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Train Platforming Problem Solving

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I hereby confirm that:

I have written this dissertation thesis independently. All the reference literature and information used in this work are quoted in the list of reference literature.

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Annotation

This dissertation addresses train platforming problem in busy complex stations at peak times which poses a great challenge for railway network controllers. The aim of the dissertation is to propose decision-making tool for an assignment plan of one-day timetable without conflicts. In this context, a special version of train platforming problem is described in detailed way. The characteristics of problem belongs to Prague main railway station. For the purpose of overcome the problem, two solution methods are applied: a mixed integer programming model and a matheuristic algorithm. The objective is to minimize total weighted delays in which weight is synonymous with importance level of each train.

Train platforming problem can be solved easily for small railway stations with very few trains and platform tracks. By the help of mixed integer mathematical model that is called M1 in this context, it can be reached optimal solutions for these kind of railway stations. However, M1 model is not capable for large railway stations due to the Np-hard nature of problem. So, a matheuristic algorithm is presented and it consists of three stages: *(i)* platform track assignment algorithm, *(ii)* calculation of total weighted delay, *(iii)* improvement algorithm. In the proposed matheuristic, the algorithms and the sub-problem (M2) work cooperatively. Platform track assignment algorithm for allocation of track for each train. M2 model that is reduced version of M1 calculates all decision variables. Finally, improvement algorithm is enhancing the quality of solutions in each step.

The mixed integer model and matheuristic algorithm have been implemented in GAMS/Cplex solver and validated using real-world data from Prague main railway station. One day timetable for a weekday in 2016/2017 year is divided into the time intervals. There are approximately 700 arriving and departing trains from/to the station for one day. In each interval, assignments of 36 trains are determined and allocation of the trains which are in intersection time for two consecutive intervals are transferred to next interval. Based on these rules, computational results are presented and solution of two methods are compared.

Keywords: Matheuristic Algorithm, Mixed integer linear programming model, Platform track assignment, Railway infrastructure capacity, Railway optimization models, Train platforming problem.

Anotace

Tato disertační práce je zaměřena na problém přidělování dopravních kolejí (s nástupištní hranou) v rozsáhlých železničních stanicích, což je problém, který vytváří ve špičkových časech velké nároky na výpravčí nebo dispečery. Cílem disertační práce je navrhnout rozhodovací nástroj, který (pokud je možné) umožní nalezení bezkonfliktního přiřazení kolejí vlakům, tzv. plánu obsazení kolejí, v rámci jednoho provozního dne. V tomto kontextu je detailně charakterizována speciální varianta problému přidělování kolejí vlakům. Aplikace je provedena v železniční stanici Praha hl. n. Jsou uplatněny dvě varianty řešení problému: smíšený celočíselný model matematického programování a matheuristický algoritmus. Kritériem je minimalizovat součet vážených zpoždění vlaků. Váhy jsou stanoveny na základě důležitosti vlaku.

Problém přidělování dopravních kolejí (s nástupištní hranou) vlakům je možné snadno řešit ve stanicích s velmi malým počtem kolejí a vlaků. Při využití smíšeného celočíselného modelu matematického programování, tento model, označený jako M1, umožňuje dosáhnout optimálního řešení pro tyto stanice. Nicméně, model M1 není možné využít pro rozsáhlé železniční stanice s velkým rozsahem provozu, neboť model je NP-složitý. Pro tyto případy je navržen druhý – matheuristický model, který je se sestává ze tří kroků: (i) přiřazení dopravních kolejí (s nástupištní hranou) vlakům, (ii) stanovení součtu vážených zpoždění vlaků (pro výchozí řešení), (iii) aplikace algoritmu zlepšujícího řešení. Navržený matheuristický algoritmus, algoritmy a dílčí problém (M2) jsou provázány. Problém přiděluje dopravní koleje (s nástupištní hranou) vlakům. Model M2 je zjednodušenou verzí modelu M1, umožňujícího výpočet všech rozhodovacích proměnných. Celkově, zlepšující algoritmus v každém kroku zvyšuje kvalitu dosaženého řešení.

Smíšený celočíselný model matematického programování i matheuristický alogismus byly implementovány v GAMS/Cplex řešiteli a validovány s využitím dat ze železniční stanice Praha hl. n. Jízdní řád pro jeden pracovní den (2016/17) je rozdělen do časových úseků. Zahrnuto je přibližně 700 příjezdů a odjezdů vlaků za jeden den. V každém časovém úseku je dopravní kolej přiřazována 36 vlakům. Vlaky, jejichž obsazení zasahuje do více časových úseků, jsou do dalšího úseku přenášeny. Na základě těchto předpokladů dosažené výsledky jsou prezentovány a oba přístupy k řešení porovnány.

Klíčová slova: Metaheuristický algoritmus, smíšený celočíselný model matematického programování, přidělování kolejí s nástupištní hranou, kapacita železniční dopravní infrastruktury, optimalizační modely na železnici, přidělování dopravních kolejí vlakům.

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INTRODUCTION

Since the first steam European railway line was opened between Stockton and Darlington in Great Britain in 1825 (Hruban, 2014), railway industries have rapidly grown over the past 200 years in many countries in the world. Especially in European countries, investments in railways increase considerably because railway industries play a big role for freight and passenger transportation. However, complexity of railway system is bringing a lot of problems which are related to subject of operation research. These problems consist of complex and difficult decision making processes that is generally categorized into three levels: strategic planning, tactical planning and operational planning. These levels are given as below:

- *Strategic Planning* contains a comprehensive planning horizon of several years or decades based on objective of organization. In strategic planning level, decisions focus generally on capacity planning. In railway systems, making decisions at this level are related, for example, construction of new infrastructure or purchase of trains.
- *Tactical Planning* consists of distribution of the available capacity within time and space while satisfying operational constraints. The planning period at tactical planning level is between two months and one year such as train platforming plan.
- *Operational Planning* is realization of plans in tactical level. It is not completely possible to implement plans since the operations in the system are dynamic. It is needed to modifications or re-scheduling. In railway stations, train platforming plans can be failed because of unexpected situations such as incidents.

In this dissertation, our motivation is the train platforming problem in large railway stations. Given the detailed layout of the involved railway station, and given the scheduled arrival and departure times of a set of trains, we aim at assignment of the trains to the platforms through the station taking into account the capacity of the station, the safety system, and several operational constraints. The objective of problem is to minimize the total delay of incoming and outgoing trains.

Train platforming problem can be handled in both tactical and operational planning level. We concentrate fundamentally on the problem in tactical planning level. It includes preparation of regular time schedule for one year and proposed method is designed for verification of timetable based on line and network technology from the point of view of station operation. For instances, if trains are waiting at home signals, this waiting can cause delay. If these delays can be identified, timetable should be revised until regular conflicts and delays are not occurred. If the regular timetable should generate delays and conflicts itself, there will be increased probability that the real operation will be unstable. Timetable going to be applied must be without conflicts due to mentioned reason.

This dissertation is organized as follows. In Chapter 1, a brief literature review is provided about train platforming problem in different decision making levels. In Chapter 2, objectives of dissertation are given in detail. The next chapter describes the considered problem, as viewed here, in details. A mixed integer mathematical model is presented in Chapter 4. In the chapter that follows, proposed matheuristic algorithm are reported. In Chapter 6, verification and validation of mathematical model and matheuristic algorithm are examined and computational experiments of methods are discussed. Contributions of dissertation are given in Chapter 7. Finally, last chapter summarizes dissertation and discusses subjects for further research.

1 LITERATURE REVIEW

This study aims to provide a remarkable improvement in allocation of platform tracks to the trains on the basis of development of modelling approaches and solution techniques. In recent years, there has been an increasing amount of literature on train platforming problem. These are summarized as follows.

Recent surveys such as that conducted by Caprara et al. (2010) and Lusby et al. (2011) have taken into consideration train platforming problem. Zwaneveld et al. (1996) and Lusby et al. (2011) refer to an important classification about train platforming problem deals with a system that is proposed for use at the strategic, tactical or operational level.

Planning on strategic level in railway systems includes future infrastructure of railway stations. In this context, Zwaneveld (1996) handled train routing problem. A multi objective integer programming model is formulated regarding given certain scenarios for the future capacity of the Dutch railway infrastructure. The first objective is to maximize the number of trains, the second objective is to minimize the number of shunting movements and the last one is to assign the trains to their most preferred platforms. In the next study, Zwaneveld (2001), also took into consideration routing trains through railway station in strategic planning level. Authors considered this problem as a weighted node packing problem and propose an algorithm based on preprocessing, valid inequalities, and a branch-and-cut approach. The algorithm implemented in the railway stations Arnhem, Hoorn and Utrecht in the Netherlands.

In train platforming problem literature, several studies have been carried out both strategic and tactical planning level. Carey and Carville (2003) address train platforming problem for large, busy, complex train stations, which are common in Europe. They develop mathematical model, but it is too computationally complex to solve by standard combinatorial search or integer programming methods. Because of this, they propose a scheduling heuristic based on train planner’s manual methods. Sels et. al. (2013) proposed a mixed integer programming model that is implemented as software tool for finding platform and routing plans. All TPP problems for 232 stations are solved in a short time. In follow-up study, Sels et al. (2014) handled the train platforming problem on tactical and strategic level for ten different stations in Belgium. This paper also took into consideration some factors together, i.e., infrastructure, desired departure and arrival times, buffer times, routing durations.

Publications that concentrate on train platforming problem more frequently focused on tactical planning level. The basic versions of the problem considered by Cardillo and Mione (1998) and Billionet (2003). Cardillo and Mione (1998) developed an efficient meta-heuristic algorithm based on graph coloring. Proposed algorithm is applied to train platforming problem for six stations with different number of trains and platform tracks. Billionet(2003) deals with assigning of trains to the platform tracks by using integer programming software. The computational results reported that train platforming problem with 200 trains and 14 tracks can be easily solved with integer-programming technique. Carey(1994a) points out the model to a more general network, introducing explicit choices among multiple lines in each direction between stations (or other points), choice of platforms to use at stations, shared platforms, various types of intersections, trains in one or both directions, etc. They extend model and algorithms from the previous paper for more complex rail context. Carey and Lockwood (1995) present model, algorithms and strategies for the train pathing and timetabling problem for rail lines in Britain and Europe. In follow-up paper, Carey (1994b) extended this paper to allow for choice of lines, station platforms, routes, etc. It is assumed that all trains on the line are travelling in the same direction. Ghoseiri et al. (2004) propose a multi-objective optimization model for train scheduling problem which include scheduling of railroad network with single and multiple tracks and railway stations with multiple platforms. To solve the problem, a method with two steps developed. First one, Pareto frontier, is based on ε constraint method. With the help of solution of Pareto frontier, second one solved the problem using distance based method with three types of distances. Rodriguez (2007) regards routing and scheduling of trains in point of railway junctions. He presents a constraint programming model and it is implemented in real case study of traffic on the Pierrefitte-Gonesse node, North of Paris. Efficient solutions were obtained in reasonable time. Galli (2009) dealt with operational scheduling of railway station in her PhD thesis and addressed two important scheduling problems: train platforming and rolling stock planning. A general formulation of integer programming model is illustrated to solve TPP. An exact algorithm based on a branch-and-cut-and-price is presented due to the fact that the number of variables and constraints is too large. An efficient method is proposed to linearize objective function. Caprara (2010) present general formulation of train platforming problem that is case study from the Italian Infrastructure manager. They formulate a general quadratic function which is linearize by using a small number of new variables and a set of constraints. They claim that the new heuristic based on this relaxation produces better solutions than a simple heuristic currently in use and solutions are generally (nearly-)optimal. Krempl (2012) suggested a linear mathematical model for obtaining an efficient track occupancy plan. The objective of the model is to minimize total occupation time in the railway station. Janosikova and Krempl (2014) suggested a bi-criterion mathematical model to determine the platforms to serve. First objective is to minimize the deviations from arrival and departure times stated by the timetable. The second one is to maximize the desirability of the platform for the train assignment. In order to solve the model, lexicographic approach and local branching algorithm was performed. Bai et al. (2014) points out routing and scheduling trains to generate conflict free schedules. They compare continuous time and discrete time models. This study proposes to extend their earlier study given in Bai et al. (2013), which defines and formulates the problem as integer programming model. Sels et al. (2016) proposed a tool called Leopard, a platforming and routing model, is implemented in any station in Belgium and it produces platforming plans quickly.

In recent years, there has been an increasing amount of study on operational planning level in train platforming problems. The work of Chakroborty and Vikram (2007) took into consideration the train platforming problem with three special features: delays, allocation of non-preferred platforms and last minute reassignment of platforms. The scheduling period is divided into two hours' phase. They presented a linear mixed integer programing formulation for a busy railway station in India. Optimal results are obtained for problem with up to 110 trains and 9 platforms. Janosikova et al. (2014) developed a multi-objective mathematical model that deals with assignment plan of arriving trains and waiting duration for connecting trains. In addition, the proposed mathematical model has same specific criteria which are highlighted in Bazant and Kavicka (2009). performed two-layered artificial neural network for solving assignment problem of platform tracks to delayed arriving trains. To obtain most suitable solutions, authors took into consideration specific criteria which include technical and technological preferences.

In this dissertation, it is focused on train platforming problem while satisfying operational constraints of Prague main railway station. The objective of problem is to minimize total weighted delay with specific constraints belonging to Prague main railway station. Assignment of train to the platform tracks is handled with two different methods: mixed integer mathematical model and a matheuristic algorithm.

As mentioned above, a lot of studies in the literature have drawn attention to train platforming problem. Especially, doctoral works of Galli and Bai are two comprehensive studies in the literature. Galli deals with not only train platforming problem but also other railway optimization problems. The aim of dissertation is investigation of performance of combinatorial and robust optimization models to solve railway problems. Another major study is presented in dissertation work of Bai. In a similar manner with our dissertation, the author focus on train platforming problem for busy and complex stations to generate conflict free assignment plan. The objective is minimization number of cancelled trains and they report successful solution for one day timetable. However, number of trains in the concerned timetable is approximately 250 and in this dissertation, with almost 700 trains, number of trains for a full day timetable is considerably more than dissertation work of Bai. Overall, no relevant studies are detailedly modelled and analyzed train platforming problem like in this study. To best of our knowledge, there is no study that proposed matheuristic algorithm from the point view of solution methods.

2 DISSERTATION OBJECTIVES

The dissertation focusses on assignment of arrival and departure trains in busy railway stations. The track assignment plan in railway stations is designed by railway planner in tactical level. Scheduling of the stations that include several platforms and a few trains is done by hand using dispatcher's experience and a set of rules determined by a railway company. However, it may be significant problem at complex stations. The main goal of this dissertation is to develop an efficient approach that would serve as a dispatcher's decision supporting tool and results in a better platform track assignment plan for large stations. The objectives of the dissertation can be listed as follows:

1. *Analysing train platforming problem*: Train platforming problem is certainly difficult problem for busy and complex stations. Before proposing effective methods, it should be comprehensively analysed. To avoid the conflicts, the reason of waiting of trains should be determined. Waiting at home signal and waiting at platform track should be taken into account separately. In addition to this, position of trains which are allocated to same track should be considered since capacity of some platform tracks is two trains.

2. *Mathematical programming formulation*: A mixed integer programming model mathematical model is developed for obtaining optimal solution of train platforming problem. Objective of the mathematical model are to minimize the total delay of incoming and outgoing trains. The model is implemented in GAMS/Cplex solver.

3. *Proposing a matheuristic algorithm*: Large station deals with approximately five or six hundred trains per day. Due to the Np-hard nature, it would be hard to find a feasible solution in a reasonable time by using mathematical model for large stations. Consequently, an efficient matheuristic algorithm could be performed and best solutions can be obtained.

4. *Validation:* In order to run the proposed mathematical model and matheuristic algorithm properly, validation process is necessary. Validation of mathematical model and matheuristic algorithm is achieved by one day timetable that belongs to Prague main railway station. Matheuristic algorithm is also implemented in GAMS/Cplex solver. To show the performance of proposed matheuristic, results of mathematical model is compared with results of matheuristic algorithm.

3 PROBLEM STATEMENT

In this chapter, firstly we represent basic definitions of train platforming problem and then in the following section, general description of our case study is given in details.

3.1 Basic Definitions

The major components of train platforming problem consist of trains, platform tracks and railway junctions. There is a set N of trains that arrive to the station and/or departure from the station and a collection L of platform tracks. Furthermore, a set of R includes railway junctions between all platform tracks and home/exit signals. To better understand description of the problem, some terms are explained with the help of an example of railway station topology which is illustrated in Figure 3.1.

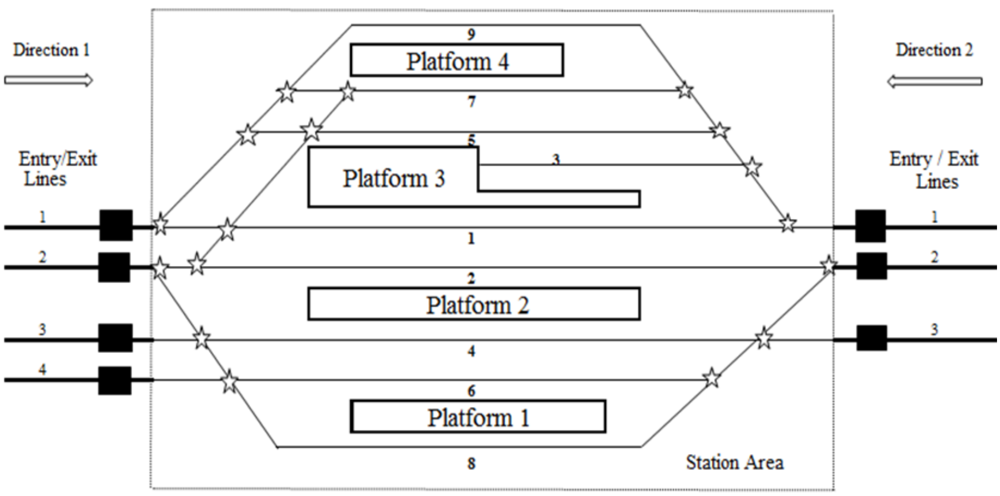

Figure 3-1 An example of railway station topology

Trains are basic components of problem. The set of trains is denoted by N= {1, 2, ... n} and subscript j refers to a train. Each train $j, q, k \in N$ can be classified according to what they carry, can be divided into two main categories: passenger trains and freight trains. Another classification about train types can be made on basis of dwell time at the considered station. First one is *transit train*. Train j arrives at the railway station and completes the boarding. Then, it departs. Some of the trains can be *connecting trains* which passenger use for changing. *Originating train* defines as a train that pulled out of the station where it starts trip. *Non-stop train* refers to train which does not stop for boarding on mentioned station and continues its trip. Cargo trains can be categorized as non-stop trains.

Platforms are several points at which a train stops within the station for boarding; these points are called *platforms* and can be different type and length. *Platform tracks* may be described as corridors between platforms in station area. The set of platform tracks is defined by $L = \{1, 2, ... pt\}$ and subscript $l, m \in L$ refers to a platform track. There are two basic kinds of platform tracks: regular platform tracks and dead-end platform tracks. Regular platform tracks are easily accessible from both direction. However, it is not possible to reach dead-end platform tracks in both directions.

Lines are used by a train for entering to the station or depart from station. A railway station can be entered by a train at a number of entry lines, and it can be left through a number of exit lines. Usually, each entry line can also serve as an exit line, and vice versa. Furthermore, each of these lines corresponds to a direction of travel(arrival/departure) and these are marked in bold in Figure 3.1. The figure shows that there are seven entry/exit lines from/to a station and four platforms.

Main Signals control the train movements and shunting movements. It also puts maximal speed for this movement. There are several types of main signals according to their purpose. *Home signal* is placed to the entry of station and controls train arrivals to the station. *Exit signal* is a main signal which is put at the exit from station. It may be classified depending on where they are placed into group exit signal and common exit signal. Group signal is for group of tracks at the station. Common signal is for whole station.

Railway junction is a place at which two or more tracks are connected. In the point of railway infrastructure, junctions are of two types: switches and crossing points. (1) Switches are elements for allowing train pass from one track to another. (2) Crossing points are intersection of two tracks. Railway junctions are common resource for trains. However, only one train operates a railway junction at a time.

3.2 Considered Problem

In this part of dissertation, a description of the operational processes within railway station and its specific topology are given. The characteristics of these belong to the railway system in the Prague, Czech Republic and it is explained as follows:

- **Train Length**

To decide of assignment of a train, train length and platform length should be seriously considered. Train length must be smaller than length of platform. Therefore, capacity of platform track can be changed based on length of train *j* and its platform length. For instances, it is assumed that all international trains are long. In case one of these kind of trains assigns a platform track, it is not possible to encounter another train at this platform track.

- **Capacity of Platform Tracks**

In the busy and complex railway stations, platform tracks can be divided into two parts – southern and northern – for possibility of manipulation (boarding and alighting of passengers) of two trains at one platform edge$(cap_l = 1)$. This division is realized by route signals placed almost in the middle of platforms. However, some kinds of platforms are designed as dead-end platform tracks and the capacity of them is one train$(cap_l = 0)$.

- **Eligibility Restrictions**

There are three significant considerations about assignment of platform tracks to the trains in railway stations. (1) All tracks cannot be connected to all entry/exit lines. With regard to this, trains are only assigned to a platform track which is satisfied for these conditions. For example, in Figure 3.1, it is not possible to assign a train that comes from line 1 in direction 2 to the platform track 8. (2) Train length should be seriously considered for decision of assignment of a train. Train length must be smaller than length of platform. (3) In railway station, dead-end platform tracks only serve to the particular trains that its arrival and departure direction are same. In Figure 3.1, platform track 3 is only allocated to the trains which arrive from direction 2 and depart to this direction. These limitations can be called as eligibility restrictions.

- **Coupled Trains**

If train j and train q are occupied the same platform track at the same time$(cap_l = 1)$, these are called as "*coupled trains*" in our context. When one of the couples left and available capacity of track is one, other one can share the platform track with other trains multiple times at the different time intervals. This implies that a train has couples more than ones. However, it should be noticed that directions of coupled trains are seriously critical. There are eight possible combination directions of train j and train q and shown in Figure 3.2.

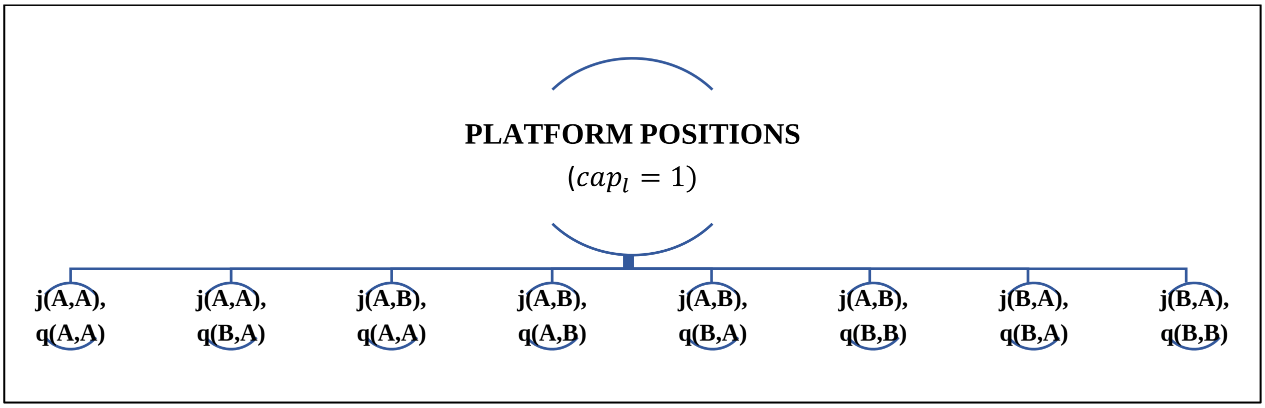

Figure 3-2 Combination directions of train *j* and train *q*

- *Case 1: j (A, A), q (A, A)*

Case 1 are based on three situations: (1) Arrival and departure directions of train *j* and *q* are direction *A*, (2) They encounter at the platform track, (3) Train *j* arrived firstly. In this situation, there is no chance that train *q* has a couple. However, train *j* has coupled more than one trains that arrival and departure directions are *A*. In left side of Figure 3.3, Case *1* is illustrated. All directions are from/through *B* so train *j* has coupled the train(s) which arrives from direction *B.* In left side of Figure 3.3, it is illustrated.

