A Variant of the Larmor Frequency Calculation

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***Abstract*—The Larmor frequency is a basic parameter in the design and calculation of elements, especially in microwave technology. Its value is usually derived using moments, but this calculation is both laborious and lengthy and does not lead to the correct result for ferrites. The article describes a more elementary procedure for finding the Larmor frequency based not on moments, but on the equality of force action. The latter is much shorter, clearer, and simpler, and it also gives a result that is in agreement with the experimentally verified quantum theory.**

Keywords—Larmor frequency, gyromagnetic ratio

I. INTRODUCTION

Each electron has a certain mass and charge and rotates around its axis. Therefore, an electron has a mechanical and magnetic moment. When placed in a static magnetic field, it takes a position such that the magnetic moment has a direction identical to the direction of the intensity of the static magnetic field. Another high-frequency intensity of the magnetic field of the electromagnetic wave, which is transverse to the static intensity, excites the rotating electron. Then, the axis of the electron begins to perform a precession movement.

If the electromagnetic wave has the same sense of rotation as the precession movement and if the frequency of the precession movement is identical to the frequency of the exciting electromagnetic wave, the gyromagnetic resonance will occur. This wave is associated with a strong interaction between the high-frequency wave and the electrons. At the same time, the excitation wave supports the precession movement and transfers its energy to the electrons. Therefore, it is heavily damped. Determining the frequency of this so-called Larmor precession is of considerable importance.

Microwave elements such as ferrite insulators, ferrite circulators, and many other elements are based on this principle.

II. CLASSIC PROCEDURE

According to the concepts of classical physics, electrons rotate around their axis, so they have their moment of momentum or spin [1]. Moment of force L acting on the loop is according to Fig. 1 [2], [3], [4].

$$L = F \cdot r \cdot 2 = I \cdot B \cdot l \cdot r \cdot 2 = I \cdot B \cdot S = \\ = B \cdot I \cdot S = B \cdot m , \quad (1)$$

where: L is the moment of force, F is the force, r is the arm of the force, I is the current, B is the magnetic field induction, l is the length of the active part of loop, S is the area of the loop, m is the magnetic moment of an electron.

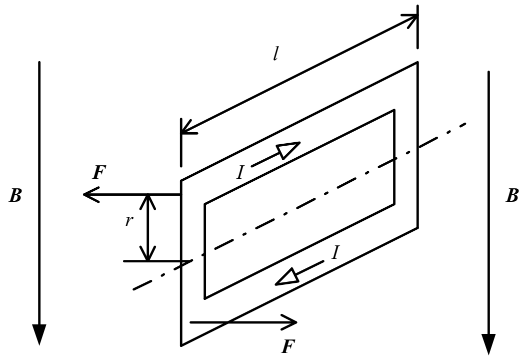

Fig. 1. A loop through which the current flows.

The magnetic moment of an electron is (2).

$$m = I \cdot S \quad (2)$$

In vector form is (3).

$$\boldsymbol{m} = I \cdot \boldsymbol{S} \quad (3)$$

The result is (4).

$$\boldsymbol{L} = \boldsymbol{m} \times \boldsymbol{B} \quad (4)$$

It follows from classical electrodynamics that electrons also have their magnetic moment of size m.

For a circular orbital path of an electron, the current I per one revolution is defined (5).

$$I = \frac{Q}{t} = \frac{e}{t} = e \cdot f = e \cdot \frac{\omega}{2 \cdot \pi} , \quad (5)$$

where: I is the current, Q is the charge, t is the time, e is the charge of an electron, f is the frequency, ω is the angular frequency.

According to Fig. 2, the magnetic moment of an electron is defined as (6).

$$m = I \cdot S = e \cdot \frac{\omega}{2 \cdot \pi} \cdot \pi \cdot r^2 = \frac{1}{2} \cdot e \cdot \omega \cdot r^2 = \\ = \frac{1}{2} \cdot e \cdot \omega \cdot r \cdot r = \frac{1}{2} \cdot e \cdot v \cdot r , \quad (6)$$

where: v is the speed.

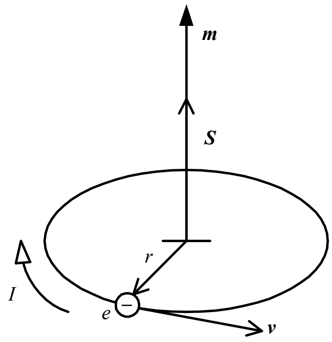

Fig. 2. The path of an electron.

