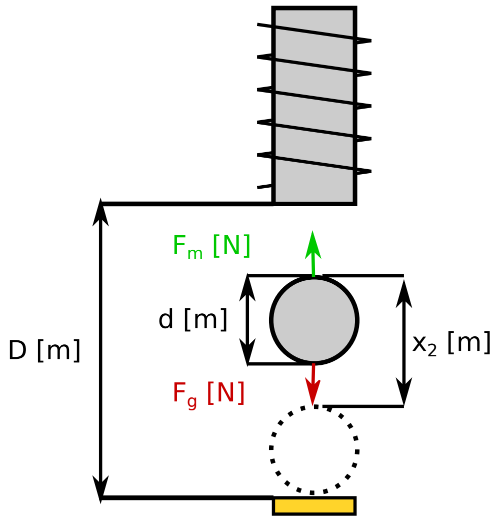

Fig. 2. Selected convention of the plant model.

The selected convention of the model and position x_2 definition is shown in Figure 2. The rectangle at the bottom (pad) is position sensor.

According to [8], the forces are determined as in (2). This approach is one of the most frequent, described in [9, 11, 12, 15, 16], etc.

$$m\frac{d^2x_2}{t^2} = k_c\left(\frac{i}{x_2+x_0}\right)^2 - mg \tag{2}$$

The parameters of the model are shown in Table I. Briefly, some of the constants were measured by direct measurements. k_c and x_0 parameters were measured with advantage in steady states where $F_g = F_c$. The whole identification procedure is comprehensively described in [8].

TABLE I
CE 152 PLANT MODEL PARAMETERS.

Parameter	**Sym.**	**Value**
U/I converter gain	k_i	$2.970 \cdot 10^{-1}\ A \cdot V^{-1}$
Gravitation constant	g	$9.807\ m \cdot s^{-2}$
Ball weight	m	$8.280 \cdot 10^{-3}\ kg$
Ball diameter	d	$1.270 \cdot 10^{-2}\ m$
Space between coil and sensor	D	$1.840 \cdot 10^{-2}\ m$
Position sensor gain	k_x	$8.549 \cdot 10^{2}\ V \cdot m^{-1}$
Position sensor offset	y_0	$4.700 \cdot 10^{-2}\ V$
U/X convertor gain	a	$-2.563 \cdot 10^{-3}\ m \cdot V^{-1}$
U/X convertor offset	b	$8.335 \cdot 10^{-3}$
Coil constant	k_c	$6.049 \cdot 10^{-6}\ N \cdot kg^2 \cdot A^{-2}$
Coil offset	x_0	$-8.335 \cdot 10^{-3}\ m$

It should be noted that the voltage measured on the position sensor represents position indirectly. For this purpose is used (3) with an assumption that the sensor is almost linear (it is declared by the manufacturer).

$$y = k_x x_2 + y_0 \tag{3}$$

Also, a conversion for input current is needed. The current amplifier is considered as fast enough to simplify the first-order system representing the U/I converter by

$$i = k_i u. \tag{4}$$

Next, in order to be able to apply a controller, it is necessary to know how to convert output position x_2 to control input u. Static model (when $F_g = F_m$) was used for this purpose. The conversion is then described by equation (5) (more comprehensively it is described in [8]).

$$x_2 = au + b \tag{5}$$

Finally, the model can be described by the state-space representation (6).

$$\dot{x}_1 = \frac{k_c}{m}\left(\frac{i}{x_2+x_0}\right)^2 - g \tag{6a}$$

$$\dot{x}_2 = x_1 \tag{6b}$$

$$y_n = x_2 \tag{6c}$$

Where y_n represents output of the system (transformed from real-sensor voltage by (3)), x_1 represents velocity and x_2 position. The input is current i.

III. Discretization and Linearization of Non-Linear Plant

A. Linearization of a plant

Linearization is a procedure of finding linear approximation in a given point of the former function. The linear approximation is the first-order Taylor expansion. Taylor expansion of a general non-linear multi-variable function f_n is given by

$$f_n(\vec{x}) \approx f_n(\vec{p}) + \nabla f_n(\vec{x})|_{\vec{p}}(\vec{x}-\vec{p}). \tag{7}$$

Where $\vec{x}$ is vector of the function f_n parameters and $\vec{p}$ is a selected point for linearization.