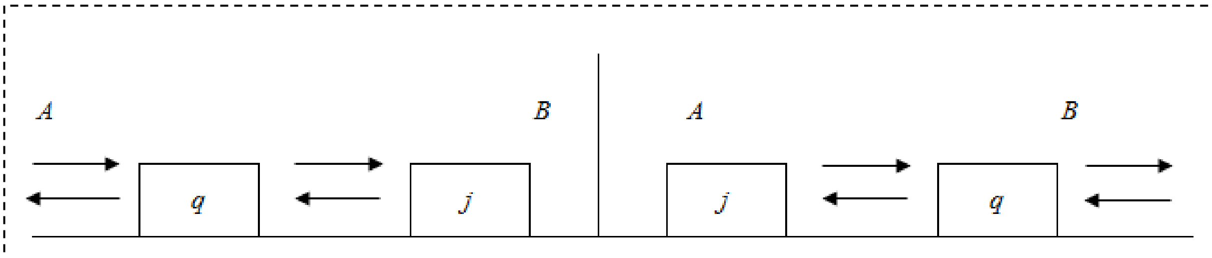

Figure 3-3 Illustration of Case 1

- *Case 2: j (A, A), q (B, A)*

Case 2 refers that arrival and departure directions of train *j* is *A* and train *q* come from direction *B* and depart through direction *A*. If trains share the same platform track, only train *q* can be coupled another train *k* on condition that direction of this train should be *A*. In Figure 3.4, Case 2 is illustrated.

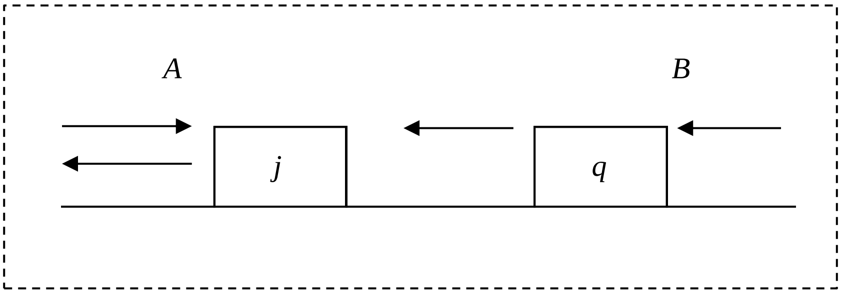

Figure 3-4 Illustration of Case 2

- *Case 3: j (A, B), q (A, A)*

In Case 3, train *j* and *q* are coupled if and only if train *j* arrived first. Otherwise, it is impossible to encounter at the platform track. Both trains can be also coupled by other trains, when one of them departs. In case of departure of train *j*, train *q* can share the track with a train which is coming from direction *B*. Train *j* can be coupled the train *k* on condition that train *q* departs and arrival direction of train *k* is *A*. Position of trains in Case 3 are set out in Figure 3.5.

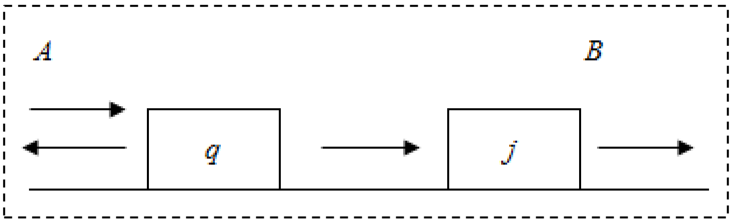

Figure 3-5 Position of Trains for Case 3

- *Case 4: j (A, B), q (A, B)*

Case 4 states that trains come from direction *A* and depart through direction *B*. Both trains can have couples no matter which train arrives before. The significant point is that arrival direction of train *k* should be direction *A*. This situation is illustrated in Figure 3.6.

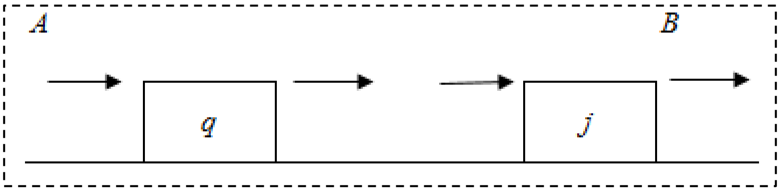

Figure 3-6 Position of Trains for Case 4

○ *Case 5: j (A, B), q (B, A)*

In *Case 5*, it is not possible to share same platform track because their arrival and departure directions are opposite. So, it can be said that train *j* and *q* cannot be coupled.

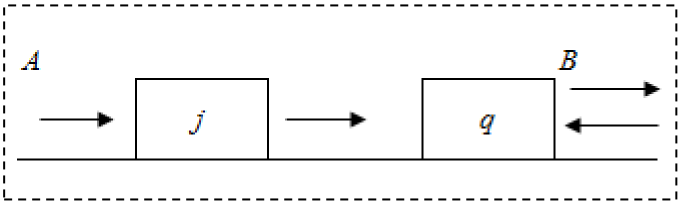

Figure 3-7 Illustration of Case 6

○ *Case 6: j (A, B), q (B, B)*

When train *j* and *q* departure through the direction *B* and one of them arrive from direction *A* and they also encounter platform track, only train *j* can be coupled another train *k* on condition that direction of this train should be *B*. Case 6 are shown in Figure 3.7.

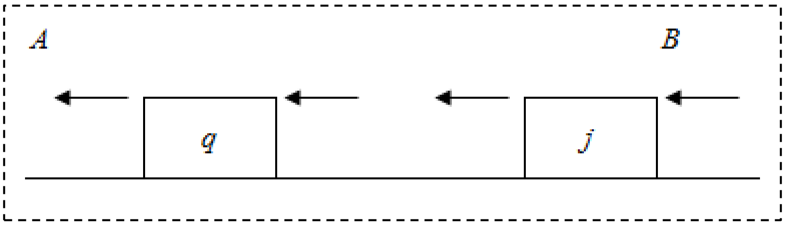

Figure 3-8 Illustration of Case 7

○ *Case 7: j (B, A), q (B, A)*

Case 7 refers that trains come from direction *B* and depart through direction *A*. Both trains can have couples no matter which train arrives before as Case 4. However, arrival direction of train *k* should be direction *B*. It is set out in Figure 3.8.

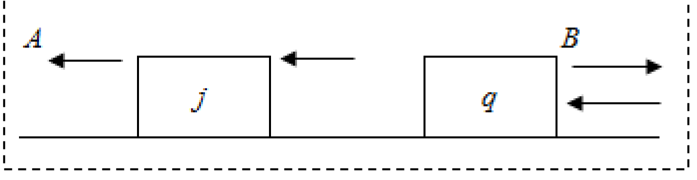

Figure 3-9 Illustration of Case 8

- *Case 8: j (B, A), q (B, B)*

In Case 8, train *j* should arrive firstly. Otherwise, they do not share same platform track at the same time. Train *j* also can have couple trains that arrives from direction *B*. In Figure 3.9, Case 8 is illustrated.

Time Intervals

In railway systems, safety rules are ensured by time between trains. These time intervals are given as follows:

Headway Interval: It is expressed as the time interval between train *j* and train *q* that are running along the same line and denoted by "*ht*".

Platform Interval: It is defined as necessary minimum time for arriving of train *j* after platform track is released by train *q* and this parameter denoted by "*pt*".

Switch Interval: It is described as necessary minimum time for reutilization after railway junction is released and denoted by "*st*".

4 MATHEMATICAL MODEL

Train platforming problem completely depends on topology of station infrastructure and on timetable. In this study case, these parameters belong to the Prague main railway station. The objective of the problem is to minimize total weighted delay of trains. It is based on the following assumptions:

- All platform tracks are available for the scheduling horizon.
- Shunting operations are neglected.
- All parameters are deterministic.
- Arrival times of trains are based on timetable.
- All time data are shown in minutes.

4.1 Notations

The following sets, indices, parameters and decision variables will be used in the mathematical model:

Sets:

$N=\{1, 2,\ldots, n\}$ trains set

$L=\{1, 2,\ldots, pt\}$platform tracks set

$R=\{1, 2,\ldots, g\}$ railway junctions set

$D=\{A, B\}$ directions set

Indices:

$j, q, k \in N$ are indices used to show a particular train.

$l, m \in L$ are indices used to show a particular platform track.

$r, s \in R$ is indices used to show a particular railway junction.

Parameters:

n: number of trains

pt: number of platform tracks

g : number of railway junctions

p_j : sojourn time of train j

rs_j : scheduled arrival time of train j to the home signals

d_j : scheduled departure time of train j

h_j: duration of train j arrival from home signals to platform tracks

sp_j: duration of train j departure from platform to out of station

wt_j : importance degree of train j

MK: a large positive number

gA_j: arrival line of train j, if it arrives to the platform track from direction A.

gB_j: arrival line of train j, if it arrives to the platform track from direction B.

bA_j: departure line of train j, if it departures to the platform track from direction A.

bB_j: departure line of train j, if it departures to the platform track from direction B.

$delay_j$: $\begin{cases} 1, & \text{if train } j \text{ terminates at considered station} \\ 0, & \text{otherwise} \end{cases}$

int_j: $\begin{cases} 1, & \text{if train } j \text{ is international train} \\ 0, & \text{otherwise} \end{cases}$

cap_l: $\begin{cases} 1, & \text{if capacity of platform track } l \text{ is two trains} \\ 0, & \text{otherwise} \end{cases}$

$res1_{jlr}$: $\begin{cases} 1, & \text{if train } j \text{ uses railway junction } r \text{ to arrive to the platform track } l \\ 0, & \text{otherwise} \end{cases}$

$res2_{jlr}$: $\begin{cases} 1, & \text{if train } j \text{ uses railway junction } r \text{ to leave from the platform track } l \\ 0, & \text{otherwise} \end{cases}$

b_{jl} : $\begin{cases} 1, & \text{if train } j \text{ can be assigned to the platform track } l \\ 0, & \text{otherwise} \end{cases}$

st: necessary minimum time for reutilization after railway junction is released

pt: necessary minimum time for arriving of next train after platform track is released

ht: necessary minimum time between two trains that are running along the same line

α: a small number

Decision variables:

C_j: arrival time of train j to the exit signals

x_{jl} : $\begin{cases} 1, & \text{if train } j \text{ is assigned to the platform track } l, \\ 0, & \text{otherwise} \end{cases}$

$e1_{jq}$, $e2_{jq}$, $e3_{jq}$, $e4_{jq}$, $e5_{jq}$, $e6_{jq}$, $e7_{jq}$, $e8_{jq}$, $e9_{jq}$ $e10_{jq}$, $e11_{jq}$, $e12_{jq}$, $e13_{jq}$, $e14_{jq}$, $e15_{jq}$, $e16_{jq}$, $e17_{jq}$, $e18_{jq}$, $e19_{jq}$, $e20_{jq}$, $e21_{jq}$, $e22_{jq}$, $e23_{jq}$, $e24_{jq}$, $e25_{jq}$, $e26_{jq}$, $e27_{jq}$, $e28_{jq}$, $e29_{jq}$, $e30_{jq}$: 0-1 integer variables for ensuring particular constraints

$w1_j$: waiting time of train j at home signals

$w2_j$: waiting time of train j at platform track

T_j : delay of train j

a_j : departure time of train j from home signals to the platform track

v_j : arrival time of train j to the platform track

z_j : departure time of train j from platform track

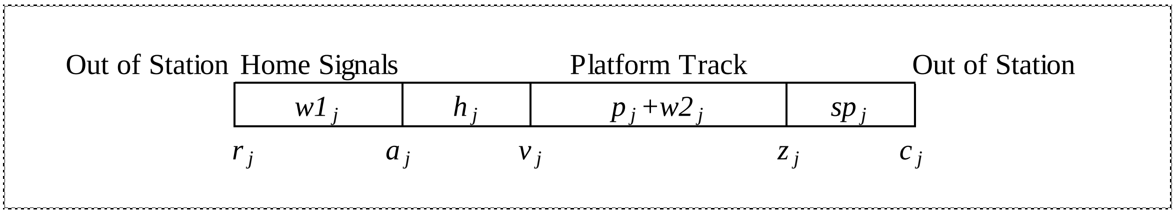

Figure 4-1 Time schedule for train j

To understand better, decision variables and parameters of train $j \in N$ are depicted in time schedule in Figure 4.1. The rs_j is defined as arrival of train $j \in N$ to the home signals. Departure time of train j through platform track is associated to waiting time at home signals $w1_j$ and rs_j. The h_j is travel time of train $j \in N$ between home signals and platform track. The sum of rs_j , $w1_j$ and h_j is equal to actual arrival time to the platform track (v_j). Train j should stay on the platform track at least boarding time. To avoid conflicts, train j can wait in platform track l and waiting time is expressed by $w2_j$. The sum of v_j , p_j and $w2_j$ is equal to actual departure time z_j of train $j \in N$. The sp_j is travel time of train $j \in N$ between platform track and exit signals. Arrival time of train $j \in N$ to the exit signals is denoted by c_j that is equal to sum of z_j and sp_j.

4.2 Objective Function

The objective function (4.1) is to minimize sum of total weighted delay of all trains. First part of equation represents minimizing total weighted delays of originating and transit trains that are strictly not allowed to wait not only at the home signal, but also at the platform track. Second and last part of equation are formulated for terminating trains. When such trains wait at the home signal, delay occurs. On the other hand, if it waits at the platform track, it should depart as soon as possible but there is no delay in this situation.

$$Min\ Z = \sum_{j=1, delay_j=0}^{n} wt_j * T_j + \sum_{j=1, delay_j=1}^{n} wt_j * w1_j + \alpha * \sum_{j=1, delay_j=1}^{n} wt_j * w2_j \quad (4.1)$$

4.3 Constraints

The constraints of the presented mixed integer mathematical model are explained in detail in the following subsections.

4.3.1 Calculation of Decision Variables

$$a_j = rs_j + w1_j \qquad \forall j\ , j \in N \tag{4.2}$$

$$v_j = a_j + h_j \qquad \forall j, j \in N \tag{4.3}$$

$$z_j = v_j + p_j + w2_j \qquad \forall j, j \in N \tag{4.4}$$

$$c_j = z_j + \ sp_j \qquad \forall j, j \in N \tag{4.5}$$

All the movements of the trains in the railway station can be calculated in a classical way as above. Constraint (4.2) indicates that departure time of a train from home signals towards to the platform should be equal to the sum of arrival time of train to the home signals and waiting time at the home signals. Constraints (4.3), (4.4) and (4.5) are formulated for actual arrival time of train j to the platform track, actual departure time of train j from platform track and arrival time of train j to out of station.

4.3.2 Assignment Constraint

$$\sum_{l=1}^{s} x_{jl} = 1 \qquad \forall j, j \in N \tag{4.6}$$

Constraint (4.6) ensures that all trains should be assigned to platform track.

4.3.3 Platform Eligibility Restrictions

$$b_{jl} \geq x_{jl} \qquad \forall j, l \quad j \in N \quad l \in L \tag{4.7}$$

Constraint (4.7) is formed in order to ensure that trains are only assigned to a platform which is available from the point view of technical conditions.

4.3.4 Encountering Constraints

Mathematical model should ensure that train j and train q satisfy safety conditions. Possible position of train j and q that encounter in direction A couple of constraints are presented in below for possible positions of encountering trains.

✓ **Entry - Entry**

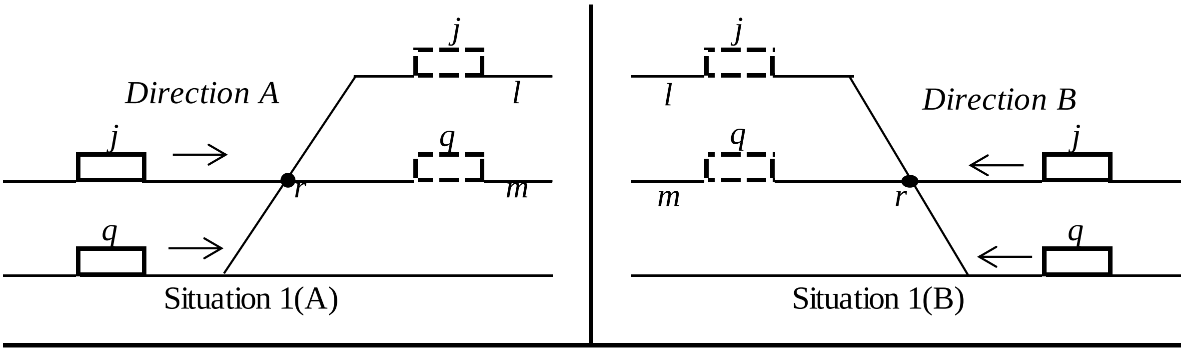

Figure 4-2 Positions of arriving trains that have railway junction(s) in common

In Figure 4.2, possible positions of two arriving trains at the home signals are presented. Situation 1(A) occurs that train *j* and train *q* arrive to the home signal from direction *A* and they are assigned to the different platform tracks with at least one common railway junction. On the right side of Figure 4.2, Situation 1(B) represents the same position of trains in direction *B*. Constraints (4.8) and (4.9) impose that these trains cannot go through arrival path at the same time and one of them should wait for other one.

$$v_q + st \leq a_j + MK * \left(2 - x_{jl} - x_{qm} + e1_{jq}\right) \;\; \forall j,q,l,m,\; j<q,\; l \neq m, (\left(gA_j \neq 0, gA_q \neq 0,\; \textstyle\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0\right) \; or \; (gB_j \neq 0, gB_q \neq 0, \textstyle\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0)) \quad (4.8)$$

$$v_j + st \leq a_q + MK * \left(3 - x_{jl} - x_{qm} - e1_{jq}\right) \;\; \forall j,q,l,m,\; j<q, l \neq m, (\left(gA_j \neq 0, gA_q \neq 0, \textstyle\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0\right) \; or \; (gB_j \neq 0, gB_q \neq 0, \textstyle\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0)) \quad (4.9)$$

✓ **Entry - Exit** *(l≠m)*

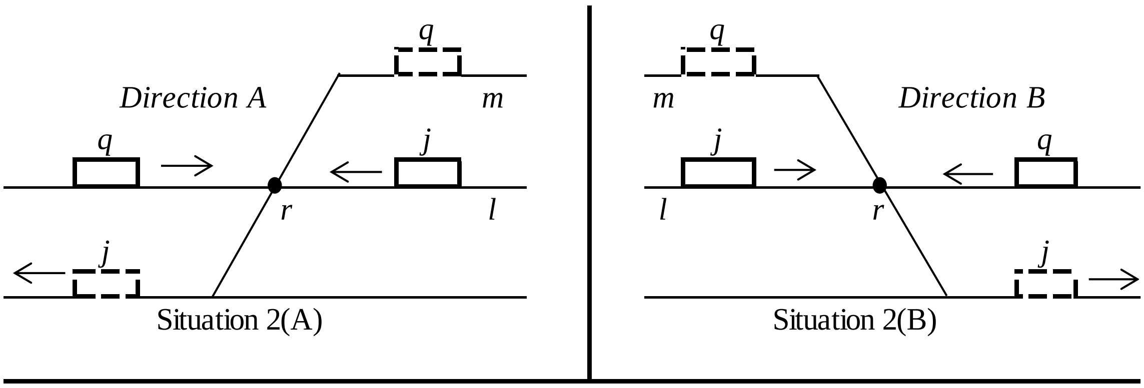

Figure 4-3 Possible positions of two trains that encounter at the arrival path

In Figure 4.3, possible positions of two trains that encounter at the arrival path are shown. Train *q* arrives to the home signal, train *j* departs from station and they are assigned to the platform track *m* and *l*, respectively. Note that one of these trains should wait other one since they pass through railway junction *r* in common. Situation 2(A) shows that trains are arriving from direction *A* and departure from platform towards to the direction *A*. Situation 2(B), as depicted on right side of Figure 4.3, is presented for possible positions of the encountering trains which use shared railway junction on direction *B* of the railway station. Constraints (4.10) - (4.15) are formulated for both situations if arrival and departure directions of two trains are same and they have at least one common railway junction.

$$v_j \leq v_q + MK * \left(2 - x_{jl} - x_{qm} + e2_{jq}\right) \quad \forall j,q,l,m,\ l \neq m,\ j \neq q, ((bA_j \neq 0, gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0, bA_q \neq 0, gA_j \neq 0,\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0)\ or\ (bB_j \neq 0,\ gB_q \neq 0,\ \sum_{r=1}^{g} res2_{jlr} \cdot res1_{qmr} > 0,\ bB_q \neq 0,\ gB_j \neq 0, \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0\ )) \tag{4.10}$$

$$v_q + st \leq z_j + MK * \left(2 - x_{jl} - x_{qm} + e2_{jq} + e3_{jq}\right) \quad \forall j,q,l,m,\ l \neq m,\ j \neq q, ((bA_j \neq 0, gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0, bA_q \neq 0, gA_j \neq 0,\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0)\ or\ (bB_j \neq 0,\ gB_q \neq 0,\ \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ bB_q \neq 0,\ gB_j \neq 0, \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0\ )) \tag{4.11}$$

$$c_j + st \leq a_q + MK * \left(3 - x_{jl} - x_{qm} + e2_{jq} - e3_{jq}\right) \quad \forall j,q,l,m,\ l \neq m,\ j \neq q, ((bA_j \neq 0, gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ bA_q \neq 0,\ gA_j \neq 0,\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0)\ or\ (bB_j \neq 0,\ gB_q \neq 0,\ \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ bB_q \neq 0,\ gB_j \neq 0, \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0\ )) \tag{4.12}$$

$$c_j + st \leq z_q + MK * \left(2 - x_{jl} - x_{qm} + e2_{jq} + e3_{jq} + e31_{jq}\right) \ \forall j,q,l,m,\ l \neq m,\ j \neq q, ((bA_j \neq 0,\ gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ bA_q \neq 0,\ gA_j \neq 0,\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0)\ or\ (bB_j \neq 0,\ gB_q \neq 0,\ \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ bB_q \neq 0,\ gB_j \neq 0, \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0\ )) \tag{4.13}$$

$$c_q + st \leq z_j + MK * \left(3 - x_{jl} - x_{qm} + e2_{jq} + e3_{jq} - e31_{jq}\right) \quad \forall j,q,l,m,\ l \neq m,\ j \neq q, ((bA_j \neq 0,\ gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ bA_q \neq 0,\ gA_j \neq 0,\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0)\ or\ (bB_j \neq 0,\ gB_q \neq 0,\ \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ bB_q \neq 0,\ gB_j \neq 0, \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0\ )) \tag{4.14}$$

$e2_{jq} + e2_{qj} = 1 \quad \forall j,q,l,m,\ \ l \neq m,\ \ j \neq q,\ \ ((bA_j \neq 0,\ \ gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0,\ \ bA_q \neq 0,\ \ gA_j \neq 0,\ \ \sum_{r=1}^{g} res1_{jlr} \cdot res2_{qmr} > 0)\ or\ (bB_j \neq 0,\ \ gB_q \neq 0,\ \sum_{r=1}^{g} res2_{jlr} \cdot res1_{qmr} > 0,\ \ bB_q \neq 0,\ \ gB_j \neq 0, \sum_{r=1}^{g} res1_{jlr} \cdot res2_{qmr} > 0\ )) \quad (4.15)$

In Situation 2(A) and Situation 2(B), if arrival and departure directions of each train are not same or they have not common railway junction when train *j* arrives and train *q* departs, Constraint (4.16) and (4.17) are used.