In vector form is (7).

$$\boldsymbol{m} = \frac{1}{2} \cdot e \cdot \boldsymbol{r} \times \boldsymbol{v} \tag{7}$$

The momentum p of an electron with the mass m_e is (8).

$$\boldsymbol{p} = m_e \cdot \boldsymbol{v} \tag{8}$$

Therefore, the moment of force $\boldsymbol{K}$ of an electron is (9).

$$\boldsymbol{K} = \boldsymbol{r} \times \boldsymbol{p} = m_e \cdot \boldsymbol{r} \times \boldsymbol{v} \tag{9}$$

Comparing (7) with (9) we get (10).

$$\boldsymbol{r} \times \boldsymbol{v} = \frac{\boldsymbol{K}}{m_e} = \frac{2 \cdot \boldsymbol{m}}{e} \tag{10}$$

Therefore, the magnetic moment m of an electron is (11).

$$\boldsymbol{m} = \frac{e}{2 \cdot m_e} \cdot \boldsymbol{K} \tag{11}$$

We modify this resulting relation by defining the gyromagnetic ratio γ (12) and then obtain the result in a simplified form (13).

$$\gamma = \frac{e}{2 \cdot m_e} \tag{12}$$

$$\boldsymbol{m} = \gamma \cdot \boldsymbol{K} \tag{13}$$

The gyromagnetic ratio γ (12) is equal to the ratio of the magnetic moment of a charged particle to its moment of force.

If we substitute the values of $e = 1.602176634 \times 10^{-19}$ C and $m_e = 9.1093837015 \times 10^{-31}$ kg into relation (12), we calculate the theoretical value of the gyromagnetic ratio $\gamma = 8.7941 \times 10^{10}$ C/kg approx.. However, by measuring the ferrites, the value of the gyromagnetic ratio was experimentally determined to be $\gamma = 1.76 \times 10^{11}$ C/kg approx.. Thus, the experimentally determined value is roughly twice as high as the theoretically determined value.

Therefore, we can state that the value of the gyromagnetic ratio determined by the method described above cannot lead to the correct result. Thus, this procedure does not even lead to the determination of the correct value of the Larmor frequency.

We can explain the above contradiction using quantum theory. Each electron creates a spin current loop in addition to the orbital current loop. Because the electron orbits around the nucleus on the one hand, and on the other hand, according to the mechanical idea, it rotates around its axis, i.e. it has its spin. From the principles of quantum mechanics, we can deduce that the relationship (14) holds between the spin magnetic moment of electrons m_S and their spin moment of momentum K_S.

$$\boldsymbol{m}_S = \frac{e}{m_e} \cdot \boldsymbol{K}_S \tag{14}$$

The resultant moments of an atom or molecule are equal to the sum of the orbital and spin moments. Therefore, the relationship between the total magnetic moment and the angular momentum takes the form (15).

$$\boldsymbol{m} = \eta \cdot \boldsymbol{K} \tag{15}$$

Now, the gyromagnetic ratio η is no longer a universal constant but depends on the interrelationship between the orbital and spin momenta of the atom. Therefore, we must correct the value of the coefficient between the total magnetic moment and the moment of force found by the above calculation. For ferrites, the total moment is given practically only by the spin moment of the electrons. Thus, the gyromagnetic ratio of ferrites is (16).

$$\eta = \frac{e}{m_e} \tag{16}$$

However, the method described above does not lead to this relationship.

III. Proposed Procedure

Analogous to the simplified solution of Poisson's equation using Coulomb's law [3], [5], [6], [7], [8], we can simplify this calculation as well. If we assume a dominant spin, the magnetic moment and the angular momentum can be bypassed by using the Lorenz force, thus simplifying the calculation for ferrite.

Each electron with charge $e = 1.602176634 \times 10^{-19}$ C rotates about its axis (creating a spin magnetic moment). At the same time, the electron has a mass $m_e = 9.1093837015 \times 10^{-31}$ kg (and thus a moment of momentum), so it behaves like a flywheel with a magnetic moment, i.e. with a magnetization vector $\boldsymbol{m}$ or by the intensity of the magnetic field $\boldsymbol{H}$.

After being placed in a magnetostatic field with induction $\boldsymbol{B_0} = \mu \cdot \boldsymbol{H_0}$, the electron takes a position in which the magnetization vector has $\boldsymbol{m}$ or intensity $\boldsymbol{H}$ direction the same as the intensity of the magnetostatic field $\boldsymbol{H_0}$. The angular rate of rotation of the axis of rotation ω_0 is proportional to the induction B_0 (for zero B_0 the axis does not rotate).