For the plant specified by (6) it is possible to find linearized model

$$\dot{x}(t) \approx Ax + Bu_n = \begin{bmatrix}0 & -K\\ 1 & 0\end{bmatrix}x + \begin{bmatrix}K\\ 0\end{bmatrix}u_n, \tag{8a}$$

$$y_n(t) \approx Cx = \begin{bmatrix}0 & 1\end{bmatrix}x, \tag{8b}$$

$$K = \frac{2g}{x_{2p}+x_0}, \tag{8c}$$

Where x_{2p} is working point for linearization, u_n is control input of linearized system and y_n is its output.

It should be noted, (4) was used as a substitution for i and (5) approximation was used as a substitution for u which create new control input u_n in meters. It simplifies expressions and removes linearization offset. Also, this linearized plant has input u_n and output y_n in meters with the same meaning instead of real-plant input u with a meaning of voltage on the input of U/I converter and output y represented by the position sensor voltage which cannot be compared due to different meanings.

B. *Discretization of a linear continuous system*

Discretization of a continuous linear state-space model into a discrete form was performed using MATLAB *c2d* (default method "Zero-order hold") command.

IV. EXTENDED KALMAN FILTER

EKF is used for the estimation of non-linear plant states. In this work, the standard EKF algorithm described in [17–19] is applied.

However, process noise R_{ekf}, measurement noise Q_{ekf} and initial P_0 covariance matrices should be determined. The correlation matrix N_{ekf} of above-mentioned noises is not used in this work, and it would be assumed as zero.

It is assumed a linearized discrete stochastic system given by state equation (9a) and output (measurement) equation (9b).

$$x[k+1] = F_x x[k] + F_u u[k] + w[k] \tag{9a}$$

$$y[k] = H_x x[k] + H_u u[k] + v[k] \tag{9b}$$

Using the control circuit (with PID) described in [8], the steady-state real-world data was measured for Q_{ekf} and R_{ekf} specification. The dispersion of input noise D_u can be estimated by (10a) and the output noise dispersion D_y by (10b).

$$D_u = \frac{1}{n}\sum_{i=1}^{n}\left(u[i] - u_0\right)^2 \tag{10a}$$

$$D_y = \frac{1}{n}\sum_{i=1}^{n}\left(y[i] - y_0\right)^2 \tag{10b}$$

Where u_0 and y_0 is input and output mean value for the steady state of the plant. $u[i]$ and $y[i]$ are set input and measured output. n is number of experimental data samples.

The process noise is the noise on the plant states, thus input noise with dispersion D_u is multiplied by F_u, and the process noise covariance matrix can be expressed by (11a). The output noise covariance matrix would be equal to the output noise dispersion D_y (11b).

$$Q_{ekf} = \operatorname{diag}\left(D_u I_{2,2} F_u\right)_{2\times 2} \tag{11a}$$

$$R_{ekf} = D_y \tag{11b}$$

Furthermore, it is obvious that if a system is in a steady state, a covariance matrix P inside the EKF is equal to Q_{ekf} [17–19]. Thus, this fact is with advantage used for proper initialization of P which is

$$P_0 = Q_{ekf}. \tag{12}$$

V. DISCRETE MODEL PREDICTIVE CONTROLLER

A. *Motivation*

MPC is the main part of the designed control circuit which represents a control strategy. In this work, the MPC algorithm accepts linearized (see Chapter III-A) and discretized (see Chapter III-B) version of the non-linear CE 152 plant model (6) as a reference model for predictions.