$v_q + st \leq z_j + MK * \left(2 - x_{jl} - x_{qm} + e4_{jq}\right)\ \forall j,q,l,m,l \neq m,j \neq q,((\ bA_j \neq 0, gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0\ ,((\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} = 0,\ \ gA_j \neq 0,\ \ bA_q \neq 0)\ or\ (gA_j \neq 0, bB_q \neq 0)\ or\ (gB_j \neq 0, bB_q \neq 0)\ or\ (gB_j \neq 0, bA_q \neq 0)))\ or\ (\ \ bB_j \neq 0, gB_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0\ ,\ ((\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} = 0,\ \ gB_j \neq 0,\ \ bB_q \neq 0)\ or\ (gB_j \neq 0, bA_q \neq 0)\ or\ (gA_j \neq 0, bA_q \neq 0)\ or\ (gA_j \neq 0, bB_q \neq 0)))) \quad (4.16)$

$c_j + st \leq a_q + MK * \left(3 - x_{jl} - x_{qm} - e4_{jq}\right) \quad \forall j,q,l,m,\ \ l \neq m,\ \ j \neq q,((\ bA_j \neq 0, gA_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0\ ,\ ((\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} = 0,\ \ gA_j \neq 0,\ \ bA_q \neq 0)\ or\ (gA_j \neq 0, bB_q \neq 0)\ or\ (gB_j \neq 0, bB_q \neq 0)\ or\ (gB_j \neq 0, bA_q \neq 0)))\ or\ (\ \ bB_j \neq 0, gB_q \neq 0, \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0\ ,\ ((\ \sum_{r=1}^{g} res1_{jlr} * res2_{qmr} = 0,\ \ gB_j \neq 0,\ \ bB_q \neq 0)\ or\ (gB_j \neq 0, bA_q \neq 0)\ or\ (gA_j \neq 0, bA_q \neq 0)\ or\ (gA_j \neq 0, bB_q \neq 0)))) \quad (4.17)$

✓ **Entry - Exit** *(l=m)*

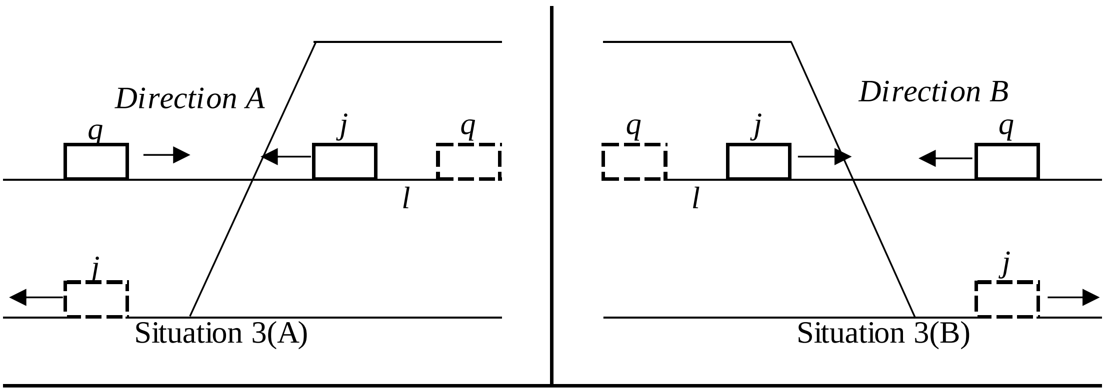

Figure 4-4 Possible positions of encountering trains that are assigned to the platform *l*

In Figure 4.4, Situation 3(A) shows that arrival direction of train *q* and departure direction of train *j* are *A* and they are assigned to the same platform track *l*. To prevent overlapping of these trains at the platform and arrival path, constraints (4.18) -(4.21) are written. Especially, constraints (4.18) and (4.19) indicate that arrival and departure directions of both of trains are *A*. If at least one of the directions of trains is different from *A*, this is expressed by constraints (4.20) and (4.21). Similarly, the constraints (4.18) - (4.21) are also valid for Situation 3(B).

$$c_j + st \leq a_q + MK \cdot \left(2 - x_{jl} - x_{ql} + e5_{jq}\right) \ \forall j, q, l, j < q, cap_l = 0, ((bA_j \neq 0, gA_q \neq 0, bA_q \neq 0, gA_j \neq 0) \ or \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0)) \tag{4.18}$$

$$c_q + st \leq a_j + MK \cdot \left(3 - x_{jl} - x_{ql} - e5_{jq}\right) \ \forall j, q, l, j < q, cap_l = 0, ((bA_j \neq 0, gA_q \neq 0, bA_q \neq 0, gA_j \neq 0) \ or \ (bB_j \neq 0, gB_q \neq 0, \ bB_q \neq 0, gB_j \neq 0)) \tag{4.19}$$

$$c_j + st \leq a_q + MK \cdot \left(2 - x_{jl} - x_{ql} + e6_{jq}\right) \ \ \forall j, q, l, cap_l = 0, \ j \neq q, ((bA_j \neq 0, gA_q \neq 0, \left(\left(gA_j \neq 0, \ bB_q \neq 0\right) or \left(gB_j \neq 0, bB_q \neq 0\right) or \left(gB_j \neq 0, bA_q \neq 0\right)\right)) \ or \ (bB_j \neq 0, gB_q \neq 0, \left(\left(gB_j \neq 0, \ bA_q \neq 0\right) or \left(gA_j \neq 0, bA_q \neq 0\right) or \left(gA_j \neq 0, bB_q \neq 0\right)\right))) \tag{4.20}$$

$$z_q + pt \leq v_j + MK \cdot \left(3 - x_{jl} - x_{ql} - e6_{jq}\right) \ \ \forall j, q, l, cap_l = 0, \ j \neq q, ((bA_j \neq 0, gA_q \neq 0, \left(\left(gA_j \neq 0, \ bB_q \neq 0\right) or \left(gB_j \neq 0, bB_q \neq 0\right) or \left(gB_j \neq 0, bA_q \neq 0\right)\right)) or \ (bB_j \neq 0, gB_q \neq 0, \left(\left(gB_j \neq 0, \ bA_q \neq 0\right) or \left(gA_j \neq 0, bA_q \neq 0\right) or \left(gA_j \neq 0, bB_q \neq 0\right)\right))) \tag{4.21}$$

- ✓ **Exit-Exit**($bB_j = bBq \ or \ bA_j = bA_q$)

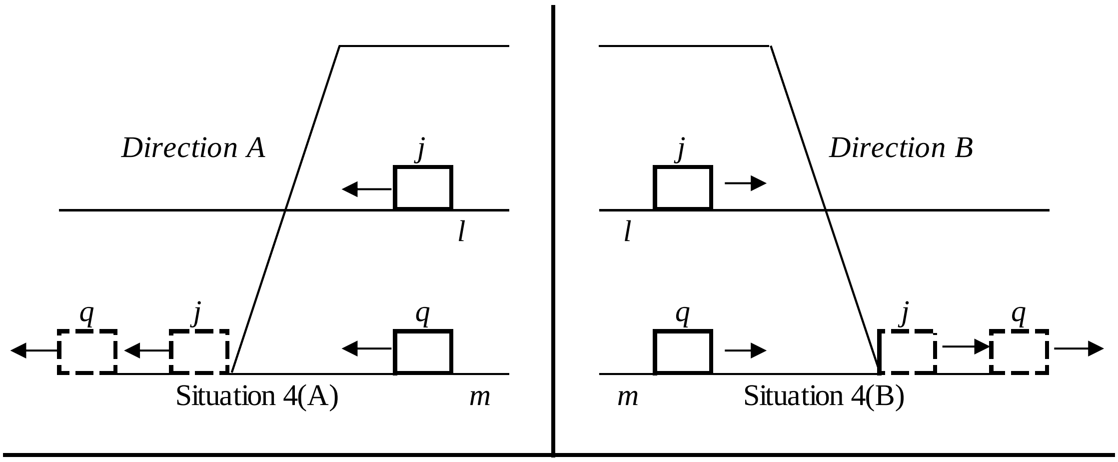

Figure 4-5 Possible positions of trains which departure from platform tracks in same line

In Figure 4.5, each situation shows that train *j* and train *q* will depart from platform tracks in same line. There must be an additional time that is called *headway interval* parameter for safety reasons. Technically, it is related to way of line segment blocking by applied line interlocking system and naturally also to train speeds, how long the train will occupy line segment. Constraints (4.22) and (4.23) are formulated for providing headway interval between two trains.

$c_j + ht \leq c_q + MK * \left(e18_{jq} \right)\ \forall j,q,j < q,\ ((bA_j \neq 0,\ bA_q \neq 0,\ bA_j = bA_q) \ or\ (bB_j \neq 0,\ bB_q \neq 0,\ bB_j = bB_q))$ (4.22)

$c_q + ht \leq c_j + MK * \left(1 - e18_{jq} \right)\ \forall j,q,j < q, ((bA_j \neq 0,\ bA_q \neq 0, bA_j = bA_q)\ or\ (bB_j \neq 0,\ bB_q \neq 0,\ bB_j = bB_q))$ (4.23)

- ✓ **Exit - Exit**$(bB_j \neq bBq\ or\ bA_j \neq bA_q)$

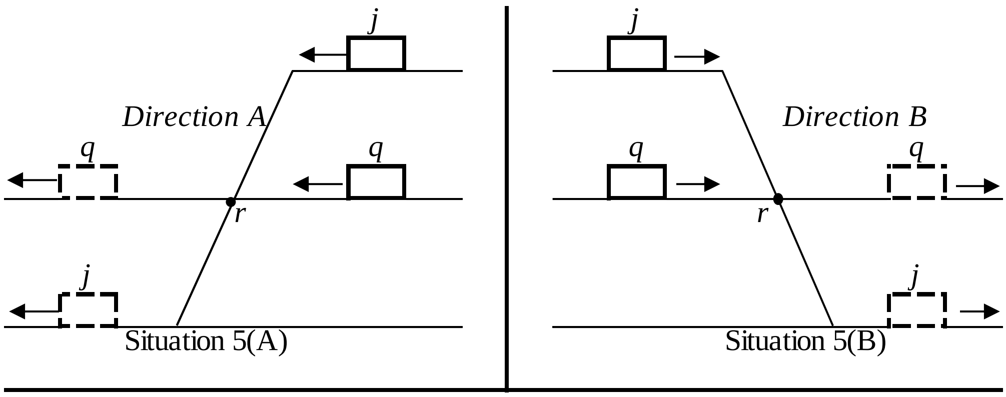

Figure 4-6 Possible positions of trains which will depart from platform tracks to different lines

Train *j* and train *q* that departure from different platform tracks to different lines towards to the same direction are set out in Figure 4.6. Constraints (4.24) and (4.25) are concentrated on Situation 5(A) and Situation 5(B). In each direction, for every couple of trains train *j* and *q* these constraints say that if *j* departs before *q*, and if *j* and *q* are leaving from the different line and they have at least one common railway junction, then train *q* should wait a time that is equal to sum of the arrive time of train *j* to the exit signals and railway junction interval.

$c_j + st \leq z_q + MK * \left(2 - x_{jl} - x_{qm} + e18_{jq}\right)\ \forall j,q,l,m,l \neq m,j < q, (\left(bA_j \neq 0, bA_q \neq 0, bA_j \neq bA_q, \sum_{r=1}^{g} res2_{jlr} * res2_{qmr} > 0 \right) or\ \ (\sum_{r=1}^{g} res2_{jlr} * res2_{qmr} > 0, bB_q \neq bB_j, bB_j \neq 0, bB_q \neq 0))$ (4.24)

$c_q + st \leq z_j + MK * \left(3 - x_{jl} - x_{qm} - e18_{jq}\right)\ \forall j,q,l,m,l \neq m,j < q, (\left(bA_j \neq 0, bA_q \neq 0, bA_j \neq bA_q, \sum_{r=1}^{g} res2_{jlr} * res2_{qmr} > 0 \right) or\ \ (\sum_{r=1}^{g} res2_{jlr} * res2_{qmr} > 0, \mathrm{b}B_q \neq bB_j, bB_j \neq 0, bB_q \neq 0))$ (4.25)

4.3.5 Platform Position Constraints

In busy and complex railway stations, platform position constraints should be formulated based on capacity of platform track. If the platform track is divided into two parts, it can be said that capacity of platform track is two trains($cap_l = 1$). If not or it is dead end platform track, capacity is one train($cap_l = 0$).

- ***One platform -One train (cap_l=0)***

$$z_j + pt \le v_q + MK * \left(2 - x_{jl} - x_{ql} + e19_{jq}\right) \;\forall j, q, l, j < q \tag{4.26}$$

$$z_q + pt \le v_j + MK * \left(3 - x_{jl} - x_{ql} - e19_{jq}\right) \;\forall j, q, l, j < q \tag{4.27}$$

Constraints (4.26) and (4.27) say that a platform track($cap_l = 0$) cannot be assigned to more than one train at the same time. In addition, there must be a platform interval between train *j* and *q* that are assigned to the same platform track.

- ***One platform -Two trains (cap_l=1)***

To formulate one platform-two train constraints, it should be noted that arrival and departure directions of trains are taken seriously into consideration. There are eight different combination directions of train *j* and *q* and set out in Figure 3.3. Based on eight different cases that are declared previous pages, constraints are formulated.

✓ ***Case 1: j (A, A), q (A, A) / j (B, B), q (B, B)***

If arrival and departure directions of train *j* and *q* are same (Direction *A/B*) and they are assigned to same platform track, following constraints (4.28-4.51) are represented. In Figure 4.7, modelling steps of Case 1 are illustrated.

Case 1-Part I $(\boldsymbol{v_j \le v_q})$

$$v_j \le v_q + MK * \left(2 - x_{jl} - x_{ql} + e7_{jq}\right) \;\forall j, q, l, j < q, cap_l = 1, ((bA_j \ne 0, bA_q \ne 0, gA_q \ne 0, gA_j \ne 0) \;\; or \;\; (bB_j \ne 0, gB_q \ne 0, bB_q \ne 0, gB_j \ne 0)) \tag{4.28}$$

$$v_q \le z_j + MK * \left(2 - x_{jl} - x_{ql} + e7_{jq} + e21_{jq}\right) \;\forall j, q, l, j < q, cap_l = 1, ((bA_j \ne 0, bA_q \ne 0, gA_q \ne 0, gA_j \ne 0) \;\; or \;\; (bB_j \ne 0, gB_q \ne 0, bB_q \ne 0, gB_j \ne 0)) \tag{4.29}$$

$$c_q + st \le z_j + MK * \left(2 - x_{jl} - x_{ql} + e7_{jq} + e21_{jq}\right) \;\forall j, q, l, j < q, cap_l = 1, ((bA_j \ne 0, bA_q \ne 0, gA_q \ne 0, gA_j \ne 0, bA_j \ne bA_q) \;\; or \;\; (bB_j \ne 0, gB_q \ne 0, bB_q \ne 0, gB_j \ne 0, bB_j \ne bB_q)) \tag{4.30}$$

$$c_q + ht \leq c_j + MK * \left(2 - x_{jl} - x_{ql} + e7_{jq} + e21_{jq}\right)\ \forall j,q,l,j < q, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0\ , bA_j = bA_q)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0,\ bB_j = bB_q\ )) \quad (4.31)$$

$$c_j + st \leq a_q + MK * \left(3 - x_{jl} - x_{ql} + e7_{jq} - e21_{jq}\right)\ \forall j,q,l,j < q, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0\ )) \quad (4.32)$$

Platform position constraints should be written based on arrival sequence of trains to the platform track. When train *j* arrives at the platform track before train *q*, there are two possibilities: (1) train *j* and *q* occupy platform track *l* at the same time. In this situation, train *j* cannot departure before train *q* leaves and waiting time of train *j* depends on whether external lines of trains are same or not. (2) Another possibility is that train *j* has already left before train *q* arrived. These situations are expressed by the Constraints (4.28) -(4.32).

$$v_q \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e7_{jq} + e21_{jq} + e8_{qk}\right)\ \forall j,q,l,k,\ j < q, j \neq k, q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0)) \quad (4.33)$$

$$c_q + st \leq a_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e7_{jq} + e21_{jq} + e8_{qk}\right)\ \forall j,q,l,k,\ j < q, j \neq k, q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gA_k \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gB_k \neq 0)) \quad (4.34)$$

$$z_j \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e7_{jq} + e21_{jq} + e8_{qk}\right)\ \forall j,q,l,k,\ j < q, j \neq k, q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gB_k \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gA_k \neq 0)) \quad (4.35)$$

If train *j* and *q* are coupled at the platform track and train *k* also assigned to the platform track *l*, it is not possible to occupy by train *k* before coupled trains depart. Conflicts between trains are avoided by constraints (4.33) -(4.39) that are formulated based on arrival time of train *k*.

$$v_k \leq v_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e7_{jq} + e21_{jq} - e8_{qk}\right)\ \forall j,q,l,k,\ j < q, j \neq k, q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0)) \quad (4.36)$$

$$c_k + st \leq a_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e7_{jq} + e21_{jq} - e8_{qk}\right)\ \forall j,q,l,k,\ j < q, j \neq k, q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gA_k \neq 0, bA_k \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gB_k \neq 0, bB_k \neq 0,)) \quad (4.37)$$

$c_k + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e7_{jq} + e21_{jq} - e8_{qk}\right)\ \forall j, q, l, k,\ j < q, j \neq k,$
$q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, ((\ gA_k \neq 0, bB_k \neq 0) or (gB_k \neq 0,$
$bA_k \neq 0)))\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, ((\ gB_k \neq 0, bA_k \neq 0) or (\ gA_k \neq 0,$
$bB_k \neq 0)))))$ (4.38)

$z_k \leq v_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e7_{jq} + e21_{jq} - e8_{qk}\right)\ \forall j, q, l, k,\ j < q, j \neq k,$
$q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gB_k \neq 0, bB_k \neq 0))\ or\ (bB_j \neq 0,$
$gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gA_k \neq 0, bA_k \neq 0))$ (4.39)

Case I-Part II ($\boldsymbol{v_q \leq v_j}$)

As was mentioned in the previous pages, platform position constraints for *Case 1* depend entirely on arrival sequence of trains to the platform track *l*. Constraint (4.28) -(4.39) impose that train *j* arrives at the platform track before train *q*. On the contrary, Constraint (4.40) -(4.44) indicate that train *q* reaches to the platform track firstly. Set of the constraints present avoiding the conflicts between trains.

$v_q \leq v_j + MK * \left(3 - x_{jl} - x_{ql} - e7_{jq}\right)\ \forall j, q, l, j < q, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,$
$gA_q \neq 0, gA_j \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0))$
(4.40)

$v_j \leq z_q + MK * \left(3 - x_{jl} - x_{ql} - e7_{jq} + e21_{jq}\right)\ \forall j, q, l, j < q, cap_l = 1, ((bA_j \neq 0, bA_q \neq$
$0,\ gA_q \neq 0, gA_j \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0))$ (4.41)

$c_j + st \leq z_q + MK * \left(3 - x_{jl} - x_{ql} - e7_{jq} + e21_{jq}\ \right)\ \forall j, q, l, j < q, cap_l = 1, ((bA_j \neq 0,$
$bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0\ , bA_j \neq bA_q)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0,\ bB_j \neq$
$bB_q\ ))$ (4.42)

$c_j + ht \leq c_q + MK * \left(3 - x_{jl} - x_{ql} - e7_{jq} + e21_{jq}\ \right)\ \forall j, q, l, j < q, cap_l = 1, ((bA_j \neq 0,$
$bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0\ , bA_j = bA_q)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0,\ bB_j =$
$bB_q\ ))$ (4.43)

$c_q + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - e7_{jq} - e21_{jq}\ \right)\ \forall j, q, l, j < q, cap_l = 1, ((bA_j \neq 0,$
$bA_q \neq 0, gA_q \neq 0, gA_j \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0\ ))$ (4.44)

The relation between coupled trains (*j* and *q*) and the other train *k* is presented by Constraint (4.45) -(4.51) that are on the basis of arrival time of train *k* in a similar way to Constraint (4.33)-(4.39).