If, in addition, the time-varying intensity $\boldsymbol{H_{HF}}$ of an electromagnetic wave with frequency ω_{HF} acts in the direction transverse to $\boldsymbol{H_0}$, an electron is excited by it. Its axis of rotation of intensity $\boldsymbol{H}$ does the so-called Larmor precessional motion around $\boldsymbol{H_0}$. This movement fills the envelope of the cone of rotation, which causes the perpendicular component of the vector $\boldsymbol{H_n}$ to appear. Intensity $\boldsymbol{H_n}$ moves in a circle according to Fig. 3 and rotates with the Larmor angular frequency ω_0. By mechanical inertia, the axis tries to keep in the direction $\boldsymbol{B_0}$, the direction of the current I determines the (positive) direction m, and the direction $\boldsymbol{B_0}$ determines the positive direction ω_0 that ω_0 has the direction I according to Fig. 3.

We determine the angular frequency of Larmor precession ω_0 in ferrite by comparing the relations for the mechanical force F acting on a particle of mass dm rotating at the speed ω_0 (17) with the relation to the electric force acting on this particle moving at a speed v and carrying a charge dq (18).

The mechanic force (a is the acceleration) is (17).

$$F = dm \cdot a = dm \cdot \omega_0 \cdot v \tag{17}$$

The electric force is (18).

$$F = dq \cdot v \cdot B_0 \tag{18}$$

We compare both forces and derive a relation for the Larmor frequency (19).

$$\begin{aligned}
dm \cdot \omega_0 \cdot v &\approx dq \cdot v \cdot B_0 \;/\; : v \\
dm \cdot \omega_0 &\approx dq \cdot B_0 \;/ \int \\
\int \omega_0 dm &\approx \int B_0 dq \\
m \cdot \omega_0 &\approx q \cdot B_0 \\
\omega_0 &\approx \frac{q}{m} \cdot B_0 \\
\omega_0 &= \frac{q_e}{m_e} \cdot B_0 \\
\omega_0 &= \gamma \cdot B_0
\end{aligned} \tag{19}$$

The symbol γ is the so-called gyromagnetic ratio, the unit is C/kg. The value of the gyromagnetic ratio after substituting is (20).

$$\gamma = \frac{q_e}{m_e} = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}} = 1.76 \times 10^{11}\ C/kg \tag{20}$$

The obtained result agrees with the result found using quantum theory, which was confirmed for ferrite by experimental measurement.

By directly using the relationship between the centrifugal force and the Lorentz force, we also get (21).

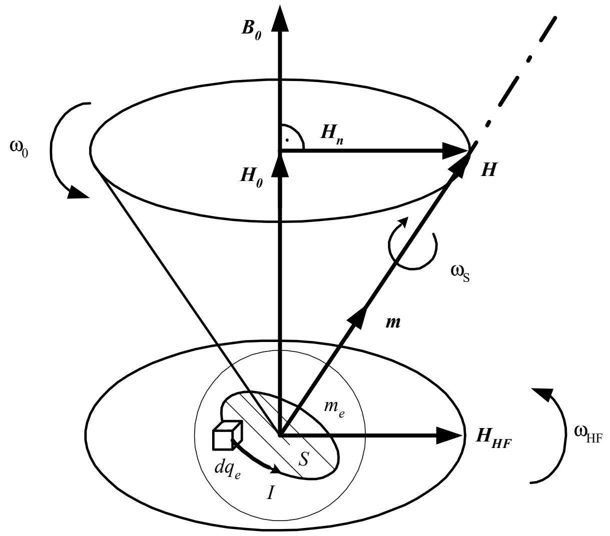

Fig. 3. The situation of the rotating electron.

$$\begin{aligned}
m \cdot \frac{v^2}{r} &= q \cdot v \cdot B \\
m \cdot \frac{v}{r} &= q \cdot B \\
m \cdot \omega &= q \cdot B \\
\omega &= \frac{q}{m} \cdot B \\
\omega &= \gamma \cdot B
\end{aligned} \tag{21}$$

CONCLUSION

In the described procedure, we use elementary relations analogously to the solution of Poisson's equation by elementary application of Coulomb's law. As was shown and proved above, we obtain the same results for ferrites as when solving using quantum theory.

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