B. *Algorithm*

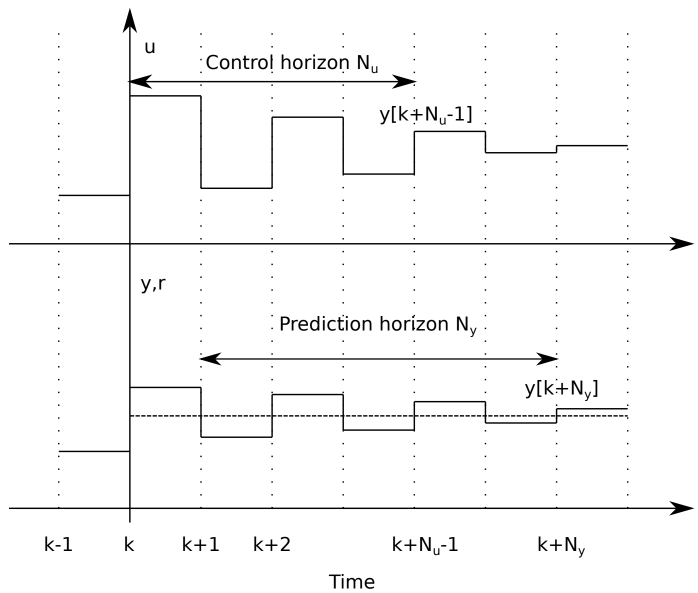

Fig. 3. MPC strategy - horizons

It is considered a general discrete SISO Linear Time-Invariant (LTI) plant model with two state variables (x_1 - velocity, x_2 - position) given by (13).

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = A \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + Bu[k] \tag{13a}$$

$$y[k] = C \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} \tag{13b}$$

The model (13) is augmented by a new state $\hat{x}$ into deviation model (14). A new input is then $\Delta u = u[k] - u[k-1]$.

$$\hat{x}[k+1] = \hat{A}\hat{x}[k] + \hat{B}\Delta u[k] \tag{14a}$$

$$y[k] = \hat{C}\hat{x}[k] \tag{14b}$$

$$\hat{x}[k] = \begin{bmatrix} x[k] \\ u[k-1] \end{bmatrix} \tag{14c}$$

$$\hat{A} = \begin{bmatrix} A & B \\ 0_{1\times 2} & 1 \end{bmatrix} \tag{14d}$$

$$\hat{B} = \begin{bmatrix} B \\ 1 \end{bmatrix} \tag{14e}$$

$$\hat{C} = \begin{bmatrix} C & 0 \end{bmatrix} \tag{14f}$$

The augmented state is possible to express in any sample as j-step prediction by (15a). Thus, it is possible to express output by

$$\hat{x}[k+j] = \hat{A}^j \hat{x}[k] + \sum_{i=0}^{j-1} \hat{A}^{j-i-1} \hat{B} \Delta u[k+i], \quad (15a)$$

$$\hat{y}[k+j] = \hat{C} \hat{x}[k+j]. \quad (15b)$$

The whole prediction can be expressed by

$$y_{N_y} = G \Delta u_{N_u} + F \hat{x}[k] = GU + f, \quad (16a)$$

$$y_{N_y} = \begin{bmatrix} y[k+1] \\ y[k+2] \\ \vdots \\ y[k+N_y] \end{bmatrix}, \quad (16b)$$

$$G = \begin{bmatrix} \hat{C}\hat{B} & 0 & \dots & 0 \\ \hat{C}\hat{A}\hat{B} & \hat{C}\hat{B} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \hat{C}\hat{A}^{N_y-1}\hat{B} & \hat{C}\hat{A}^{N_y-2}\hat{B} & \dots & \hat{C}\hat{A}^{N_y-N_u}\hat{B} \end{bmatrix}, \quad (16c)$$

$$\Delta u_{N_u} = \begin{bmatrix} \Delta u[k] \\ \Delta u[k+1] \\ \vdots \\ \Delta u[k+N_u-1] \end{bmatrix}, \quad (16d)$$

$$F = \begin{bmatrix} \hat{C}\hat{A} \\ \hat{C}\hat{A}^2 \\ \vdots \\ \hat{C}\hat{A}^{N_y} \end{bmatrix}. \quad (16e)$$

In (16c) is $\hat{A}$ present only if its exponent is greater than zero, e.g. $N_y - N_u > 0$.

A criterion for MPC can be expressed by (17a).

$$J = e_{N_y}^T Q_{mpc} e_{N_y} + \Delta u_{N_u}^T R_{mpc} \Delta u_{N_u} \quad (17a)$$

$$e_{N_y} = y_{N_y} - w_{N_y} \quad (17b)$$

Where N_u is control horizon and N_y is prediction horizon in samples (see Figure 3). Q_{mpc} and R_{mpc} are user defined weighting matrices for cumulated squares of future control errors and cumulated squares of future control inputs differences. w_{N_y} represents future reference.