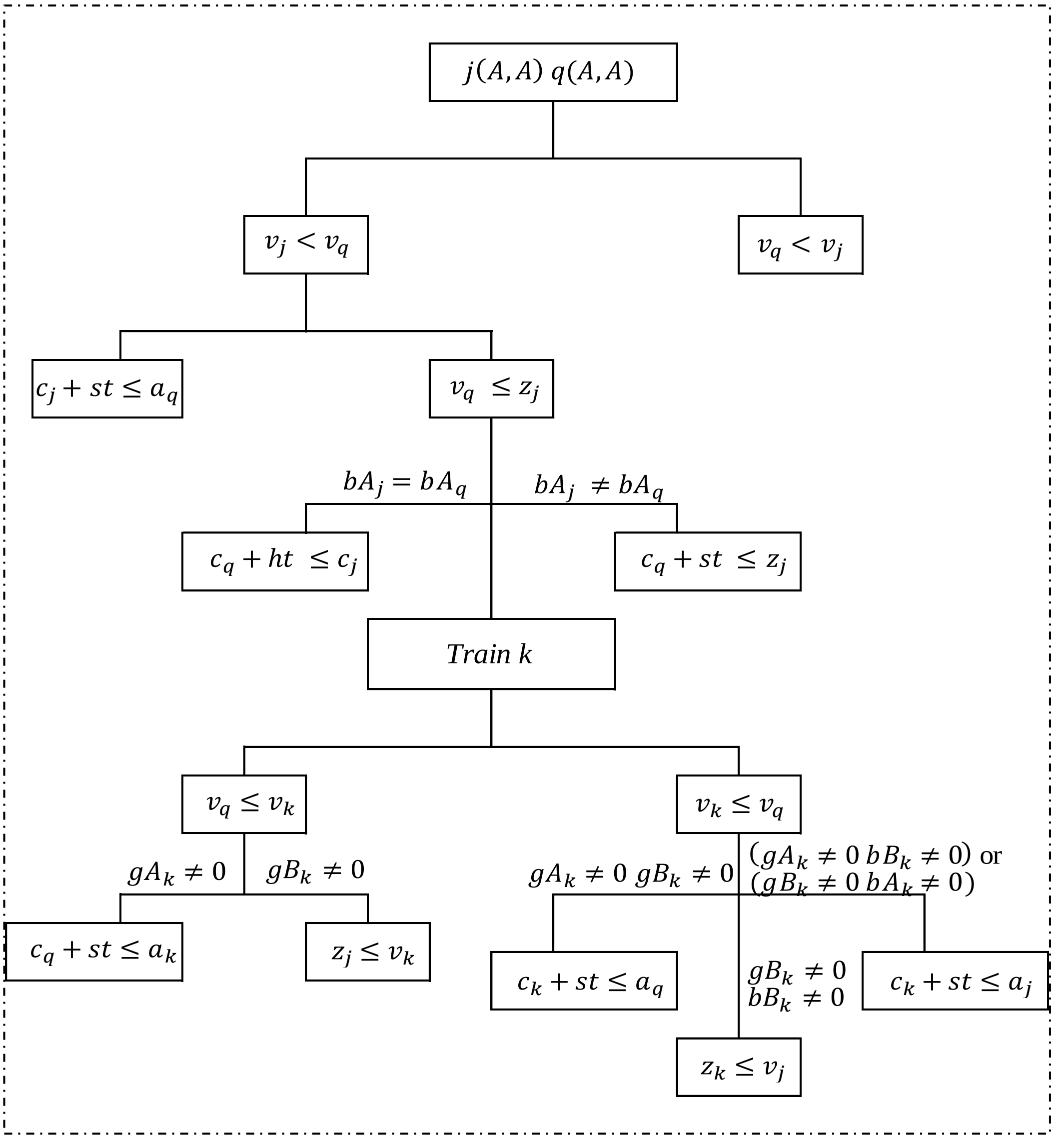

Figure 4-7 Modelling steps for Case 1

$v_j \le v_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e7_{jq} + e21_{jq} + e8_{jk}\right)\ \forall j, q, l, k,\ j < q, j \ne k,$
$q \ne k, cap_l = 1, ((bA_j \ne 0, bA_q \ne 0,\ gA_q \ne 0, gA_j \ne 0)\ or\ (bB_j \ne 0, gB_q \ne 0, bB_q \ne$
$0, gB_j \ne 0))$ (4.45)

$c_j + st \le a_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e7_{jq} + e21_{jq} + e8_{jk}\right)\ \forall j, q, l, k,\ j < q, j \ne k,$
$q \ne k, cap_l = 1, ((bA_j \ne 0, bA_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, gA_k \ne 0)\ or\ (bB_j \ne 0, gB_q \ne$
$0, bB_q \ne 0, gB_j \ne 0, gB_k \ne 0))$ (4.46)

$z_q \le v_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e7_{jq} + e21_{jq} + e8_{jk}\right)\ \forall j, q, l, k,\ j < q, j \ne k,\ q \ne$
$k, cap_l = 1, ((bA_j \ne 0, bA_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, gB_k \ne 0)\ or\ (bB_j \ne 0, gB_q \ne 0,$
$bB_q \ne 0, gB_j \ne 0, gA_k \ne 0))$ (4.47)

$v_k \leq v_j + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e7_{jq} + e21_{jq} - e8_{jk})\ \forall j, q, l, k,\ j < q, j \neq k,$
$q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0,$
$gB_j \neq 0))$ (4.48)

$c_k + st \leq a_j + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e7_{jq} + e21_{jq} - e8_{jk})\ \forall j, q, l, k,\ j < q, j \neq k,$
$q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gA_k \neq 0, bA_k \neq 0)\ or\ (bB_j \neq$
$0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gB_k \neq 0, bB_k \neq 0,))$ (4.49)

$c_k + st \leq a_q + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e7_{jq} + e21_{jq} - e8_{jk})\ \forall j, q, l, k,\ j < q, j \neq k,$
$q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, ((\ gA_k \neq 0, bB_k \neq 0) or(\ gB_k \neq$
$0, bA_k \neq 0)))\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, ((\ gB_k \neq 0, bA_k \neq 0) or(\ gA_k \neq$
$0, bB_k \neq 0)))))$ (4.50)

$z_k \leq v_q + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e7_{jq} + e21_{jq} - e8_{jk})\ \forall j, q, l, k,\ j < q, j \neq k,$
$q \neq k, cap_l = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gB_k \neq 0, bB_k \neq 0))\ or\ (bB_j \neq 0,$
$gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gA_k \neq 0, bA_k \neq 0))$ (4.51)

- ✓ **Case 2: *j (A, B), q (A, A)***

If train *j* and *q* are arriving from direction *A* and one of them is running through direction *B*, this situation can be expressed by set of constraints (4.52) -(4.62). In Figure 4.8, modelling steps of Case 2 are illustrated.

Case 2-Part I $(\boldsymbol{v_q \leq v_j})$

In case train q arrives before, there is no chance to share same platform track l so departure directions of trains are opposite. Constraints (4.52) and (4.53) enforce that train *j* should wait for a while that is equal to sum of arrival time of train *q* to the exit signal and switch interval.

$v_q \leq v_j + MK * (3 - x_{jl} - x_{ql} - e9_{jq})\ \forall j, q, l, j \neq q, cap_l = 1,\ bB_j \neq 0, bA_q \neq 0,\ gA_q \neq$
$0, gA_j \neq 0$ (4.52)

$c_q + st \leq a_j + MK * (3 - x_{jl} - x_{ql} - e9_{jq})\ \forall j, q, l, j \neq q, cap_l = 1,\ bB_j \neq 0, bA_q \neq 0,$
$gA_q \neq 0, gA_j \neq 0$ (4.53)

Case 2-Part II $(v_j \leq v_q)$

When train *j* reaches to the platform track *l* before train *q*, it can be possible to occupy platform track during same time interval. However, their movements do not depend on each other: they can encounter at the track or not. This situation expressed by Constraint (4.54) - (4.56).

$$v_j \leq v_q + MK * \left(2 - x_{jl} - x_{ql} + e9_{jq}\right) \ \forall j, q, l, j \neq q, cap_l = 1, \ bB_j \neq 0, bA_q \neq 0, \ gA_q \neq 0, gA_j \neq 0 \quad (4.54)$$

$$v_q \leq z_j + MK * \left(2 - x_{jl} - x_{ql} + e9_{jq} + e23_{jq}\right) \ \forall j, q, l, j \neq q, cap_l = 1, bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0 \quad (4.55)$$

In case trains encounter at the platform track and train *k* which is assigned to the same track, conflicts between trains are avoided by Constraint (4.56) -(4.62). To calculate departure time through the platform track of train *k*, arrival time of train *k* and its direction should be seriously considered. If it is direction *A* and arrives after train *q*, it should wait for a while that is equal to sum of arrival time to the exit signal of train *q* and switch interval. (Constraint (4.57) and (4.58)). If it is direction *B* and arrives after train *q*, this constraint is valid for train *j* and it is expressed by Constraint (4.59). Constraints (4.60) -(4.62) state that train *k* reaches to the platform track *l* before train *q*.

$$z_k \leq v_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e9_{jq} + e23_{jq} - e22_{qk}\right) \quad \forall j, q, l, k, \quad j \neq q, \ j \neq k, k \neq q, cap_l = 1, \ bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0, ((\ gA_k \neq 0, bB_k \neq 0) or \ (\ gB_k \neq 0, bB_k \neq 0)) \quad (4.56)$$

$$v_q \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e9_{jq} + e23_{jq} + e22_{qk}\right) \quad \forall j, q, l, k, \quad j \neq q, \ j \neq k, k \neq q, cap_l = 1, \ bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0 \quad (4.57)$$

$$c_q + st \leq a_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e9_{jq} + e23_{jq} + e22_{qk}\right) \forall j, q, l, k, \ j \neq q, j \neq k, k \neq q, cap_l = 1, \ bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0, \ gA_k \neq 0 \quad (4.58)$$

$$c_j + st \leq a_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e9_{jq} + e23_{jq} + e22_{qk}\right) \quad \forall j, q, l, k, \quad j \neq q, \ j \neq k, \ k \neq q, cap_l = 1, \ bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0, \ gB_k \neq 0 \quad (4.59)$$

$$v_k \leq v_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e9_{jq} + e23_{jq} - e22_{qk}\right) \quad \forall j, q, l, k, \quad j \neq q, \ j \neq k, k \neq q, cap_l = 1, \ bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0 \quad (4.60)$$

$$c_k + st \leq a_q + MK * (4 - x_{jl} - x_{ql} - x_{kl} + e9_{jq} + e23_{jq} - e22_{qk}) \quad \forall j,q,l,k, \quad j \neq q,\ j \neq k,\ k \neq q, cap_l = 1,\ bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0, bA_k \neq 0, gA_k \neq 0 \tag{4.61}$$

$$c_k + st \leq a_j + MK * (4 - x_{jl} - x_{ql} - x_{kl} + e9_{jq} + e23_{jq} - e22_{qk}) \quad \forall j,q,l,k, \quad j \neq q,\ j \neq k,\ k \neq q, cap_l = 1,\ bB_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0, bA_k \neq 0, gB_k \neq 0 \tag{4.62}$$

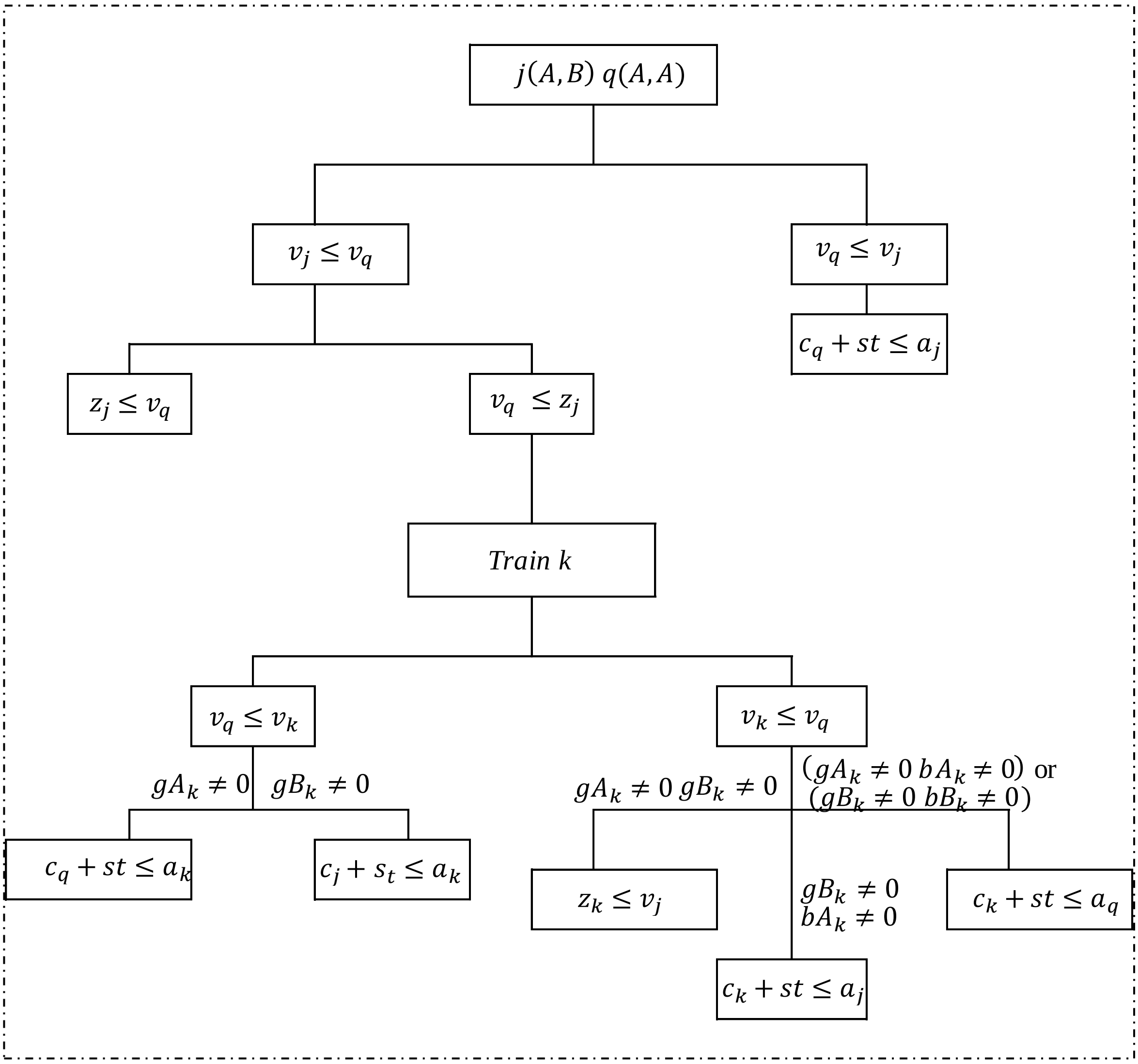

Figure 4-8 Modelling steps for Case 2

- ✓ **Case 3: *j (A, A), q (B, A)***

When train *j* and *q* departure from direction A and one of them arrive from direction B, Constraints (4.63)- (4.86) are formulated for relation between train *j* and *q* based on which train arrived first. In addition, by the help of constraints (4.68) -(4.74) and (4.80) -(4.86), avoiding conflicts between trains are guaranteed according to arrival time and direction of train *k*. In Figure 4.9, modelling steps of Case 3 are given.

Case 3-Part I $(\boldsymbol{v_j \leq v_q})$

$v_j \leq v_q + MK * \left(2 - x_{jl} - x_{ql} + \ e10_{jq}\right)\ \forall j, q, l, j \neq q,\ cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0,$
$gB_q \neq 0, gA_j \neq 0$ (4.63)

$v_q \leq z_j + MK * \left(2 - x_{jl} - x_{ql} + \ e10_{jq} + e11_{jq}\right)\ \forall j, q, l,\ j \neq q,\ cap_l = 1,\ bA_j \neq 0,$
$bA_q \neq 0, gB_q \neq 0, gA_j \neq 0$ (4.64)

$z_j \leq v_q + MK * \left(3 - x_{jl} - x_{ql} + \ e10_{jq} - e11_{jq}\right)\ \forall j, q, l, j \neq q,\ cap_l = 1, bA_j \neq 0, bA_q \neq$
$0,\ gB_q \neq 0, gA_j \neq 0$ (4.65)

$c_j + st \leq z_q + MK * \left(2 - x_{jl} - x_{ql} + \ e10_{jq} + e11_{jq}\right)\ \forall j, q, l,\ j \neq q,\ cap_l = 1,\ bA_j \neq 0,$
$bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, bA_j \neq bA_q$ (4.66)

$c_j + ht \leq c_q + MK * \left(2 - x_{jl} - x_{ql} + \ e10_{jq} + e11_{jq}\right)\ \forall j, q, l,\ j \neq q,\ cap_l = 1,\ bA_j \neq 0,$
$bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, bA_j = bA_q$ (4.67)

$v_q \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + \ e10_{jq} + e11_{jq} + e24_{qk}\right)\ \forall j, q, l, k\ \ j \neq q,\ j \neq k,$
$q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0$ (4.68)

$z_q \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + \ e10_{jq} + e11_{jq} + e24_{qk}\right)\ \forall j, q, l, k\ \ j \neq q,\ j \neq k,$
$q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gB_k \neq 0$ (4.69)

$c_j + st \leq a_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + \ e10_{jq} + e11_{jq} + e24_{qk}\right)\ \forall j, q, l, k\ \ j \neq q,\ j \neq$
$k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gA_k \neq 0$ (4.70)

$v_k \leq v_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + \ e10_{jq} + e11_{jq} - e24_{qk}\right)\ \forall j, q, l, k\ \ j \neq q,\ j \neq k,$
$q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0$ (4.71)

$c_k + st \leq a_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + \ e10_{jq} + e11_{jq} - e24_{qk}\right)\ \forall j, q, l, k\ \ j \neq q, j \neq$
$k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gB_k \neq 0, bB_k \neq 0$ (4.72)

$c_k + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + \ e10_{jq} + e11_{jq} - e24_{qk}\right)\ \forall j, q, l, k\ \ j \neq q,\ j \neq$
$k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, ((gB_k \neq 0, bA_k \neq 0)\ or\ (gA_k \neq$
$0, bA_k \neq 0))$ (4.73)

$z_k \leq v_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + \ e10_{jq} + e11_{jq} - e24_{qk}\right)\ \forall j, q, l, k\ \ j \neq q, j \neq k,$
$q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gA_k \neq 0, bB_k \neq 0$ (4.74)

Case 3-Part II $(\boldsymbol{v_q \leq v_j})$

Constraints (3.75)- (3.86) are written for *Case 3* which train *q* arrived first. In similar way with *Case 3- Part 1*, avoiding conflicts between trains are guaranteed based on arrival time and direction of train *k*.

$$v_q \leq v_j + MK * \left(3 - x_{jl} - x_{ql} - e10_{jq}\right) \ \forall j,q,l,j \neq q,\ cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0 \quad (4.75)$$

$$v_j \leq z_q + MK * \left(3 - x_{jl} - x_{ql} - e10_{jq} + e11_{jq}\right) \ \forall j,q,l,j \neq q,\ cap_l = 1,\ bA_j \neq 0, bA_q \neq 0,\ gB_q \neq 0, gA_j \neq 0 \quad (4.76)$$

$$c_j + st \leq z_q + MK * \left(3 - x_{jl} - x_{ql} - e10_{jq} + e11_{jq}\right) \ \forall j,q,l,\ j \neq q,\ cap_l = 1,\ bA_j \neq 0, bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, bA_j \neq bA_q \quad (4.77)$$

$$c_q + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - e10_{jq} - e11_{jq}\right) \ \forall j,q,l,\ j \neq q,\ cap_l = 1,\ bA_j \neq 0, bA_q \neq 0, gB_q \neq 0, gA_j \neq 0 \quad (4.78)$$

$$c_j + ht \leq c_q + MK * \left(3 - x_{jl} - x_{ql} - e10_{jq} + e11_{jq}\right) \ \forall j,q,l,\ j \neq q,\ cap_l = 1,\ bA_j \neq 0, bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, bA_j = bA_q \quad (4.79)$$

$$v_j \leq v_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e10_{jq} + e11_{jq} + e24_{jk}\right) \ \forall j,q,l,k\ \ j \neq q,\ j \neq k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0 \quad (4.80)$$

$$c_j + st \leq a_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e10_{jq} + e11_{jq} + e24_{jk}\right) \ \forall j,q,l,k\ \ j \neq q,\ j \neq k,\ q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gA_k \neq 0 \quad (4.81)$$

$$c_q + st \leq a_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e10_{jq} + e11_{jq} + e24_{jk}\right) \ \forall j,q,l,k\ \ j \neq q,\ j \neq k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gB_k \neq 0 \quad (4.82)$$

$$v_k \leq v_j + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e10_{jq} + e11_{jq} - e24_{jk}\right) \ \forall j,q,l,k\ \ j \neq q,\ j \neq k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0 \quad (4.83)$$

$$z_k \leq v_q + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e10_{jq} + e11_{jq} - e24_{qk}\right) \ \forall j,q,l,k\ \ j \neq q, j \neq k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gB_k \neq 0, bA_k \neq 0 \quad (4.84)$$

$$c_k + st \leq a_j + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e10_{jq} + e11_{jq} - e24_{jk}\right) \ \forall j,q,l,k\ \ j \neq q, j \neq k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, gA_k \neq 0, bA_k \neq 0 \quad (4.85)$$

$c_k + st \leq a_q + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e10_{jq} + e11_{jq} - e24_{jk})\ \forall j,q,l,k\ \ j \neq q,\ j \neq k, q \neq k, cap_l = 1,\ bA_j \neq 0,\ bA_q \neq 0, gB_q \neq 0, gA_j \neq 0, ((gB_k \neq 0, bB_k \neq 0)\ or\ (gA_k \neq 0, bB_k \neq 0))$ (4.86)

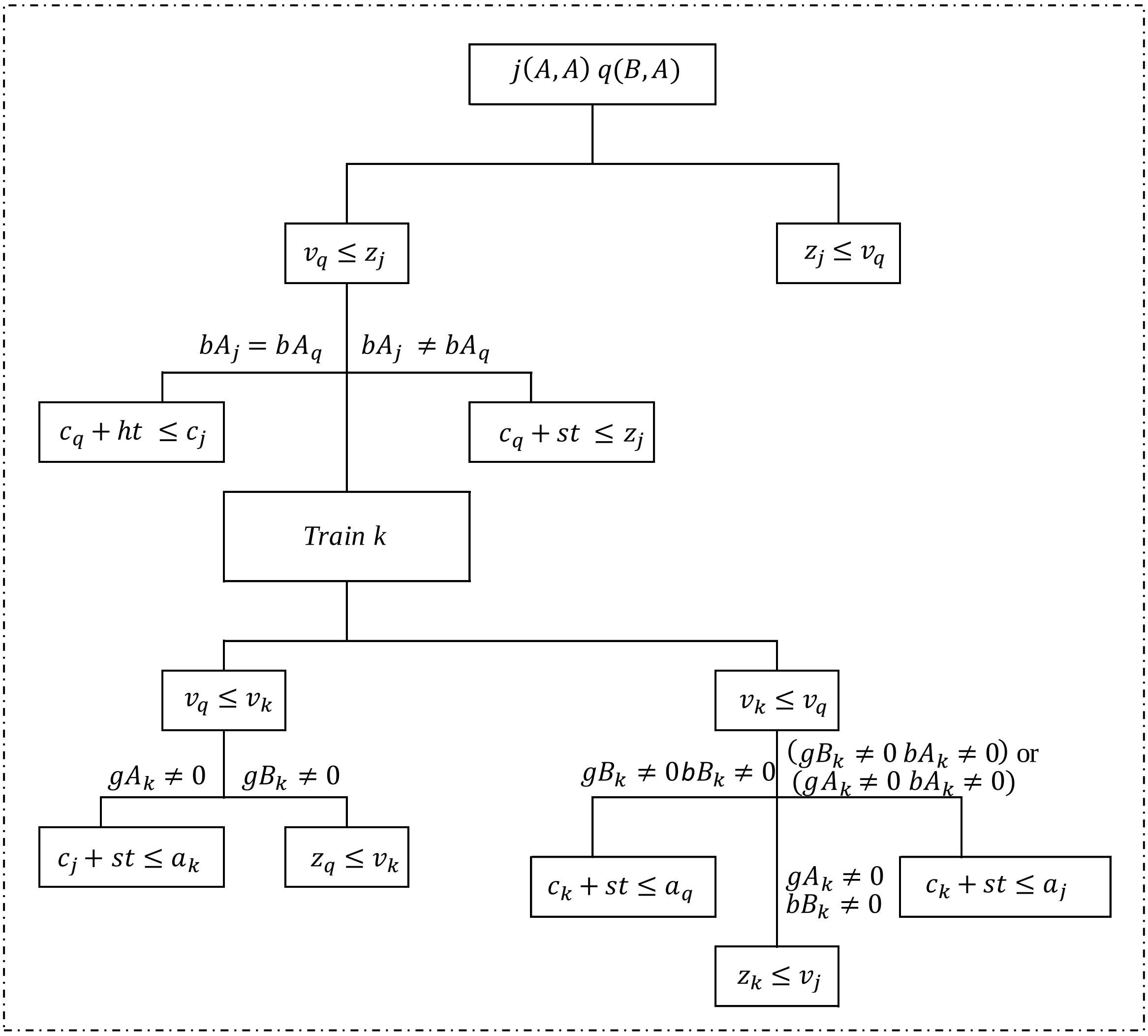

Figure 4-9 Modelling steps for Case 3

- ✓ **Case 4:** ***j (A, B) q (A, B)***

Case 4 states that train *j* and train *q* arrive to the platform track *l* from direction *A* and both run through direction *B*. Constraints (4.87) -(4.110) are written for *Case 4*. If train *j* arrives to the platform track firstly, this is expressed by Constraint (4.87)- (4.98). Besides that, Constraints (3.92) -(3.98) are formulated for preventing conflicts of trains including train *k* that is assigned to the same platform track. In Figure 4.10, modelling steps of Case 4 are given.