It is possible to substitute from (16) to criterion (17a) and get (18a).

$$J = \Delta u_{N_u}^T H \Delta u_{N_u} + 2g^T \Delta u_{N_u} + k \quad (18a)$$

$$H = G^T Q_{mpc} G + R_{mpc} \quad (18b)$$

$$g^T = (f - w_{N_y})^T Q_{mpc} G \quad (18c)$$

$$k = (f - w_{N_y})^T Q_{mpc} (f - w_{N_y}) \quad (18d)$$

In order to minimize the criterion J by changing future control inputs differences Δu_{N_u} it is possible to find global minimum of J by finding such a Δu_{N_u} that satisfies

$$\frac{\mathrm{d}J}{\mathrm{d}\Delta u_{N_u}} = 2(H \Delta u_{N_u} + g) = 0. \quad (19)$$

Finally, the optimal Δu_{N_u} is

$$\Delta u_{N_u} = -H^{-1} g. \quad (20)$$

It is noteworthy that only the actual control input is applied and the remaining are forgotten, and finding the global minimum using (19) and (20) is possible only if there are no constraints for u, x and y.

VI. Control results

A. Designed control circuit

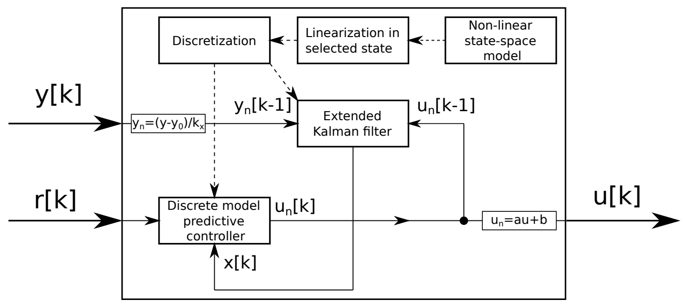

Fig. 4. Control circuit block scheme.

A block scheme of the whole control circuit is shown in Figure 4 . Linearization according to Chapter III was selected in $\frac{(D-d)}{2}$ working point and sampling period T_s was selected as small as the PC allowed because the magnetic levitation plant has very quick responses. The linearized state-model matrices are shown in Table II.

TABLE II
Linearized discretized state-space model plant parameters

Parameter	**Sym.**	**Value**
Sampling period	T_s	$2.000 \cdot 10^{-4}\ s$
Working point for linearization	x_{2p}	$4.275 \cdot 10^{-3}\ m$
State matrix	A	$\begin{bmatrix} 1.000 & 9.662 \cdot 10^{-1} \\ 2.000 \cdot 10^{-4} & 1.000 \end{bmatrix}$
Input matrix	B	$\begin{bmatrix} -9.662 \cdot 10^{-1} \\ -9.662 \cdot 10^{-5} \end{bmatrix}$
Output matrix	C	$\begin{bmatrix} 0.000 & 1.000 \end{bmatrix}$

Next, the PID controller with parameters shown in Table III connected in same circuit as in [8].

The approach of setting EKF matrices was mentioned in Chapter IV and the final parameters are shown in Table IV.

The MPC parameters were set for simulation and experiment with values shown in Table V. Where the $[300, \dots, 7]$ is logarithmically spaced vector. It is such a vector that decadical logarithm of this $\log_{10}([300, \dots, 7])$ is linearly spaced vector.

TABLE III
PID PARAMETERS

Parameter	Sym.	Value
Proportional constant	P_{pid}	2.000
Integral constant	I_{pid}	3.000
Derivative constant	D_{pid}	$3.000 \cdot 10^{-2}$
Filter coefficient	N_{pid}	$1.000 \cdot 10^{3}$

TABLE IV
EKF PARAMETERS

Parameter	Sym.	Value
Meas. noise cov. matrix	R_{ekf}	4.492
Proc. noise cov. matrix	Q_{ekf}	$\begin{bmatrix} -2.799 \cdot 10^{-8} & 0 \\ 0 & -6.997 \cdot 10^{-12} \end{bmatrix}$
Initial. cov. matrix	P_0	Q_{ekf}

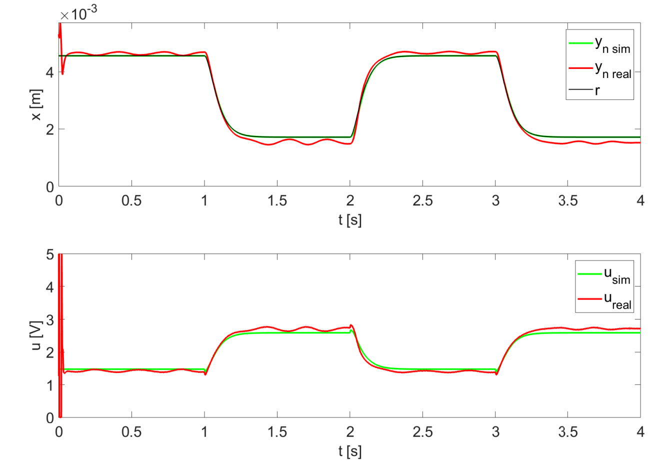

Fig. 5. Simulation and experiment comparison of the MPC-based controller.