Case 4-Part I $(\boldsymbol{v_j \leq v_q})$

$v_j \leq v_q + MK * \left(2 - x_{jl} - x_{ql} + e12_{jq}\right)\ \forall j,q,l,j < q,\ cap_l = 1,\ bB_j \neq 0,\ bB_q \neq 0,$
$gA_q \neq 0, gA_j \neq 0$ (4.87)

$v_q \leq z_j + MK * \left(2 - x_{jl} - x_{ql} + e12_{jq} + e25_{jq}\right)\ \forall j,q,l,j < q,\ cap_l = 1,\ bB_j \neq 0,$
$bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0$ (4.88)

$c_j + st \leq z_q + MK * \left(2 - x_{jl} - x_{ql} + e12_{jq} + e25_{jq}\right)\ \forall j,q,l,j < q,\ cap_l = 1,\ bB_j \neq 0,$
$bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, bB_j \neq bB_q$ (4.89)

$c_j + ht \leq c_q + MK * \left(2 - x_{jl} - x_{ql} + e12_{jq} + e25_{jq}\right)\ \forall j,q,l,j < q,\ cap_l = 1,\ bB_j \neq 0,$
$bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, bB_j = bB_q$ (4.90)

$z_j \leq v_q + MK * \left(3 - x_{jl} - x_{ql} + e12_{jq} - e25_{jq}\right)\ \forall j,q,l,j < q,\ cap_l = 1,\ bB_j \neq 0,$
$bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0$ (4.91)

$v_q \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e12_{jq} + e25_{jq} + e26_{qk}\right)\ \forall j,q,l,k,j <$
$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0$ (4.92)

$z_q \leq \mathrm{v}_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e12_{jq} + e25_{jq} + e26_{qk}\right)\ \forall j,q,l,k,j <$
$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gA_k \neq 0$ (4.93)

$c_j + st \leq a_k + MK * \left(3 - \mathrm{x}_{jl} - x_{ql} - x_{kl} + e12_{jq} + e25_{jq} + e26_{qk}\right)\ \forall j,q,l,k,j < q,$
$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gB_k \neq 0$ (4.94)

$v_k \leq v_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e12_{jq} + e25_{jq} - e26_{qk}\right)\ \forall j,q,l,k,j <$
$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0$ (4.95)

$c_k + st \leq a_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e12_{jq} + e25_{jq} - e26_{qk}\right)\ \forall j,q,l,k,j < q,$
$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gA_k \neq 0, bA_k \neq 0$ (4.96)

$z_k \leq v_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e12_{jq} + e25_{jq} - e26_{qk}\right)\ \forall j,q,l,k,j <$
$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, ((gA_k \neq 0, bB_k \neq$
$0) or\ (gB_k \neq 0,\ bB_k \neq 0))$ (4.97)

$c_k + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e12_{jq} + e25_{jq} - e26_{qk}\right) \forall j,q,l,k,\ j <$
$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gB_k \neq 0,\ bA_k \neq 0$ (4.98)

Case 4-Part II $(\boldsymbol{v_q \leq v_j})$

If train q arrives to the platform track firstly (Constraint (4.99) -(4.110)), the relations between coupled trains (j and q) and the other train k are presented by Constraint (4.103) -

(4.109) that are on the basis of arrival time of train k in a similar way to Constraint (4.92) - (4.98).

$$v_q \le v_j + MK * (3 - x_{jl} - x_{ql} - e12_{jq})\ \forall j,q,l,j<q,\ cap_l = 1,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0 \tag{4.99}$$

$$v_j \le z_q + MK * (3 - x_{jl} - x_{ql} - e12_{jq} + e25_{jq})\ \forall j,q,l,j<q,\ cap_l = 1,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0 \tag{4.100}$$

$$c_q + st \le z_j + MK * (3 - x_{jl} - x_{ql} - e12_{jq} + e25_{jq})\ \forall j,q,l,j<q,\ cap_l = 1,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, bB_j \ne bB_q \tag{4.101}$$

$$c_q + ht \le c_j + MK * (3 - x_{jl} - x_{ql} - e12_{jq} + e25_{jq})\ \forall j,q,l,j<q,\ cap_l = 1,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, bB_j = bB_q \tag{4.102}$$

$$v_j \le v_k + MK * (4 - x_{jl} - x_{ql} - x_{kl} - e12_{jq} + e25_{jq} + e26_{jk})\ \forall j,q,l,k,j<q, cap_l = 1,\ j \ne k, k \ne q,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0 \tag{4.103}$$

$$z_j \le v_k + MK * (4 - x_{jl} - x_{ql} - x_{kl} - e12_{jq} + e25_{jq} + e26_{jk})\ \forall j,q,l,k,j<q, cap_l = 1,\ j \ne k, k \ne q,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, gA_k \ne 0 \tag{4.104}$$

$$c_q + st \le a_k + MK * (4 - x_{jl} - x_{ql} - x_{kl} - e12_{jq} + e25_{jq} + e26_{jk})\ \forall j,q,l,k,j<q,\ cap_l = 1,\ j \ne k, k \ne q,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, gB_k \ne 0 \tag{4.105}$$

$$v_k \le v_j + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e12_{jq} + e25_{jq} - e26_{jk})\ \forall j,q,l,k,j<q, cap_l = 1,\ j \ne k, k \ne q,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0 \tag{4.106}$$

$$c_k + st \le a_j + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e12_{jq} + e25_{jq} - e26_{jk})\ \forall j,q,l,k,j<q,\ cap_l = 1,\ j \ne k, k \ne q,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, gA_k \ne 0, bA_k \ne 0 \tag{4.107}$$

$$z_k \le v_q + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e12_{jq} + e25_{jq} - e26_{jk})\ \forall j,q,l,k,j<q, cap_l = 1,\ j \ne k, k \ne q,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, ((gA_k \ne 0, bB_k \ne 0) or\ (gB_k \ne 0,\ bB_k \ne 0)) \tag{4.108}$$

$$c_k + st \le a_q + MK * (5 - \mathrm{x}_{jl} - x_{ql} - x_{kl} - e12_{jq} + e25_{jq} - e26_{jk}) \forall j,q,l,k,j<q, cap_l = 1,\ j \ne k, k \ne q, bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0, gB_k \ne 0, bA_k \ne 0 \tag{4.109}$$

$$z_q \le v_j + MK * (4 - x_{jl} - \mathrm{x}_{ql} - e12_{jq} - e25_{jq})\ \forall j,q,l,j<q,\ cap_l = 1,\ bB_j \ne 0,\ bB_q \ne 0,\ gA_q \ne 0, gA_j \ne 0 \tag{4.110}$$

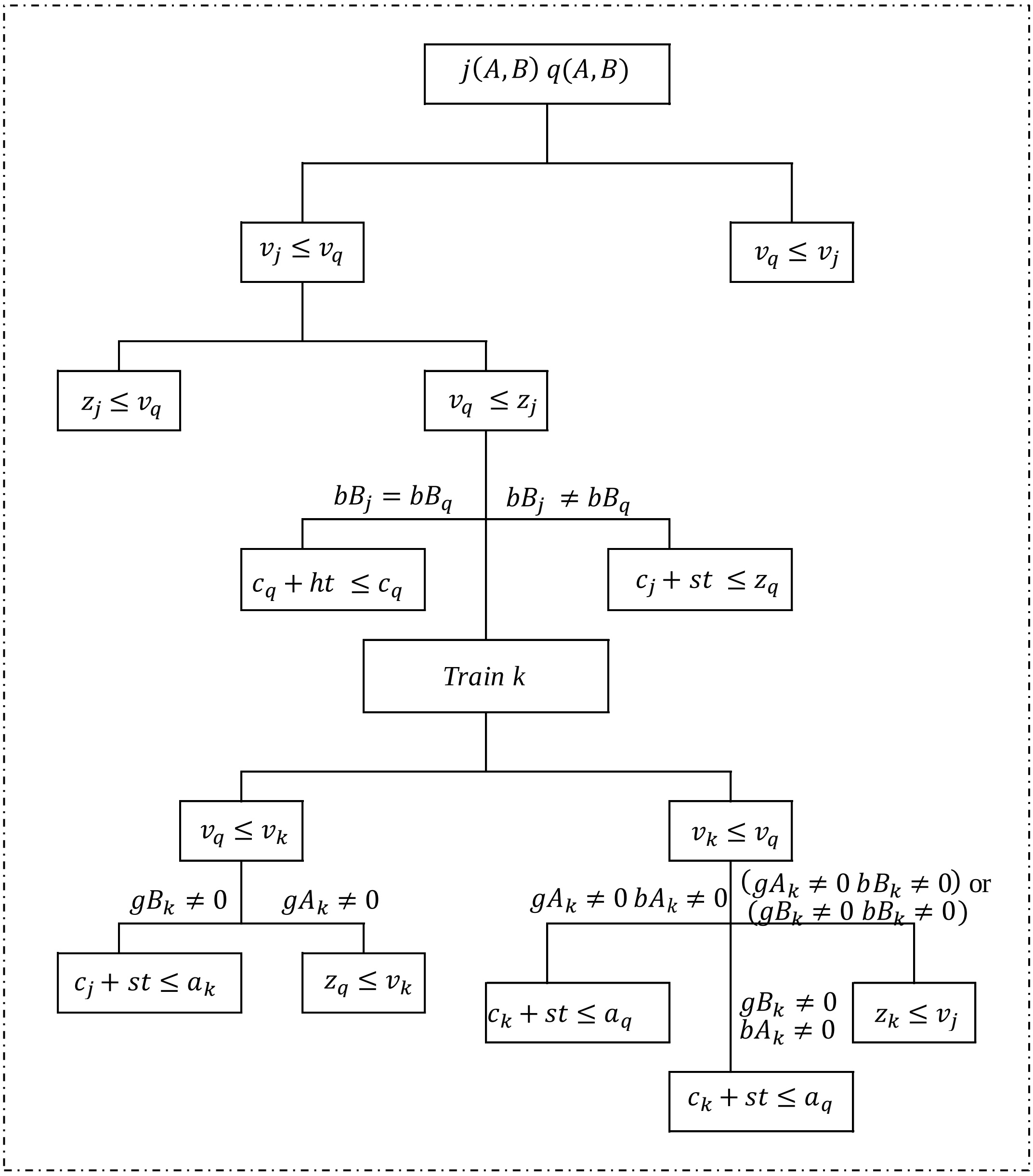

Figure 4-10 Modelling steps for Case 4

✓ ***Case 5: j(A,B) q(B,A)***

If arrival and departure direction of train *j* and *q* are opposite, it is not possible to occupy same platform track at the same time. Constraints (4.111)-(4.112) ensure that they cannot encounter at the track.

$$c_q + st \leq a_j + MK * \left(2 - x_{jl} - x_{ql} + \ e13_{jq}\right)\ \forall j,q,\mathrm{l}, j \neq q,\ cap_l = 1,\ bB_j \neq 0,\ bA_q \neq 0,\ gB_q \neq 0, gA_j \neq 0 \quad (4.111)$$

$c_j + st \leq a_q + MK * \left(3 - x_{jl} - x_{ql} - e13_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,\ bA_q \neq 0,$
$gB_q \neq 0, gA_j \neq 0$ (4.112)

- ✓ ***Case 6: j (A, B), q (B, B)***

Case 6 occurs if train *j* and *q* departure through the direction *B* and one of them arrive from direction *A*, Constraints (4.113)- (4.137) are formulated for relation between train *j* and *q* based on which train arrived first. In addition, by the help of constraints (4.117) -(4.124) and (4.130) -(4.136), avoiding conflicts between trains are guaranteed according to arrival time and direction of train *k*. In Figure 4.11, modelling steps of Case 6 are given.

Case 6-Part I $(\boldsymbol{v_q} \leq \boldsymbol{v_j})$

$v_q \leq v_j + MK * \left(2 - x_{jl} - x_{ql} + e14_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,\ bB_q \neq 0,$
$gB_q \neq 0, gA_j \neq 0$ (4.113)

$v_j \leq z_q + MK * \left(2 - \mathrm{x}_{jl} - x_{ql} + e14_{jq} + e15_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,$
$bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0$ (4.114)

$c_q + st \leq z_j + MK * \left(2 - x_{jl} - x_{ql} + e14_{jq} + e15_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,$
$bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, bB_j \neq bB_q$ (4.115)

$c_q + ht \leq c_j + MK * \left(2 - x_{jl} - x_{ql} + e14_{jq} + e15_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,$
$bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, bB_j = bB_q$ (4.116)

$v_j \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} + e27_{j\mathrm{k}}\right)\ \forall j,q,l,k,j \neq$
$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0$ (4.117)

$z_j \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} + e27_{jk}\right)\ \forall j,q,l,k,j \neq$
$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, gA_k \neq 0$ (4.118)

$c_q + st \leq a_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} + e27_{jk}\right)\ \forall j,q,l,k,j \neq q,$
$cap_l = 1,\ \mathrm{j} \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, gB_k \neq 0$ (4.119)

$v_k \leq v_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} - e27_{jk}\right)\ \forall j,q,l,k,j \neq q,$
$cap_l = 1,\ j \neq k, k \neq q,\ bB_{\mathrm{j}} \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0$ (4.120)

$c_k + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} - e27_{jk}\right)\ \forall j,q,l,k,j \neq q,$
$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, gA_k \neq 0, bA_k \neq 0$ (4.121)

$c_k + st \leq a_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} - e27_{jk}\right)\ \forall j,q,l,k,j \neq q,$
$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, bB_k \neq 0$ (4.122)

$$v_k \leq v_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} - e27_{jk}\right) \ \forall j,q,l,k,j \neq q,$$
$$cap_l = 1,\ j \neq k, k \neq q, bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, gB_k \neq 0, bA_k \neq 0 \qquad (4.123)$$
$$c_k + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e14_{jq} + e15_{jq} - e27_{jk}\right) \ \forall j,q,l,k,j \neq q,$$
$$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, gB_k \neq 0, bA_k \neq 0 \qquad (4.124)$$
$$z_q \leq v_j + MK * \left(3 - x_{jl} - x_{ql} + e14_{jq} - e15_{jq}\right) \ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,$$
$$bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0 \qquad (4.125)$$

Case 6-Part II $(\boldsymbol{v_j \leq v_q})$

Constraints (4.126)- (4.137) are written for *Case 6* that train *j* arrived first. In similar way with *Case 6*- Part I, avoiding conflicts between trains are ensured based on arrival time and direction of train *k*.

$$v_j \leq v_q + MK * \left(3 - x_{jl} - x_{ql} - e14_{jq}\right) \ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,\ bB_q \neq 0,$$
$$gB_q \neq 0, gA_j \neq 0 \qquad (4.126)$$
$$v_q \leq z_j + MK * \left(3 - x_{jl} - x_{ql} - e14_{jq} + e15_{jq}\right) \ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,$$
$$bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0 \qquad (4.127)$$
$$c_q + st \leq z_j + MK * \left(3 - x_{jl} - x_{ql} - e14_{jq} + e15_{jq}\right) \ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,$$
$$bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, bB_j \neq bB_q \qquad (4.128)$$
$$c_q + ht \leq c_j + MK * \left(3 - x_{jl} - x_{ql} - e14_{jq} + e15_{jq}\right) \ \forall j,q,l,j \neq q,\ cap_l = 1,\ bB_j \neq 0,$$
$$bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, bB_j = bB_q \qquad (4.129)$$
$$v_q \leq v_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e14_{jq} + e15_{jq} + e27_{qk}\right) \ \forall j,q,l,k,j \neq q,$$
$$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0 \qquad (4.130)$$
$$z_j \leq v_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e14_{jq} + e15_{jq} + e27_{qk}\right) \ \forall j,q,l,k,j \neq$$
$$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, gA_k \neq 0 \qquad (4.131)$$
$$c_q + st \leq a_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e14_{jq} + e15_{jq} + e27_{qk}\right) \ \forall j,q,l,k,j \neq q,$$
$$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, gB_k \neq 0 \qquad (4.132)$$
$$v_k \leq v_q + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e14_{jq} + e15_{jq} - e27_{qk}\right) \ \forall j,q,l,k,j \neq$$
$$q, cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0 \qquad (4.133)$$
$$c_k + st \leq a_j + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e14_{jq} + e15_{jq} - e27_{qk}\right) \ \forall j,q,l,k,j \neq q,$$
$$cap_l = 1,\ j \neq k, k \neq q,\ bB_j \neq 0,\ bB_q \neq 0,\ gB_q \neq 0, gA_j \neq 0, bA_k \neq 0 \qquad (4.134)$$

$c_k + st \le a_q + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - \ e14_{jq} + e15_{jq} - e27_{qk}\right)\ \forall j, q, l, k, j \ne q,$
$cap_l = 1,\ j \ne k, k \ne q,\ bB_j \ne 0,\ bB_q \ne 0,\ gB_q \ne 0, gA_j \ne 0, bB_k \ne 0, ((gB_k \ne 0, bB_k \ne 0) or\ (gA_k \ne 0, bB_k \ne 0)$ (4.135)

$v_k \le v_j + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - \ e14_{jq} + e15_{jq} - e27_{jk}\right)\ \forall j, q, l, k, j \ne q,$
$cap_l = 1,\ j \ne k, k \ne q, bB_j \ne 0,\ bB_q \ne 0,\ gB_q \ne 0, gA_j \ne 0, gA_k \ne 0, bB_k \ne 0$ (4.136)

$c_j + st \le a_q + MK * \left(4 - x_{jl} - x_{ql} - \ e14_{jq} - e15_{jq}\right)\ \forall j, q, l, j \ne q,\ cap_l = 1,\ bB_j \ne 0,$
$bB_q \ne 0,\ gB_q \ne 0, gA_j \ne 0,$ (4.137)

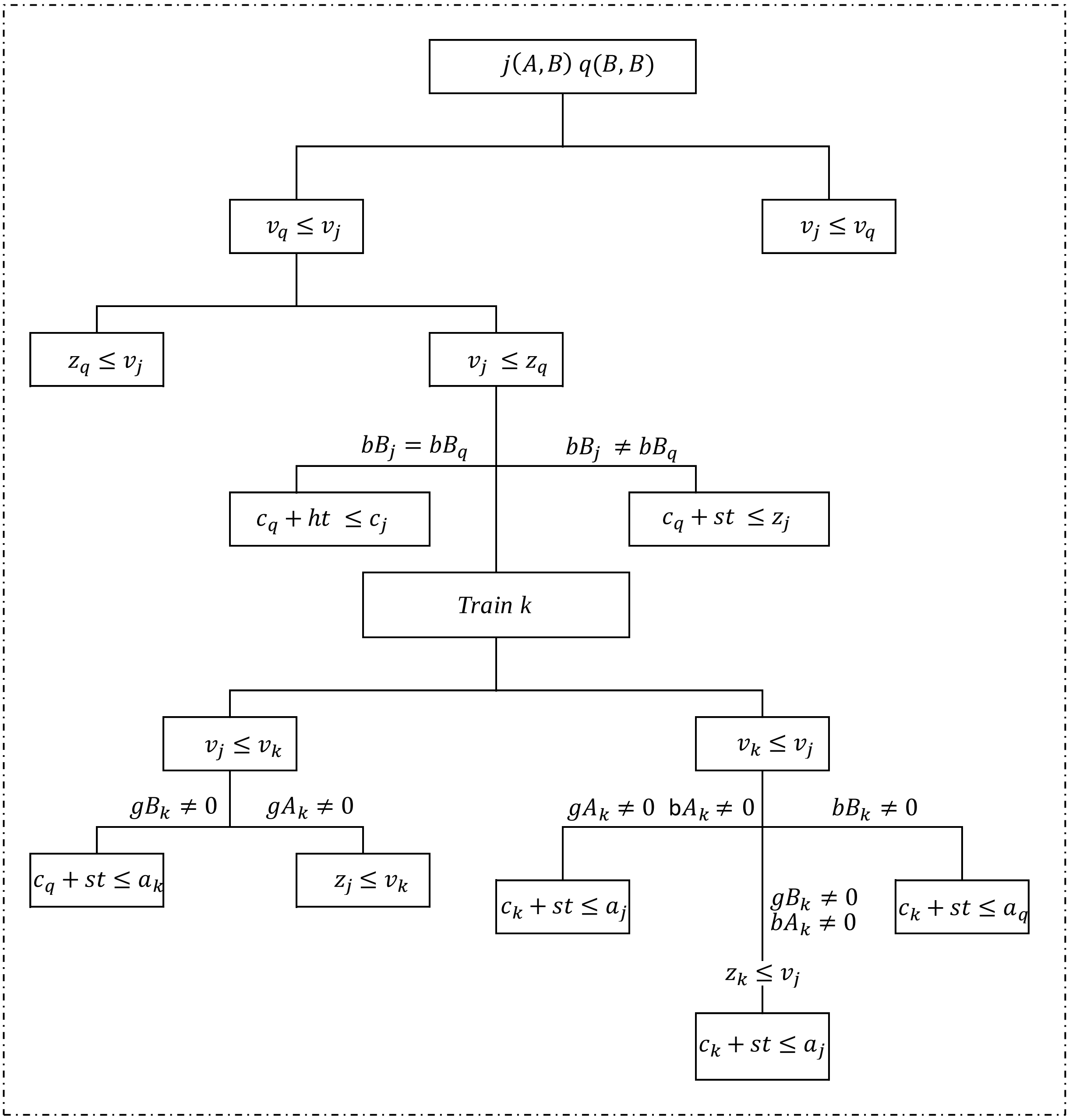

Figure 4-11 Modelling Steps for Case 6

- ✓ ***Case 7: j (B, A), q (B, A)***

Case 7 states that train *j* and train *q* arrives to the platform track *l* from direction *B* and both run through direction *A*. Constraints (4.138) -(4.161) are written for *Case 7* and its formulation is basically inspired by *Case 4*. If train *j* arrives to the platform track firstly, this is expressed by Constraint (4.138)- (4.149). Besides that, Constraints (4.142) -(4.148) are formulated for preventing conflicts of trains including train *k* that is assigned to the same platform track. In Figure 4.12, modelling steps of Case 7 are given.

Case 7-Part I $(\boldsymbol{v_j \leq v_q})$

$$v_j \leq v_q + MK * (2 - x_{jl} - x_{ql} + e16_{jq}) \ \forall j, q, l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.138)$$

$$v_q \leq z_j + MK * (2 - x_{jl} - x_{ql} + e16_{jq} + e28_{jq}) \ \forall j, q, l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.139)$$

$$c_j + st \leq z_q + MK * (2 - x_{jl} - x_{ql} + e16_{jq} + e28_{jq}) \ \forall j, q, l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, \ bA_j \neq bA_q \quad (4.140)$$

$$c_j + ht \leq c_q + MK * (2 - x_{jl} - x_{ql} + e16_{jq} + e28_{jq}) \ \forall j, q, l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, \ bA_j = bA_q \quad (4.141)$$

$$v_q \leq v_k + MK * (3 - x_{jl} - x_{ql} - x_{kl} + e16_{jq} + e28_{jq} + e29_{qk}) \ \forall j, q, l, k \ j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, \quad (4.142)$$

$$c_j + st \leq a_k + MK * (3 - x_{jl} - x_{ql} - x_{kl} + e16_{jq} + e28_{jq} + e29_{qk}) \ \forall j, q, l, k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gA_k \neq 0 \quad (4.143)$$

$$z_q \leq v_k + MK * (3 - x_{jl} - x_{ql} - x_{kl} + e16_{jq} + e28_{jq} + e29_{qk}) \ \forall j, q, l, k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gB_k \neq 0 \quad (4.144)$$

$$v_k \leq v_q + MK * (4 - x_{jl} - x_{ql} - x_{kl} + e16_{jq} + e28_{jq} - e29_{qk}) \ \forall j, q, l, k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.145)$$

$$z_k \leq v_j + MK * (4 - x_{jl} - x_{ql} - x_{kl} + e16_{jq} + e28_{jq} - e29_{qk}) \ \forall j, q, l, k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, bA_k \neq 0 \quad (4.146)$$

$$c_k + st \leq a_j + MK * (4 - x_{jl} - x_{ql} - x_{kl} + e16_{jq} + e28_{jq} - e29_{qk}) \ \forall j, q, l, k, j < q, \ k \neq q, \ j \neq k, cap_l = 1, bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gA_k \neq 0, bB_k \neq 0 \quad (4.147)$$

$$c_k + st \leq a_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e16_{jq} + e28_{jq} - e29_{qk}\right) \ \forall j,q,l,k, j < q, k \neq q, \ j \neq k, cap_l = 1, bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gB_k \neq 0, bB_k \neq 0 \quad (4.148)$$

$$z_j \leq v_q + MK * \left(3 - x_{jl} - x_{ql} + e16_{jq} - e28_{jq}\right) \ \forall j,q,l, j < q, cap_l = 1, bA_j \neq 0, bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.149)$$

Case 7-Part II ($\boldsymbol{v_q \leq v_j}$)

In *Case 7*, train q arrives before train j. The relations between coupled trains (j and q) and the other train k are presented by Constraint (4.150) -(4.161) that are on the basis of arrival time of train k in a similar way to Constraint (4.142) -(4.148).

$$v_q \leq v_j + MK * \left(3 - x_{jl} - x_{ql} - e16_{jq}\right) \ \forall j,q,l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.150)$$

$$v_j \leq z_q + MK * \left(3 - x_{jl} - x_{ql} - e16_{jq} + e28_{jq}\right) \ \forall j,q,l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.151)$$

$$c_q + st \leq z_j + MK * \left(3 - x_{jl} - x_{ql} - e16_{jq} + e28_{jq}\right) \ \forall j,q,l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, \ bA_j \neq bA_q \quad (4.152)$$

$$c_q + ht \leq c_j + MK * \left(3 - x_{jl} - x_{ql} - e16_{jq} + e28_{jq}\right) \ \forall j,q,l, j < q, \ cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, \ bA_j = bA_q \quad (4.153)$$

$$v_j \leq v_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e16_{jq} + e28_{jq} + e29_{jk}\right) \ \forall j,q,l,k \ j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, \quad (4.154)$$

$$c_q + st \leq a_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e16_{jq} + e28_{jq} + e29_{jk}\right) \ \forall j,q,l,k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gA_k \neq 0 \quad (4.155)$$

$$z_j \leq v_k + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} - e16_{jq} + e28_{jq} + e29_{jk}\right) \ \forall j,q,l,k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gB_k \neq 0 \quad (4.156)$$

$$v_k \leq v_j + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e16_{jq} + e28_{jq} - e29_{jk}\right) \ \forall j,q,l,k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.157)$$

$$z_k \leq v_q + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e16_{jq} + e28_{jq} - e29_{jk}\right) \ \forall j,q,l,k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, bA_k \neq 0 \quad (4.158)$$

$$c_k + st \leq a_q + MK * \left(5 - x_{jl} - x_{ql} - x_{kl} - e16_{jq} + e28_{jq} - e29_{jk}\right) \ \forall j,q,l,k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gA_k \neq 0, bB_k \neq 0 \quad (4.159)$$

$$c_k + st \leq a_j + MK * (5 - x_{jl} - x_{ql} - x_{kl} - e16_{jq} + e28_{jq} - e29_{jk}) \ \forall j, q, l, k, j < q, k \neq q, \ j \neq k, cap_l = 1, \ bA_j \neq 0, \ bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0, gB_k \neq 0, bB_k \neq 0 \quad (4.160)$$

$$z_q \leq v_j + MK * (4 - x_{jl} - x_{ql} - e16_{jq} - e28_{jq}) \ \forall j, q, l, j < q, cap_l = 1, bA_j \neq 0, bA_q \neq 0, \ gB_j \neq 0, gB_q \neq 0 \quad (4.161)$$

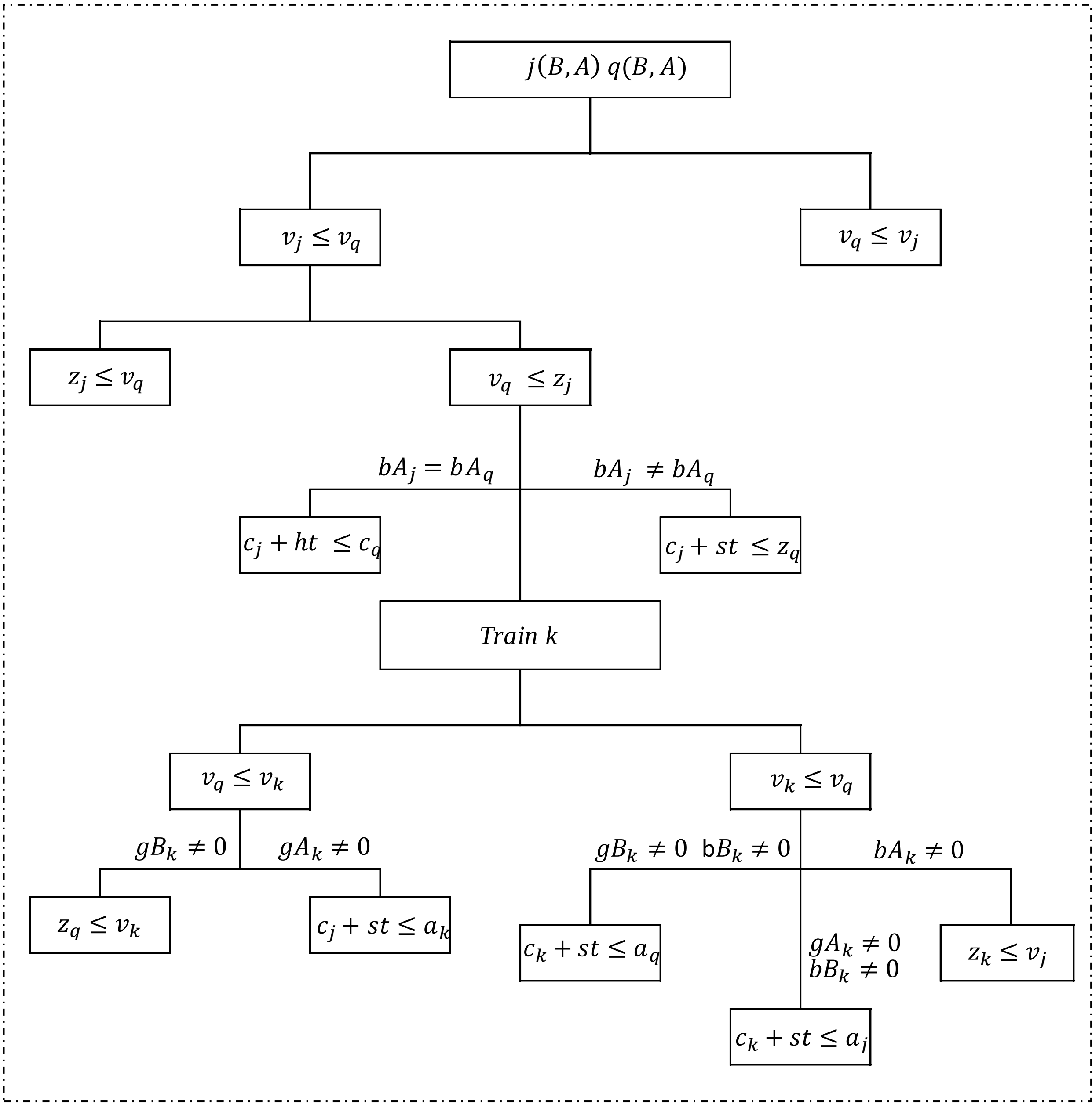

Figure 4-12 Modelling steps for Case 7

✓ ***Case 8 : j(B,A) q(B,B)***

Case 8 states that train *j* and train *q* arrives to the platform track *l* from direction *B* and their departure direction is opposite. If train *j* arrives first (Constraint 4.162) and they encounter at the platform track (Constraint 4.163), there is no restriction regarding their movement. Constraints (4.164) -(4.170) are written for preventing conflicts of trains including train *k* that occupy same platform track. In *Case 8*, it is not possible to share same platform at the same time when train q reaches platform firstly and it is formulated in Constraints (4.172) and (4.173). Because their departure directions are opposite. In Figure 4.13, modelling steps of Case 8 are given.

Case 8-Part I $(\boldsymbol{v_j \leq v_q})$

$$v_j \leq v_q + MK * \left(2 - x_{jl} - x_{ql} + e17_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0 \quad (4.162)$$

$$v_q \leq z_j + MK * \left(2 - x_{jl} - x_{ql} + e17_{jq} + e30_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0 \quad (4.163)$$

$$v_q \leq v_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e17_{jq} + e30_{\mathrm{jq}} + e31_{qk}\right)\ \forall j,q,l,k,j \neq q,\ k \neq q,\ j \neq k, cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0 \quad (4.164)$$

$$c_j + st \leq a_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e17_{jq} + e30_{jq} + e31_{qk}\right)\ \forall j,q,l,k,j \neq q,\ \mathrm{k} \neq q,\ j \neq k, cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0, gA_k \neq 0 \quad (4.165)$$

$$c_q + st \leq a_k + MK * \left(3 - x_{jl} - x_{ql} - x_{kl} + e17_{jq} + e30_{jq} + e31_{qk}\right)\ \forall j,q,l,k,j \neq q,\ k \neq q,\ j \neq k, cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0, gB_k \neq 0 \quad (4.166)$$

$$v_k \leq v_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e17_{jq} + e30_{jq} - e31_{qk}\right)\ \forall j,q,l,k,j \neq q,\ k \neq q,\ j \neq k, cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0 \quad (4.167)$$

$$c_k + st \leq a_q + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e17_{jq} + e30_{jq} - e31_{qk}\right)\ \forall j,q,l,k,j \neq q,\ k \neq q,\ j \neq k, cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0, gB_k \neq 0, bB_{\mathrm{k}} \neq 0 \quad (4.168)$$

$$c_k + st \leq a_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e17_{jq} + e30_{jq} - e31_{qk}\right)\ \forall j,q,l,k,j \neq q,\ k \neq q,\ j \neq k, cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0, gA_k \neq 0, bB_k \neq 0 \quad (4.169)$$

$$z_k \leq v_j + MK * \left(4 - x_{jl} - x_{ql} - x_{kl} + e17_{jq} + e30_{jq} - e31_{qk}\right)\ \forall j,q,l,k,j \neq q,\ k \neq q,\ j \neq k, cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0,\ bA_k \neq 0 \quad (4.170)$$

$$z_j \leq v_q + MK * \left(3 - x_{jl} - x_{ql} + e17_{jq} - e30_{jq}\right)\ \forall j,q,l,j \neq q,\ cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,\ gB_j \neq 0, gB_q \neq 0 \quad (4.171)$$

Case 8-Part II $(v_q \leq v_j)$

$v_q \leq v_j + MK * (3 - x_{jl} - x_{ql} - e17_{jq})\ \forall j, q, l, j \neq q,\ cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,$
$gB_j \neq 0, gB_q \neq 0$ (4.172)

$c_q + st \leq a_j + MK * (3 - x_{jl} - x_{ql} - e17_{jq})\ \forall j, q, l, j \neq q,\ cap_l = 1,\ bA_j \neq 0,\ bB_q \neq 0,$
$gB_j \neq 0, gB_q \neq 0$ (4.173)

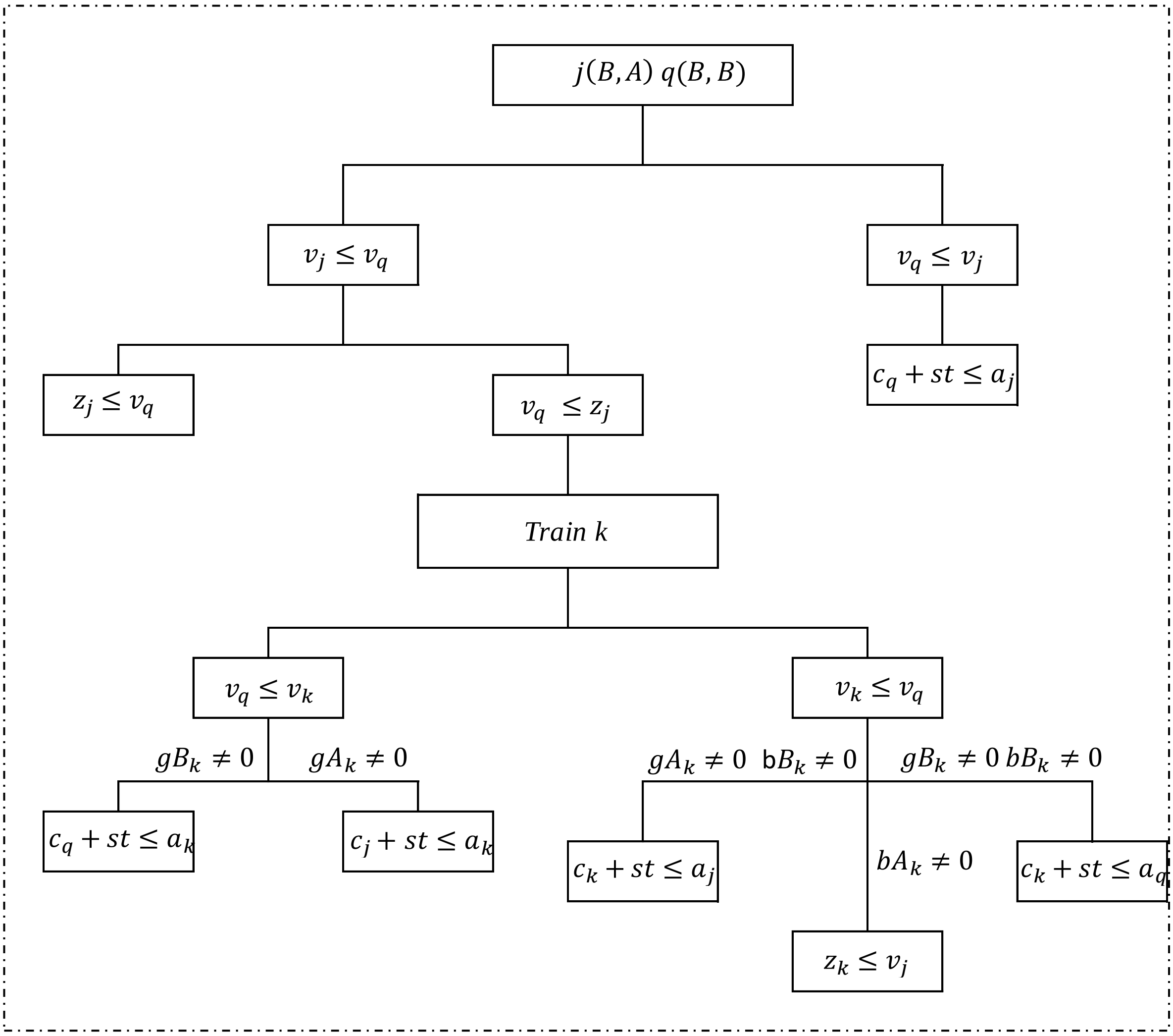

Figure 4-13 Modelling steps for Case 8

4.3.6 Long Trains

As was mentioned in the previous chapter, if track is not dead end track, capacity of platform track is two regional trains. Constraints (4.174)- (4.179) are needed to ensure that capacity of tracks for international trains is one.

$v_j \leq v_q + MK * (2 - x_{jl} - x_{ql} + e20_{jq})\ \forall j, q, l,\ int_j = 1, cap_l = 1, j \neq q$ (4.174)

$c_j + st \leq a_q + MK * \left(2 - x_{jl} - x_{ql} + \ e20_{jq}\right) \ \forall j, q, l, \ int_j = 1, cap_l = 1, j \neq$
$q, ((bB_j \neq 0, gB_q \neq 0) \ or \ (bA_j \neq 0, gA_q \neq 0)$ (4.175)

$z_j \leq v_q + MK * \left(2 - x_{jl} - x_{ql} + \ e20_{jq}\right) \ \forall j, q, l, \ int_j = 1, cap_l = 1, j \neq q, ((bA_j \neq 0,$
$gB_q \neq 0 \ ) \ or \ (bB_j \neq 0, gA_q \neq 0)$ (4.176)

$v_q \leq v_{\mathrm{j}} + MK * \left(3 - x_{jl} - x_{ql} - \ e20_{jq}\right) \ \forall j, q, l, \ int_j = 1, cap_l = 1, j \neq q$ (4.177)

$c_q + st \leq a_j + MK * \left(3 - x_{jl} - x_{ql} - \ e20_{jq}\right) \ \forall j, q, l, \ int_j = 1, cap_l = 1, j \neq$
$q, ((bB_q \neq 0, \ gB_j \neq 0 \ ) \ or \ (bA_q \neq 0, gA_j \neq 0) \ )$ (4.178)

$z_q \leq v_j + MK * \left(3 - x_{jl} - x_{ql} - \ e20_{jq}\right) \ \forall j, q, l, \ int_j = 1, cap_l = 1, j \neq q, ((bA_q \neq 0,$
$gB_j \neq 0 \ ) \ or \ (bB_q \neq 0, gA_j \neq 0) \ )$ (4.179)

4.3.7 Departure Time and Delay

Constraint (4.180) ensures that departure time of train *j* cannot be earlier than its planned departure time.

$z_j \leq d_j \qquad \forall j$ (4.180)

Constraint (4.181) calculates delay of train *j* that is leaving from the platform track.

$t_j = z_j - d_j \qquad \forall j$ (4.181)

4.3.8 Headway Interval for Entry

If trains *j* and *q* arrive to the home signal from same line, and train *j* arrives before train *q*, there must be minimum block interval between their departures from home signals to the platform track. Constraint (4.182) is used for direction *A* and direction *B*.

$a_j + ht \leq a_q \ \forall j, q, j \neq q, (\ (gA_q \neq 0, gA_j \neq 0, gA_j = gA_q, rs_j < rs_q) or \ (gB_q \neq 0, \ gB_j \neq$
$0, gB_j = gB_q, rs_j < rs_q) \ )$ (4.182)

4.3.9 Sign Constraints

$x_{jl} \in \{0,1\} \qquad \forall \ j, l, \ \ j \in N, \ \ l \in L$ (4.183)

$e1_{jq}, e2_{jq}, e3_{jq}, e4_{jq}, e5_{jq}, e6_{jq}, e7_{jq}, e8_{jq}, e9_{jq}, e10_{jq}, e11_{jq}, e12_{jq}, e13_{jq}, e14_{jq}, e15_{jq}, e16_{jq},$
$e17_{jq}, e18_{jq}, e19_{jq}, e20_{jq}, e21_{jq}, e22_{jq} e23_{jq}, e24_{jq}, e25_{jq}, e26_{jq}, e27_{jq}, e28_{jq}, e29_{jq}, e30_{jq},$
$e31_{jq} \in \{0,1\} \ \ \forall \ j, q \ \in N$ (4.184)

$$w1_j \geq 0 \qquad \forall j \in N \qquad (4.185)$$

$$w2_j \geq 0 \qquad \forall j \in N \qquad (4.186)$$

$$a_j \geq 0 \qquad \forall j \in N \qquad (4.187)$$

$$v_j \geq 0 \qquad \forall j \in N \qquad (4.188)$$

$$z_j \geq 0 \qquad \forall j \in N \qquad (4.189)$$

$$T_j \geq 0 \qquad \forall j \in N \qquad (4.190)$$

$$C_j \geq 0 \qquad \forall j \in N \qquad (4.191)$$

Finally, constraints (4.183) -(4.191) are sign constraints of the decision variables.

We called the mathematical model M1. M1 has $(n^2 + nm - n)$ binary variables, $(7n)$ positive variables and $(85n^3m - 195n^2m - 8n^2m^2 - 8nm^2 + 5n^2 - 51nm + 2n)$ constraints.

5 MATHEURISTIC FOR TRAIN PLATFORMING PROBLEM

Due to the computational complexity of integer programming methods, it is need to an effective solution method to overcome train platforming problem. In first section, Np-hard nature of problem is declared. General description and a brief literature are given in the following section. Last section is designed for proposed algorithm. Matheuristic which is consist of three stages is comprehensively explained.

5.1 Problem Complexity

Dissertation works such as that conducted by Galli (1998) and Bai (2016) have investigated complexity of train platforming problem. Galli (1998) discusses two special versions of their considered problem to demonstrate complexity of problem. In the first case, the problem is regarded to Circular Arc Graph Coloring problem. When the arrival and departure times of the trains are fixed and period of one day is considered as a circle, each train corresponds to a circular interval of this circle. To assign a platform to each train means that is assignment of a color to circular interval and objective function is to minimize number of used colors. Subject to Circular Arc Graph Coloring problem, train platforming problem is strongly NP-hard. In the second case, the author defines the problem as Interval Graph μ-Coloring. With this version, they claim that train platforming problem is strongly NP-complete. Another dissertation work made by Bai (2016) searched for an analogy between train platforming problem and flow shop scheduling problem. Jobs are synonyms with trains and each movement of a train referred as a production stage. The author defined train platforming problem as 5-stages no-wait hybrid flow shop scheduling problem with no-identical jobs and no-identical parallel machines. No identical job means that every train has a target arrival or departure time and no-identical parallel machine means that choice of internal lines and paths depends on characteristics of trains. With the help of hybrid flow shop, train platforming problem that is concerned in her dissertation is at least an NP-complete.

In this part, it is briefly explained the fact that train platforming problem is at least strongly NP-hard from our point of view. On this basis, train platforming problem with $cap_l = 0$ can be considered as parallel machine scheduling problem with resource constraints and machine eligibility restrictions. This is achieved by regarding the trains as jobs and platform tracks as machines. The arrival time to the home signals of a train can be expressed by release date that is the earliest time at which train can start its arriving to the platforms. Starting time of a train is equal to sum of the release date and waiting time at home signals. Arrival duration between entry signals and platform is referred to as starting setup time of a train. The train has to cross the particular track using switches that are shared resources for trains. In addition, trains are only assigned platform tracks in order to satisfy railway junction conditions and train length restrictions. These constraints are stated by eligibility restrictions in machine scheduling literature. Sojourn time which is needed time for boarding or other manipulations considered as processing time. Planned departure time can be referred to as due date. In Table 5.1, it is shown terms of analogy between train platforming problem and parallel machine scheduling problem.

Table 5-1 Terms of analogy between train platforming and parallel machine scheduling

Train Platforming Problem	**Parallel Machine Scheduling Problem**
Train	Job
Platform track	Identical parallel machine
Arrival time to the entry signals of a train($\boldsymbol{rs_j}$)	Release date
Departure time from entry signals towards platform($\boldsymbol{a_j}$)	Starting time
Arrival duration between entry signals and platform($\boldsymbol{h_j}$)	Starting setup time
Arrival time to the exit signals of a train($\boldsymbol{C_j}$)	Completion time
Planned departure time($\boldsymbol{d_j}$)	Due Date
Delay($\boldsymbol{T_j}$)	Tardiness
Railway Junctions	Resource
Eligibility restrictions	Machine eligibility restrictions

A scheduling problem is described by a triplet $\alpha|\beta|\gamma$(Pinedo,2012). The α and β provide the machine environment and details of processing characteristics and constraints, respectively. γ field describes the objective to be minimized. Using the three-field classification, train platforming problem with $cap_l = 0$ is denoted in the scheduling literature as

$P_m|s_j, M_j, shared\ resource|\sum w_jT_j$ where P_m designates identical parallel machines, $\sum w_jT_j$ refers to total weighted tardiness.

Pinedo (2012) classified machine scheduling problems according to whether they are polynomial time problems or NP-hard problems. In this classification, $1|r_j|\sum T_j$ and $1||\sum w_jT_j$ are strongly NP-hard. These are simplest versions of restricted train platforming problem with $cap_l = 0$. Based on these assumptions, **it can be said that considered train platforming problem with different track capacity ($cap_l = 1$) is at least strongly NP-hard**.