B. Real-world experiment

For verification of the control circuit, simulation in MATLAB environment was implemented and results were proved by real-world experiments.

The reference was generated from periodic rectangular pulse signal by filtering using discretized continuous second-order system given by transfer function $H_{filter}(s) = \frac{400}{(s+20)^2}$.

In Figures 5, 6 and 7, the ball was stabilized in steady point until time 0 s in order to remove horizontal ball oscillations. However, the signal reference was selected relatively close to limit ball positions (0 and $D - d$) which is challenging for control circuit at this plant in order to see the capability of the control circuits.

Figure 5 shows a comparison of simulation and experiment. It can be seen that the linearized non-linear CE 152 model has similar dynamic behaviour to the real plant if the reference is changing slowly. The differences between the model and the plant are compensated by EKF state estimation.

A comparison between PID based control circuit designed in previous work [8] and MPC based control circuit is shown in Figure 6. It can be noted that the PID-based circuit is very simple but has worse closed-loop performance than the MPC-based circuit.

Finally, the EKF states estimation are shown in Figure 7. The measured position y_n is one of the states and can be compared to the estimated state x_2 which has the same

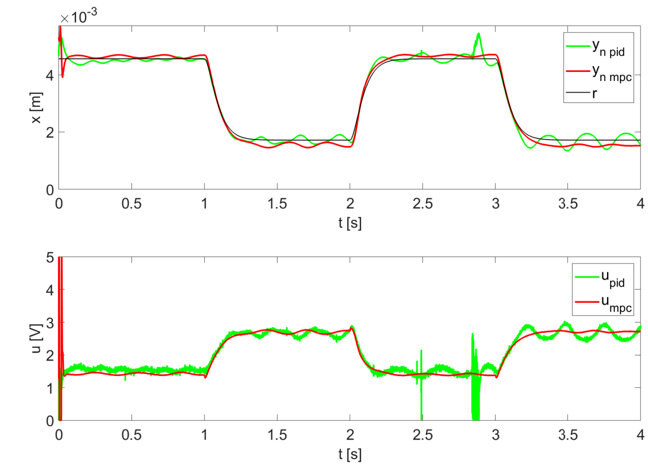

Fig. 6. Comparison of PID and MPC based control circuits responses.

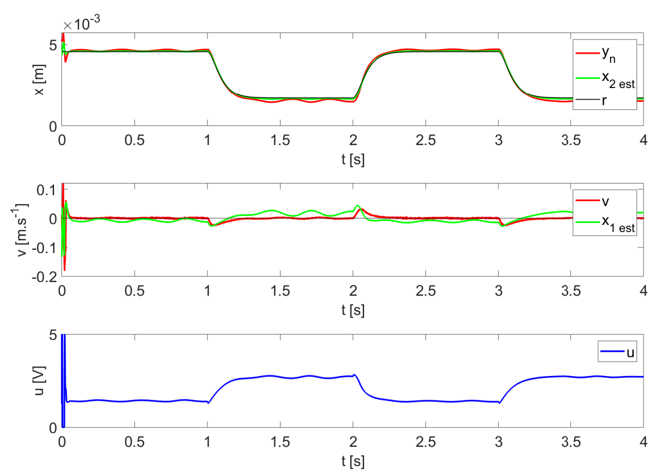

Fig. 7. EKF estimation.

TABLE V
MPC PARAMETERS

Parameter	Sym.	Value
Control horizon	N_u	$5.000 \cdot 10^{1}$
Prediction horizon	N_2	$5.000 \cdot 10^{1}$
Energy penalization matrix	R_{mpc}	I_{N_u}
Error penalization matrix	Q_{mpc}	$\mathrm{diag}\left([300, \ldots, 7]\right)_{N_2 \times N_2}$

meaning. The velocity of the ball is not directly measured but for the illustration, it is computed as $v[k] = \frac{y_n[k]-y_n[k-1]}{T_s}$ and then the computed velocity is filtered using Moving Average Filter (MAF) with windows size 10 described in [8].