5.2 Matheuristics

Matheuristic are optimization approaches that metaheuristics are hybridized with a mathematical programming technique. An essential feature of matheuristics combines the advantages of both exact methods and (meta)heuristics. Therefore, an increasing amount of literature pays particular attention to matheuristic in recent years. Some of them is given as follows: Billauta et.al. (2015) handled a scheduling problem that jobs consume a perishable resource stored in vials. The problem is modelled as a single machine scheduling problem with additional duration and consumption constraints. They proposed a two-step approach that consist of a Recovering Beam Search algorithm and a matheuristic algorithm. Guimaraes et. al. (2013) considered single machine capacitated lot sizing and scheduling problem with sequence-dependent setup times and costs. They presented a MIP-based heuristic(matheuristic) model for solving this problem. Singh et. al. (2012) focused sensor coverage scheduling in wireless sensor networks subject to Q-coverage constraints. The objective of problem is to maximize the network lifetime. A matheuristic algorithm that includes GA and linear programming model is developed. However, there are relatively only a few studies that are concerned with matheuristic and railway optimization problems together (Zhu et. Al(2014), Yaghini et. al (2015), Haahr and Lusby (2016)). Yaghini et. al (2015) proposed a set covering approach for multi-depot train driver scheduling. The authors present a matheuristic algorithm by combining a tabu search metaheuristic and neighborhood structure. Haahr and Lusby (2016) deals with hump yard block-to-track assignment problem. Proposed matheuristic approach decomposes the problem into three highly dependent sub-problems and successful results are achieved.

5.3 Proposed Matheuristics

Train platforming problem, especially if it belongs to the busy and complex railway stations such as Prague, cannot be solved easily, due to their NP-hard nature. An efficient matheuristic algorithm would be useful to tackle such a problem. Therefore, proposed matheuristic consists of three steps: (1) Platform track assignment algorithm, (2) Calculation of total weighted delay and (3) Improvement algorithm. Firstly, with the help of assignment algorithm, it is determined platform track assignment of trains on basis of eligibility restrictions. Second model calculates the all decision variables in the model. In case total weighted delay is bigger than zero ($T_j > 0$), improvement algorithm examines for elimination of train delays. Steps of matheuristic is explained in detail as below.

5.3.1 Platform Track Assignment Algorithm

The major issue for assignment algorithm generates initial solution that should be feasible. To ensure feasibility of solution completely depends on two important restrictions that is point out in Constraints (3.6) and (3.7). First one, all trains should be assigned to a platform track and second one, trains are only assigned to a platform track that guarantees platform track eligibility restrictions.

The quality of initial solution directly affects performance of matheuristic. So, it can be useful to follow a strategy for obtaining better initial solutions. If choice of platform track is not made randomly and we consider platform track preferences like Bazant and Kavicka (2014), waiting time of trains at home signal and platform track can be decreased. Hence, total delay of trains can be get smaller.

The main idea of platform track preference is to avoid crossing movement of trains(crossing) and run parallel to each other(paralleling) as far as possible. "Crossing" situation can be expressed that paths of trains are cross and one of them also wait for releasing of shared switches. It is called "paralleling" situation that lines and assigned platform tracks of train *j* and *k* are quite close to each other and their paths are parallel. They should wait to each other because of shared switches.

As indicated previously, platform track assignment algorithm aims to find an initial solution on basis of preferences list and eligibility restrictions. However, the algorithm determines value of decision variable "x_{jl}" that gives information about a particular jobs' assignment in which platform track. Pseudo code of platform track assignment algorithm is presented in Figure 5.1.

A whole day timetable may be divided into time intervals. The set of timetable intervals is marked as $TT = \{1, 2, \ldots tt\}$ and subscript *ts* refers to a time interval in a day. As was mentioned in the mathematical model chapter, the set N of trains includes all trains in the considered time interval. In the proposed matheuristic, the set of trains N consists of two subsets: assigned trains and unassigned trains. Assigned train refers that platform track assignment of this train is known at the beginning of time interval $ts \in TT$ and assigned train set is denoted by $N_A = \{1, 2, \dots k\}$. Track assignment of unassigned trains has not determined yet and set of unassigned trains is marked as $N_{UA} = \{k+1, k+2, \dots n\}$. If train $j \in N_{UA}$ is assigned to a platform track, it is no longer that belongs to the set $j \in N_A$. The union of N_c and N_e which defines all trains in the matheuristic is denoted by $N = \{1,2, \dots n\}$.

Each train $j \in N$ has an associated arrival line and an associated departure line. The parameter gbb_j refers to combination of gA_j/gB_j and bA_j/bB_j. For instances, gbb_j gets value between $LN \in [1,25]$ for each train $j \in N$ and a railway station with five arrival lines and five departure lines. The parameter $pref(lin, pr)$ is defined as preferences list of trains which reflects the desirability of the assignment of platform track *l* to train *j*. It is determined based on the parameter gbb_j that is combination of arrival and departure line of trains.

In the algorithm, decision of assignment of $j \in N_{UA}$ firstly depends on preference list of train *j*. Part 1 states that if one of the preference platform tracks has not occupied by a train yet, train *j* should assign to one of them considering order of preference. In case all track preferences of train *j* are allocated by at least one train, departure time of train $q \in N_A$ which is lastly assigned one of the tracks on preference list of train $j \in N_{UA}$ is highly important. Platform track of train $j \in N_{UA}$ are specified according to difference between departure time train *j* and train *q*(*dt1*). Part 2 is designed for this situation. Part 1 and part 2 does not meet the conditions for assignment of train j, if possible, train *j* is assigned a platform track which has not occupied by a train yet. Part 3 is written for this. The last part provides that train *j* is definitely assigned to a track no matter which one is chosen.

- If $j \in N_{UA}$, $ass_j = 0$ and $x_{jl} = 0$.
- If $j \in N_A$, $ass_j = 1$ and $x_{jl} = 1$ and $last_l = j$.

```
For j=1 : Trains do
Part 1
   if ass(j)=0 then
     for lin=1 : CombinationofArrivalandDepartureLines do
       if gbb(j)=lin then
         for pr=1 : Preferences do
           for l=1 : PlatformTracks in condition that l=pref(lin,pr) do
             if last(l)=0 and b(j,l)=1 then
             x(j,l)=1, ass(j)=1, last(l)=j,
             end if
           end for
         end for
       end if
     end for
   end if
Part 2
   if ass(j)=0 then
     for lin=1 : CombinationofArrivalandDepartureLines do
       if gbb(j)=lin then
         for pr=1 : NumberofPreferences do
           for l=1 : PlatformTracks in condition that l=pref(lin,pr) do
             for q=1 AssignedTrainsBeforeTrainj do
               if last(l)=q and b(j,l)=1 and d(q)+dt1<d(j) then
               x(j,l)=1, ass(j)=1, last(l)=j,
               end if
             end for
           end for
         end for
       end if
     end for
   end if
Part 3
   if ass(j)=0 then
     for l=1 : PlatformTracks do
         if last(l)=0 and b(j,l)=1 then
         x(j,l)=1, ass(j)=1, last(l)=j,
         end if
     end for
   end if
Part 4
   if ass(j)=0 then
     for l=1 : PlatformTracks do
       for q=1 : AssignedTrainsBeforeTrainj do
         if last(l)=q and b(j,l)=1 and d(q)+dt2<d(j) then
         x(j,l)=1, ass(j)=1, last(l)=j,
         end if
       end for
     end for
   end if
End for
```

Figure 5-1 Pseduo Code for Platform Track Assignment Algorithm where *last(l):* last train at platform track l. *pr:*order of preferences $dt1$, $dt2$: difference parameters between departure time of train j and train q. *ass(j)*: if train j is assigned 1, othewise 0. $lin \in LN$.

5.3.2 Calculation of Total Weighted Delay

The objective function of considered train platforming problem is minimization of total weighted delay. Value of decision variable "x_{jl}" in the mathematical model is designated by the means of platform track assignment algorithm. After determination the value of x_{jl}, specifying the values of the remaining decision variables is still difficult and it is still a decision problem. Based on platform track assignment algorithm, we solved a sub-problem (M2) for determining delay of train j (T_j), waiting times at home signals ($w1_j$) and platform tracks ($w2_j$) and the other parameters (a_j, v_j, z_j) under all constraints. The sub-problem (M2) is very similar to M1 but the x_{jl} are parameters and not decision variables and M1 is tightened using the x_{jl} values. So, we can easily find optimal solution in a short time when the sub-problem M2 is solved.

The sub-problem (M2) naturally includes objective function (4.1) that is minimization of total weighted delay and Constraints (4.2)-(4.5) that calculates decision variables of each movements of train j. However, assignment constraint (4.6) and eligibility of train to the tracks constraint (4.7) should remove.

$$v_q + st \leq a_j + MK * \left(e1_{jq}\right) \ \ \forall j,q,l,m,\ j < q,\ l \neq m, x_{jl} = 1, x_{qm} = 1, \left(\left(gA_j \neq 0, gA_q \neq 0,\ \textstyle\sum_{r=1}^{g} res1_{jlr} \cdot res1_{qmr} > 0\right) or\ \left(gB_j \neq 0, gB_q \neq 0, \textstyle\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0\right)\right) \quad (5.1)$$

$$v_j + st \leq a_q + MK * \left(1 - e1_{jq}\right) \ \ \forall j,q,l,m,\ j < q, l \neq m, x_{jl} = 1,\ x_{qm} = 1\left(\left(gA_j \neq 0, gA_q \neq 0,\ \textstyle\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0\right) or\ \left(gB_j \neq 0, gB_q \neq 0, \textstyle\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0\right)\right) \quad (5.2)$$

In M1 model, Constraints (4.8) -(4.25) ensure the safety conditions when the trains encounter in the switch area. M2 model also should guarantee encountering constraints. However, these constraints should convert based on parameter *"x_{jl}"*. In Constraints (5.1) and (5.2), the new version of Constraints (4.8) and (4.9) is given. In similar way, Constraints (4.10) -(4.179) should be also transformed to their new versions in M2 model.

$$v_j \leq v_q + MK * \left(e7_{jq}\right) \ \forall j,q,l,j < q, cap_l = 1, x_{jl} = 1, x_{ql} = 1\ ((bA_j \neq 0, bA_q \neq 0, gA_q \neq 0, gA_j \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0)) \quad (5.3)$$

$$v_q \leq z_j + MK * \left(e7_{jq} + e21_{jq}\right) \ \forall j,q,l,j < q, cap_l = 1, x_{jl} = 1, x_{ql} = 1((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0)\ \ or\ \ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0)) \quad (5.4)$$

$$c_q + st \le z_j + MK * \left(e7_{jq} + e21_{jq}\right) \ \forall j,q,l,j<q, cap_l = 1, x_{jl} = 1, x_{ql} = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0\ , bA_j \neq bA_q)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0,\ bB_j \neq bB_q\ )) \tag{5.5}$$

$$c_q + ht \le c_j + MK * \left(e7_{jq} + e21_{jq}\right) \ \forall j,q,l,j<q, cap_l = 1, x_{jl} = 1, x_{ql} = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0\ , bA_j = bA_q)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0,\ bB_j = bB_q\ )) \tag{5.6}$$

$$c_j + st \le a_q + MK * \left(1 + e7_{jq} - e21_{jq}\right) \ \forall j,q,l,j<q, cap_l = 1, x_{jl} = 1, x_{ql} = 1, ((bA_j \neq 0,\ bA_q \neq 0, gA_q \neq 0, gA_j \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0\ )) \tag{5.7}$$

$$v_q \le v_k + MK * \left(e7_{jq} + e21_{jq} + e8_{qk}\right) \ \forall j,q,l,k,\ j<q, j \neq k,\ q \neq k, cap_l = 1,\ x_{jl} = 1, x_{ql} = 1, x_{kl} = 1((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0)) \tag{5.8}$$

$$c_q + st \le a_k + MK * \left(e7_{jq} + e21_{jq} + e8_{qk}\right) \ \forall j,q,l,k,\ j<q, j \neq k,\ q \neq k, cap_l = 1, x_{jl} = 1, x_{ql} = 1, x_{kl} = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gA_k \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gB_k \neq 0)) \tag{5.9}$$

$$z_j \le v_k + MK * \left(e7_{jq} + e21_{jq} + e8_{qk}\right) \ \forall j,q,l,k,\ j<q, j \neq k,\ q \neq k, cap_l = 1, x_{jl} = 1,\ x_{ql} = 1, x_{kl} = 1((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gB_k \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0,\ bB_q \neq 0, gB_j \neq 0, gA_k \neq 0)) \tag{5.10}$$

$$v_k \le v_q + MK * \left(1 + e7_{jq} + e21_{jq} - e8_{qk}\right) \ \forall j,q,l,k,\ j<q, j \neq k,\ q \neq k, cap_l = 1, x_{jl} = 1, x_{ql} = 1, x_{kl} = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0)\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0,\ gB_j \neq 0)) \tag{5.11}$$

$$c_k + st \le a_q + MK * \left(1 + e7_{jq} + e21_{jq} - e8_{qk}\right) \ \forall j,q,l,k,\ j<q, j \neq k,\ q \neq k, cap_l = 1, x_{jl} = 1, x_{ql} = 1, x_{kl} = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gA_k \neq 0, bA_k \neq 0)\ or\ (bB_j \neq 0,\ gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gB_k \neq 0, bB_k \neq 0,)) \tag{5.12}$$

$$c_k + st \le a_j + MK * \left(1 + e7_{jq} + e21_{jq} - e8_{qk}\right) \ \forall j,q,l,k,\ j<q, j \neq k,\ q \neq k, cap_l = 1, x_{jl} = 1, x_{ql} = 1, x_{kl} = 1, ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, ((\ gA_k \neq 0, bB_k \neq 0) or (gB_k \neq 0,\ bA_k \neq 0)))\ or\ (bB_j \neq 0, gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, ((\ gB_k \neq 0, bA_k \neq 0) or (\ gA_k \neq 0,\ bB_k \neq 0))))) \tag{5.13}$$

$$z_k \le v_j + MK * \left(1 + e7_{jq} + e21_{jq} - e8_{qk}\right) \ \forall j,q,l,k,\ j<q, j \neq k,\ q \neq k, cap_l = 1, x_{jl} = 1, x_{ql} = 1, x_{kl} = 1,\ ((bA_j \neq 0, bA_q \neq 0,\ gA_q \neq 0, gA_j \neq 0, gB_k \neq 0, bB_k \neq 0))\ or\ (bB_j \neq 0,\ gB_q \neq 0, bB_q \neq 0, gB_j \neq 0, gA_k \neq 0, bA_k \neq 0)) \tag{5.14}$$

Constraints (4.26)-(4.173) are formulated for platform position constraints. Constraints (4.28)-(4.39) prevent the overlapping of trains when three cases occur: (i) Arrival and departure directions of train *j* and q are the same (Direction *A/B*), (ii) they are assigned to the platform track *l*, (iii) train *j* arrives at the platform track before train *q*. In M2 model, these constraints are transformed to the Constraint (5.3)-(5.14).The other constraints (4.10)-(4.27) and (4.25)-(4.173) can be easily rewritten in a similar manner as above.

$$v_j \leq v_q + M\text{K} * \left(e20_{jq} \right) \; \forall j,q,l, \; int_j = 1, cap_l = 1, j \neq q, x_{jl} = 1, x_{ql} = 1 \tag{5.15}$$

$$c_j + st \leq a_q + MK * \left(e20_{jq} \right) \; \forall j,q,l, \; int_j = 1, cap_l = 1, j \neq q, x_{jl} = 1, x_{ql} = 1, \left(\left(bB_j \neq 0, gB_q \neq 0\right) or \left(bA_j \neq 0, gA_q \neq 0\right)\right. \tag{5.16}$$

$$z_j \leq v_q + MK * \left(e20_{jq} \right) \; \forall j,q,l, \; int_j = 1, cap_l = 1, j \neq q, x_{jl} = 1, x_{ql} = 1, (\left(bA_j \neq 0, gB_q \neq 0 \;\right) or \left(bB_j \neq 0, gA_q \neq 0\right) \tag{5.17}$$

$$v_q \leq v_\text{j} + MK * \left(1 - \; e20_{jq}\right) \; \forall j,q,l, \; int_j = 1, cap_l = 1, j \neq q, x_{jl} = 1, x_{ql} = 1, \tag{5.18}$$

$$c_q + st \leq a_j + MK * \left(1 - \; e20_{jq}\right) \; \forall j,q,l, \; int_j = 1, cap_l = 1, j \neq q, x_{jl} = 1, x_{ql} = 1, (\left(bB_q \neq 0, \; gB_j \neq 0 \;\right) or \left(bA_q \neq 0, gA_j \neq 0\right)) \tag{5.19}$$

$$z_q \leq v_j + MK * \left(1 - \; e20_{jq}\right) \; \forall j,q,l, \; int_j = 1, cap_l = 1, j \neq q, x_{jl} = 1, x_{ql} = 1, (\left(bA_q \neq 0, \; gB_j \neq 0 \;\right) or \left(bB_q \neq 0, gA_j \neq 0\right)) \tag{5.20}$$

In M2 model, long trains such as international trains also use both two trains capacity of track. So, it is not possible to share the platform track with another train. Constraints (5.15)- (5.20) are formed in order to ensure mentioned conditions in similar way as M1 model. The other constraints of M2 model are completely same with M1 model. The equation number of these are given as follows:

(4. 180)- (4. 182)

(4. 184)-(4. 191)…

5.3.3 Improvement Algorithm

If the objective function of M2 model is bigger than zero, it means that some of trains are waiting at home signal and/or platform track. To eliminate waiting of trains, it should be known where it occurs and which trains cause to be delay of others. The parameter $cor(nt, j, q)$ is defined for this case where *nt* declares reason of waiting, train *j* is waited train and train *q* is the reason of waiting. Based on waiting points of trains, reason of waiting can be detected as follows:

Possible reasons of waiting at home signals ($\boldsymbol{w1_j}$)

a) If arrival time to the platform track of train *j* and q (rs_j, rs_q) does not meet headway interval constraints, waiting time at home signal of one of these trains is definitely bigger than zero. It can be referred as follows:

* *cor ("1", j, q)*: if train *j* is waiting for train *q* at home signals, *cor ("1", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
  for $q = 1$ **in condition that** $j \neq q$ **do**
    if $w1_j > 0$ **and** $a_j - a_q = ht$ **and** $((gA_j \neq 0, gA_q \neq 0, gA_j = gA_q)$
      or $(gB_j \neq 0, gB_q \neq 0, gB_j = gB_q))$ **then** $cor_{1jq} = 1$
    end if
  end for
End for

b) If train *j* and train *q* are assigned to the platform track *l* consecutively and at least one of them is international train (long train), waiting time at home signal of latter train depends on departure time of former one. This situation can be defined as follows:

* *cor ("2", j, q)*: if train *j* is waiting for train *q* at home signals, *cor ("2", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
  for $q = 1$ **in condition that** $j \neq q$ **do**
   for $l = 1$ **do**
    if $w1_j > 0$ **and** $z_q + pt = v_j$ **and** $x_{jl} = 1$ **and** $x_{ql} = 1$ **and** $((int_j = 1)$
      or $(int_q = 1))$ **then** $cor_{2jq} = 1$
    end if
   end for
  end for
End for

c) If train *j* is waiting for availability of switch(es), waiting time of train *j* is bigger than zero($w1_j > 0$). To determine of train *q* that cause to delay of train *j*, it is important to know in which position they encounter. As train *j* waits at home signal (it is entering to the station), there are two possible positions for train *q*: *(I)* Entering to the station, *(II)* Leaving from platform track. For each case, the procedures are given as follows:

(I) *Entry (j)- Entry (q)*

* *cor (“3”, j, q)*: if train *j* is waiting for train *q* at home signals, *cor (“3”, j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
for $q = 1$ **in condition that** $j \neq q$ **do**
for $l = 1$ **do**
for $m = 1$ **in condition that** $l \neq m$ **do**
if $w1_j > 0$ **and** $v_q + st = a_j$ **and** $((gA_j \neq 0, gA_q \neq 0, gA_j \neq gA_q)$ **or** $(gB_j \neq 0, gB_q \neq 0, gB_j \neq gB_q))$ **and** $x_{jl} = 1$ **and** $x_{qm} = 1$ **and** $\sum_{r=1}^{g} res1_{jlr} * res1_{qmr} > 0$ **then** $cor_{3jq} = 1$
end if
end for
end for
end for
End for

(II) *Entry (j)- Exit (q)*

* *cor (“4”, j, q)*: if train *j* is waiting for train *q* at home signals, *cor (“4”, j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
for $q = 1$ **in condition that** $j \neq q$ **do**
for $l = 1$ **do**
for $m = 1$ **in condition that** $l \neq m$ **do**
if $w1_j > 0$ **and** $c_q + st = a_j$ **and** $((gA_j \neq 0, bA_q \neq 0)$ **or** $(gB_j \neq 0, bB_q \neq 0))$ **and** $x_{jl} = 1$ **and** $x_{qm} = 1$ **and** $\sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0$ **then** $cor_{4jq} = 1$
end if
end for
end for
end for
End for

d) If two consecutive trains are assigned to platform track *l* and the capacity of track *l* is one train, latter train should wait at home signals for being available of track. However, only capacity of dead-end platform tracks is one train in considered railway station and latter train should wait for not only platform track but also it should stay for releasing shared switch(es). This case can be referred as below:

* *cor ("5", j, q)*: if train *j* is waiting for train *q* at home signals, *cor ("5", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
for $q = 1$ **in condition that** $j \neq q$ **do**
for $l = 1$ **do**
if $w1_j > 0$ **and** $c_q + st = a_j$ **and** $cap_l = 0$ **and** $x_{jl} = 1$ **and** $x_{ql} = 1$
then $cor_{5jq} = 1$
end if
end for
end for
End for

e) If capacity of platform track *l* is two trains and train *j* and *q* have opposite departure directions, it is not possible to share track *l*. Latter train should wait at home signal until former train arrives to the exit signal owing to the fact that they use common switch(es). The procedure of this case is presented as below:

* *cor ("6", j, q)*: if train *j* is waiting for train *q* at home signal, *cor ("6", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
for $q = 1$ **in condition that** $j \neq q$ **do**
for $l = 1$ **do**
if $w1_j > 0$ **and** $c_q + st = a_j$ **and** $cap_l = 1$ **and** $x_{jl} = 1$ **and** $x_{ql} = 1$
and $\left(\left(gA_j \neq 0, bA_q \neq 0, bB_j \neq 0\right)\right.$ **or** $\left.\left(gB_j \neq 0, bB_q \neq 0, bA_j \neq 0\right)\right)$
then $cor_{6jq} = 1$
end if
end for
end for
End for

f) As previously stated, we called "coupled trains" if train *q* and *k* share the same platform track at the same time. In some cases, train *q* can be also coupled train *j*. On the other hand, if direction of train *j* is not compatible with train *q*, it should wait at home signal for being available of track *l*. The procedure can be defined on basis of directions of train *j* and *q* as follows:

(I) Arrival direction of train j and departure direction of train q is <u>same</u>;

* *cor ("7", j, q)*: if train *j* is waiting for train *q* at home signal, *cor ("7", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
 for $q = 1$ **in condition that** $j \neq q$ **do**
  for $l = 1$ **do**
   if $w1_j > 0$ **and** $z_q + pt = a_j$ **and** $cap_l = 1$ **and** $x_{jl} = 1$ **and** $x_{ql} = 1$ **and** $((gA_j \neq 0, bB_q \neq 0)$ **or** $(gB_j \neq 0, bA_q \neq 0))$
   then $cor_{7jq} = 1$
   end if
  end for
 end for
End for

(II) Arrival direction of train j and departure direction of train q is <u>different</u>;

* *cor ("8", j, q)*: if train *j* is waiting for train *q* at home signal, *cor ("8", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
 for $q = 1$ **in condition that** $j \neq q$ **do**
  for $l = 1$ **do**
   if $w1_j > 0$ **and** $c_q + st = a_j$ **and** $cap_l = 1$ **and** $x_{jl} = 1$ **and** $x_{ql} = 1$ **and** $((gA_j \neq 0, bA_q \neq 0)$ **or** $(gB_j \neq 0, bB_q \neq 0))$
   then $cor_{8jq} = 1$
   end if
  end for
 end for
End for

Possible reasons of waiting at platform track ($w2_j$)

a) If train *j* is at the platform track *l* and it completes boarding and it is waiting for availability of switch(es), $w2_j > 0$. When train *j* waits at the platform track (it is leaving from station), there are two possible positions for train *q*: *(I)* Entering to the station, *(II)* Leaving from platform track. For each case, the procedures are given as follows:

(I) <u>Exit (j)- Entry (q)</u>

* *cor ("9", j, q)*: if train *j* is waiting for train *q* at the platform track, *cor ("9", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
  for $q = 1$ **in condition that** $j \neq q$ **do**
    for $l = 1$ **do**
      for $m = 1$ **in condition that** $l \neq m$ **do**
        if $w2_j > 0$ **and** $v_q + st = z_j$ **and** $((bA_j \neq 0, gA_q \neq 0)$ **or** $(bB_j \neq 0, gB_q \neq 0))$ **and** $x_{jl} = 1$ **and** $x_{qm} = 1$ **and** $\sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0$ **then** $cor_{9jq} = 1$
        end if
      end for
    end for
  end for
End for

(II) *<u>Exit (j)- Exit (q)</u>*

* *cor ("10", j, q)*: if train *j* is waiting for train *q* at the platform track, *cor ("10", j, q)* is equal to 1, otherwise 0.