VII. Conclusion

In this work, the procedure for implementing the predictive controller of the CE 152 system was outlined. The model was identified, and the non-linear model was created in the precedent article [8]. Firstly, the results of the article were summarized. The non-linear model with identified parameters was presented, and the essential features of this plant were briefly explained.

Next, a design of the predictive controller components was described. This controller is composed of the linearized discretized model, extended Kalman filter for estimating states of the non-linear system and model predictive control algorithm.

Finally, the MPC based control circuit was designed, and the functionality was verified by comparison of simulations in the MATLAB environment and real-world experiment. The control circuit was also compared with the PID based control circuit from precedent work [8]. Finally, the quality of state estimation was shown.

It can be noted, that more advanced MPC-based control circuit has better closed-loop performance than the PID-based controller. But still, there is space for non-linear control techniques, which can utilize of the non-linear model information. Due to this fact, a non-linear control circuit for this system will be designed in further work.

References

[1] L. Wang, *Model predictive control system design and implementation using MATLAB®*. Springer Science & Business Media, 2009.

[2] E. F. Camacho and C. B. Alba, *Model predictive control*. Springer science & business media, 2013.

[3] E. Fernandez-Camacho and C. Bordons-Alba, *Model predictive control in the process industry*. Springer, 1995.

[4] F. Allgöwer and A. Zheng, *Nonlinear model predictive control*. Birkhäuser, 2012, vol. 26.

[5] M. Ellis, J. Liu, and P. D. Christofides, "Economic model predictive control," *Springer*, vol. 5, no. 7, p. 65, 2017.

[6] T. Lembke, "Design and analysis of a novel low loss homopolar electrodynamic bearing," 01 2005.

[7] P. C. Nordine, J. K. R. Weber, and J. G. Abadie, "Properties of high-temperature melts using levitation," *Pure and Applied Chemistry*, vol. 72, no. 11, pp. 2127–2136, 2000. [Online]. Available: https://doi.org/10.1351/pac200072112127

[8] A. Novotnỳ, D. Honc, and F. Dušek, "Identification of magnetic levitation system," in *Proceedings of the Computational Methods in Systems and Software*. Springer, 2021, pp. 58–75.

[9] Humusoft. (1996) CE 152 Magnetic levitation model - Educational manual.

[10] D. Honc, *Modelling and Identification of Magnetic Levitation Model CE 152/Revised*, 01 2019, pp. 35–43.

[11] P. Šuster and A. Jadlovská, "Modeling and control design of magnetic levitation system," in *2012 IEEE 10th International Symposium on Applied Machine Intelligence and Informatics (SAMI)*, 2012, pp. 295–299.

[12] P. Chalupa, M. Maly, and J. Novak, "Nonlinear simulink model of magnetic levitation laboratory plant," 06 2016, pp. 293–299.

[13] M. Semelka and D. Honc, "Model soustavy humusoft ce152," Bachelor's Thesis, University of Pardubice, 2015. [Online]. Available: http://hdl.handle.net/10195/60423

[14] J. Bednařík and M. Matějásko, "Control of magnetic levitation educational model," Bachelor's Thesis, Brno University of Technology, 06 2017. [Online]. Available: http://hdl.handle.net/11012/66926

[15] V. Dolga and L. Dolga, "Modeling and simulation of a magnetic levitation system," *Fascicle of Management and Technological Engineering*, vol. 6, no. XVI, 2007.

[16] A. Pati, V. Verma, R. Negi, and S. Nagar, "Real time implementation of series expansion based digital controller for magnetic levitation system," *Intelligent Control and Automation*, vol. 07, pp. 110–128, 01 2016.

[17] H. Khalil, *Nonlinear Control*, ser. Always Learning. Pearson, 2015. [Online]. Available: https://books.google.cz/books?id=OfHhnQEACAAJ

[18] V. Havlena and J. Štecha, "Moderní teorie řízení," 2000. [Online]. Available: https://books.google.cz/books?id=T2hdAAAACAAJ

[19] MathWorks. (2022) Extended kalman filters. (accessed: 16.10.2022). [Online]. Available: https://www.mathworks.com/help/driving/ug/extended-kalman-filters.html