For $j = 1$ **do**
  for $q = 1$ **in condition that** $j \neq q$ **do**
    for $l = 1$ **do**
      for $m = 1$ **in condition that** $l \neq m$ **do**
        if $w2_j > 0$ **and** $c_q + st = z_j$ **and** $((bA_j \neq 0, bA_q \neq 0, bA_j \neq bA_q)$ **or** $(bB_j \neq 0, bB_q \neq 0, bB_j \neq bB_q))$ **and** $x_{jl} = 1$ **and** $x_{qm} = 1$ **and** $\sum_{r=1}^{g} res2_{jlr} * res2_{qmr} > 0$ **then** $cor_{10jq} = 1$
        end if
      end for
    end for
  end for
End for

b) If train *j* and *q* are leaving from station on the same line, a headway time interval should ensure. If not, waiting time at the platform track of train *j* is certainly bigger than zero. It can be referred as follows:

* *cor ("11", j, q)*: if train *j* is waiting for train *q* at the platform track, *cor ("11", j, q)*: is equal to 1, otherwise 0.

For $j = 1$ **do**
  for $q = 1$ **in condition that** $j \neq q$ **do**
      if $w2_j > 0$ **and** $c_j - c_q = ht$ **and** $((gA_j \neq 0, gA_q \neq 0, gA_j = gA_q)$ **or** $(gB_j \neq 0, gB_q \neq 0, gB_j = gB_q))$ **then** $cor_{11jq} = 1$
      end if
  end for
End for

After determination of the reason for waiting time of trains, it should be focused on elimination of delays. So, it is necessary to follow a set of strategies. The strategies that we called recovery algorithm should be based on change of assignments of platform track. Figure 5.2 provides the pseudo code of recovery algorithm. It should be noted that the algorithm should take into account separately the waiting times at the home signal$(w1_j)$ and at the platform track$(w2_j)$.

In pseudo code of algorithm, they are defined as Part 1 and Part 2 respectively on basis of possible waiting points of delayed trains. Part 1 indicates that the decision of reassignment of delayed train *j* are based on three different conditions. In the first condition, if train *j* is delayed due to train *q* and the value of cor_{1jq} and cor_{2jq} is equal to one, it is not allowed to change of assignment of trains so such delays are caused by timetable. The delay of considered train certainly does not decrease wherever trains are assigned. The second condition, if train *j* and *q* are assigned to same platform track *l* and waiting is caused by capacity or availability of platform track, one of the trains should relocate in regard to its preference list. Last condition in Part 1 states that the waiting time of train *j* depends on train *q* by reason of using shared railway junctions. To eliminate delay of train *j*, it should be allocated to one of the nearest platform track. To decide platform track assignment, it is highly important to know that in which position they encounter. Part 2 regards as waiting time at the platform track and the reason of waiting is exactly shared railway junctions. As implied in last condition of part 1, one of the trains should assign the neighbor platform tracks. In both situations, trains should be allocated to the tracks which strictly meet eligibility restrictions.

**change(j,q):if train j is delayed by train q, the algorithm is updated platform track assignment of trains, it is equal to 1, otherwise 0. At the beginning,* $change_{jq} = 0$ *for all* $j, q \in N$

```
For j = 1 do
  If w1_j > 0,
   for q = 1 in condition that j ≠ q do
     If (cor1_jq = 1 or cor11_jq = 1) and change_jq = 0
        then change_jq = 1 ;
     Else if (cor_2jq = 1 or cor_5jq = 1 or cor_6jq = 1 or cor_7jq = 1 or cor_8jq = 1)
        and change_jq = 0 then
          for l = 1 in condition that x_jl = 1 and x_ql = 1 do
            for lin = 1 do
              if gbb_j = lin then
                for pr = 1 do
                  for m = 1 in condition that m = pref(lin, pr) do
                    x_jl = 0, x_jm = 1, change_jq = 1
                  end for
1               end for
              end if
            end for
          end for
     Else if (cor_3jq = 1 or cor_4jq = 1) and change_jq = 0 then
          for l = 1 in condition that x_jl = 1 do
            for m = 1 in condition that x_qm = 1 do
              if b_jl+1 = 1 then
                x_jl = 0 , x_jl+1 = 1, change_jq = 1
              end if
            end for
          end for
     End if
   Else if w2_j > 0,
    for q = 1 in condition that j ≠ q do
       If (cor_9jq = 1 or cor_10jq = 1) and nochange_jq = 0 then
          for l = 1 in condition that x_jl = 1 do
            for m = 1 in condition that x_qm = 1 do
              if b_jm+1 = 1 then
2               x_jl = 0 , x_jm+1 = 1, change_jq = 1
              end if
            end for
          end for
        End if
     end for
  End if
End for
```

Figure 5-2 Pseudo Code of Recovery Algorithm

In previous pages, it is focused on the milestone of improvement algorithm which determination reason of delay and how we cope with it. Let us now consider all improvement algorithm. The procedure of improvement algorithm that recovery algorithm is embedded works as follows:

Improvement algorithm

Step 1: Create a new set $WW1 = \{ww1_j: ww1_j = wt_j * w1_j, \forall j \in N\}$

Step 2: Find the $tj = \arg\max_j \{ww1_j: ww1_j = wt_j * w1_j, \forall j \in N\}$

Step 3: Determine $(tj, tq) = \{nt \in TN, tq \in N : cor(nt, tj, tq) = 1\}$

Step 4: Apply *recovery algorithm* for tj and tq.

Step 5: Update of assignment of $train_{tj}$

Step 6: Solve decision model for calculation of total weighted delay. If former one is Step 7, go directly to Step 8.

Step 7: $change_{tjtq} = 1$ and if objection function is smaller, Update the platform track of considered $train_{tj}$. If not, go to Step 6 without changing of any assignment.

Step 8: Update $WW1 = \{ww1_j: ww1_j = wt_j * w1_j, \sum_q change_{jq} = 0, \forall j \in N\}$

Step 9: Create a new set $WW2 = \{ww2_j: ww2_j = wt_j * w2_j, \sum_q change_{jq} = 0, delay_j = 0, \forall j \in N\}$

Step 10: Update $tj = \arg\max_j \{ww1_j, ww2_j: WW1 \cup WW2, \forall j \in N\}$, if $tj = \emptyset$, go to Step 12.

Step 11: Determine $(tj, tq) = \{nt \in TN, tq \in N : cor(nt, tj, tq) = 1\}$

Step 12: If $\{tj, tq: w1_j > 0, w2_j > 0, change_{tjtq} = 0\} = \emptyset$, then STOP. If not, GO to Step 4.

At the beginning steps of improvement algorithm, it focusses on only waiting times at home signal since waiting at home signals can caused to waiting at platform track. After determination of index of $train_{tj}$ which has maximum weighted waiting time at home signal, index of $train_{tq}$ which is caused to waiting of $train_{tj}$ is detected in Step 3. To improve the solution, recovery algorithm is applied and according to objective function value and the $change_{tjtq}$ parameter (it shows whether assignment of $train_{tj}$ is updated before), solution is accepted or not. If it is accepted, allocated platform track to $train_{tq}$ should be updated. If not, we should continue with current solution. Based on waiting time which includes home signal and platform track, step 10 and 11 determine maximum weighted waiting time and the reason of waiting and return to Step 4 and apply recovery algorithm. While waiting times are equal to zero for $train_{tj}$ that its platform track is not updated before, apply improvement algorithm.

6 COMPUTATIONAL RESULTS

To demonstrate performance of the MIP model and matheuristic, toy examples and case study of platform track allocation problem in Prague main railway station has been realized. To enable this testing, proposed model and algorithm formulation have been coded in GAMS 24.0.2 and run using the Cplex solver embedded in GAMS on a 2.4 GHz Intel Core i7 with 8 GB RAM. In the subsections that follow, verification and validation of proposed methods are performed.

6.1 Verification

In this sub-section, mathematical model and matheuristic algorithm are verified with toy problems. At first, we will show the performance of mathematical model and then we will test matheuristic algorithm.

6.1.1 Verification of Mathematical Model

It can be difficult to verify for each constraint since train platforming problem concerned in this dissertation has a complex structure. However, some constraint groups resemble each other in many aspects. By means of several toy problems, accuracy and effectiveness of mathematical model and algorithm can be proved.

The railway station topology for toy problems is shown in Figure 6.1 and it consists of 3 platform tracks, 8 railway junctions and 4 lines. The capacity of track II and III is two trains, however track I is one train. This topology is prepared with an effort to show various possibilities and limitations of the infrastructure e.g. that is impossible to go from the platform track I to the line 2B.

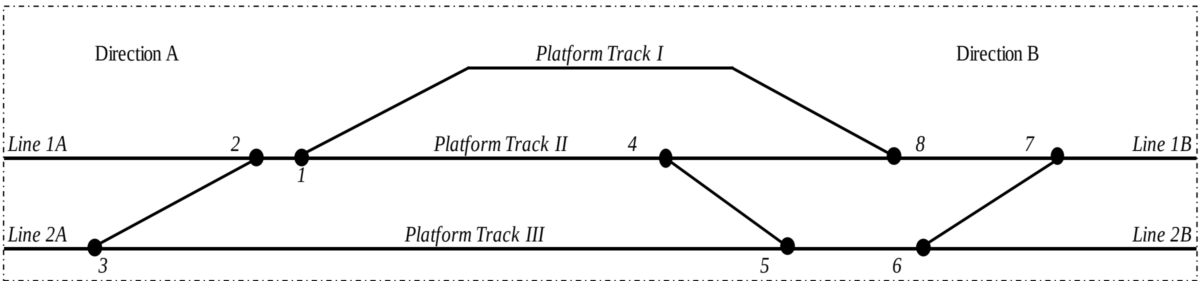

Figure 6-1 Railway station topology

There are seven arriving trains to the railway station. The parameters that are associated with necessary minimum time for assignment of trains are given: *st*=1, *pt*=0 and *ht*=2 Arrival duration of train *j* from home signal to platform*(h_j)* and arrival duration of train *j* from platform to the out of the station*(sp_j)* are given as follows: $h_j = \{1,2,1,2,1,2,2\}$, $sp_j = \{1,1,2,1,1,1,1\}$ where $j = 1, \ldots, 7$. The other required parameters are shown in Table 6.1. Train *3* and *5* are international trains and they terminate in considered station so that the parameter $delay_3 = 1$ and $delay_5$ is equal to 1 and waiting time of train *6* at the platform track cannot affect the value of objective function.

Table 6-1 Required parameters for toy problem 1

Train	***Weights of Trains(wt_j)***	***Arrival Direction (gA_j, gB_j)***	***Departure Direction (bA_j, bB_j)***	***Actual Arrival Time to the Home Signals (rs_j)***	***Sojourn Time(p_j)***	***Planned Departure Time(d_j)***	***Suitable Platform Tracks (b_{jl})***
1	*4*	*2A*	*2B*	09:00	5	09:06	*III*
2	*5*	*2A*	*2B*	09:01	3	09:06	*II, III*
3	*1*	*1B*	*1A*	09:05	6	09:12	*II*
4	*2*	*1B*	*2B*	09:07	5	09:14	*II, III*
5	*5*	*2A*	*2B*	09:07	6	09:14	*III*
6	*1*	*1B*	*1A*	09:13	5	09:20	*I, II*
7	*1*	*1B*	*1A*	09:23	3	09:28	*I, II*

The toy problem is implemented with the GAMS/Cplex solver. Optimal solution is found in 0.52 seconds. The total weighted delay of the optimum schedule is 16 and total delay is 5 minutes. The Gantt Scheme of the obtained results is shown in Figure 6.2.

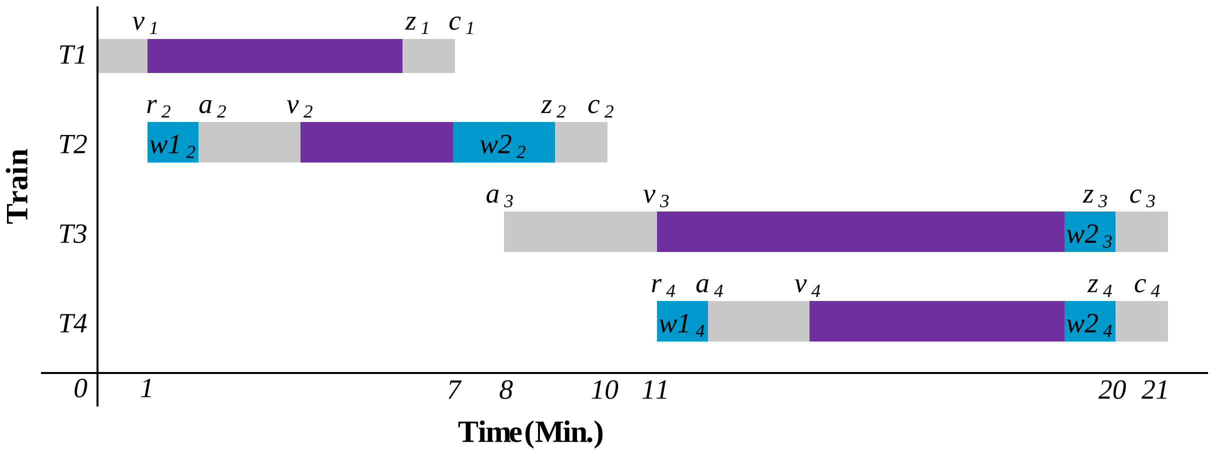

Figure 6-2 Gantt scheme of toy example 1

With respect to numerical results of example, train *1, 2, 3, 4, 5, 6* and *7* are assigned to platform track *3, 3, 2, 2, 3, 1* and *1* respectively. Train 1 and 2 are coupled trains and they are allocated the track at the same time. Train *2* is waiting at home signal and platform track one minutes for each to meet headway constraints. Train 4 is also waiting three minutes for leaving of train *3* which cannot share the platform track because it is international(long) train. Train 5 is also waiting five minutes at the platform track. Besides, the objective function does not consider because it terminates there ($delay_5 = 1$).

Toy problem 2 is a version of toy problem 1 in which train 3 is no longer international train with the same parameters. So, int_3 is equal to zero. The toy problem 2 is solved by GAMS/Cplex solver. Optimal solution is found in 0.44 seconds. The total weighted delay of the optimum schedule is 10 and total delay is 2 minutes. The Gantt Scheme of the toy problem 2 is shown in Figure 6.3.

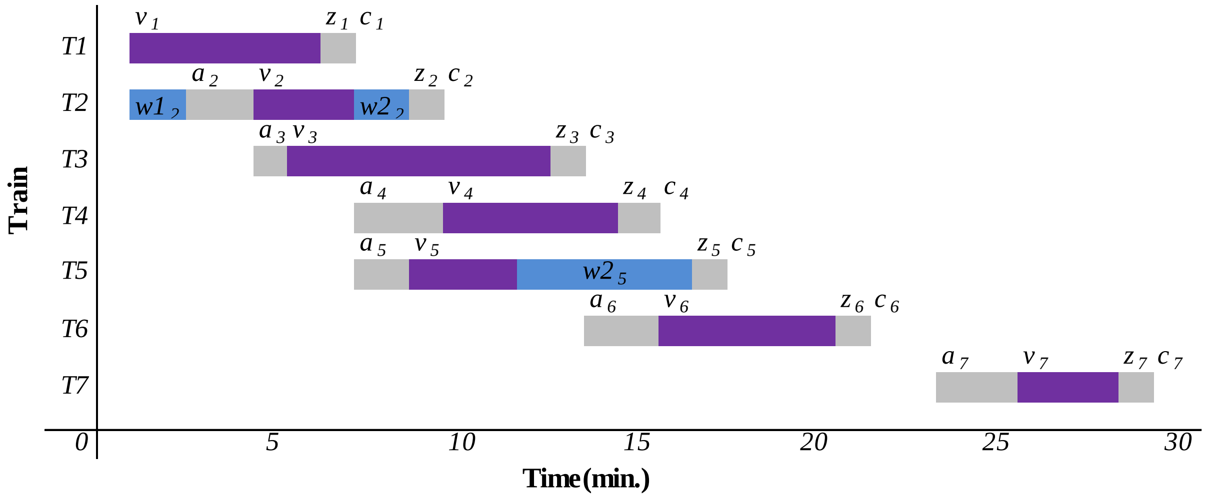

Figure 6-3 Gantt scheme of toy example 2

Comparing of these two results, it can be noticed that delay of train *4* disappears so train *3* and *4* is couple trains. Compatibility of direction of train *3* and *4* is highly important to share same platform track. For instances, directions of train *3* and *4* are respectively *(1B,1A)* and *(1B,2B)*. If direction of latter train was *(1A, 1B)*, it could be not possible to share the track.

Toy problem 3 is a different version of toy problem 2. In this version, several modifications are done on parameters. They are given as follows: (1) Boarding time and departure time of train *4* are changed as ($p_j = 10, d_j = 19$). (2) Train 6 are arriving from line 2A and it can be assigned only platform track 2.

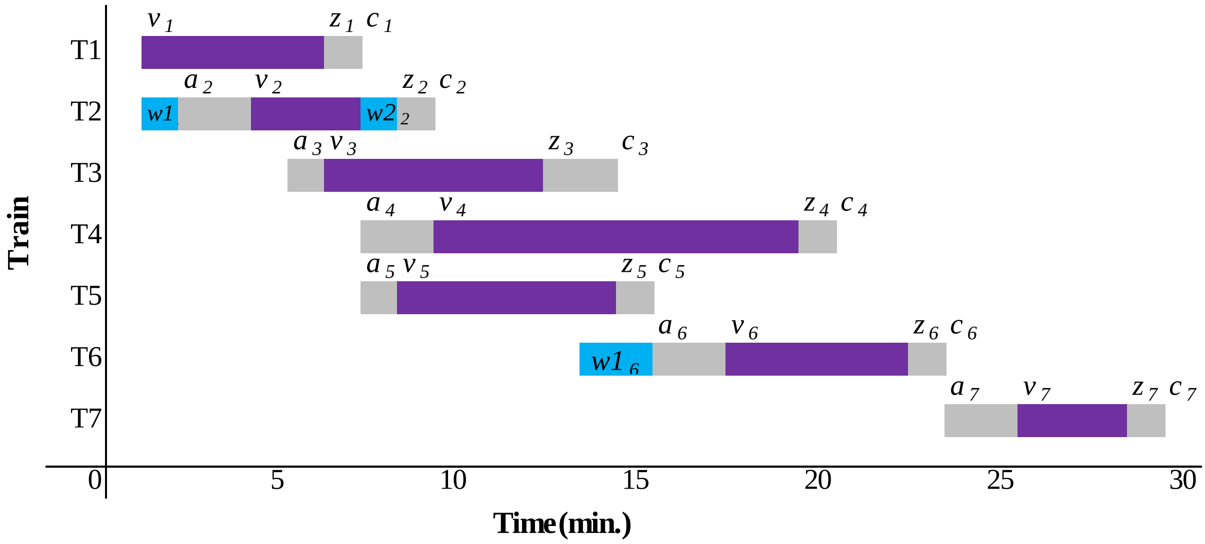

Figure 6-4 Gantt scheme of toy example 3

The Gantt Scheme of the toy problem 3 is set out in Figure 6.4. Optimal solution of toy problem 3 is found in 0.56 seconds. The total weighted delay of the optimum schedule is 12 and total delay is 4 minutes. In this version, train 3 are 4 are still couple trains but train 4 is also coupled with train 6 when train 3 leaves from platform. As previously stated in considered problem section, a train can be coupled with other trains more than one times in condition that directions are compatible.

6.1.2 Verification of Matheuristic

To verification of matheuristic, it should be applied to complex station topology. For this reason, we prefer that verification of matheuristic is done by real time interval. It is given in validation of matheuristic section and explained in detail.

6.2 Validation of Real Problem

In the first subsection, a more detailed characteristics of Prague main railway station is given. In the following sub-sections, obtained results of the GAMS/Cplex solver and the matheuristic algorithm are represented.

6.2.1 Characteristic of Prague Main Railway Station

Prague is the most complex passenger railway station area in the Czech Republic. The station is consisted of more than 100 railway junctions. There is 5 incoming and 5 outcoming tracks from the station leading to neighboring stations Vysočany, Vršovice, Holešovice, Smíchov and Libeň (all are located in the area of Prague) by 5 double-tracked lines. There are two special cases – station Vršovice is connected by 4-tracked line and on the other hand, stations Holešovice and Vysočany by one double-tracked lines which is divided into two lines at Balabenka Branch. Different numbers of arrival and departure trains are operated on each line. In the Prague main railway station, there are 9 main platforms which are consist of available 16 platform tracks. Thirteen platform tracks which can be used in both directions are regular platform tracks and there are three dead-end platform tracks. For busy and complex railway stations, such as Prague, regular platforms are generally constructed by considering that two trains can be allocated the track at the same time. However, capacity of dead-end platform tracks is one train.

Several train categories are operated in Prague railway station. They are explained in below:

- **Local trains (Os)** – trains for travelling short distances usually stopping at all stations and stops,
- **Limited-stop trains (Sp)** – trains for travelling medium distances usually stopping only at more important stations and stops,
- **Fast trains (R)** – trains for travelling long distances mainly within the Czech Republic, stopping only at more important stations,
- **Express trains (Ex)** – trains for travelling long distances within the Czech Republic and internationally, stopping only at the most important stations,
- **InterCity (IC)** – trains for comfortably travelling long distances within the Czech Republic and internationally, stopping only at the most important stations,
- **EuroCity (EC)** – trains for comfortably travelling long distances mainly internationally, stopping only at the most important stations,
- **EuroNight (EN)** – trains for comfortably travelling long distances overnight mainly internationally, stopping only at the most important stations,
- **Rail jet (rj)** – trains for comfortably travelling long distances internationally, stopping only at the most important stations,

- **SuperCity (SC)** – trains for comfortably travelling long distances in the Czech Republic and internationally, stopping only at the most important stations with above-standard services.

The trains that are given above are operated by Czech Railways. Besides that, there are two private company which operates their trains **Leo Express (LE)** and **RegioJet (RJ)**. These are also international trains such as EC, rj, IC, SC and Ex.

6.2.2 Determination of Parameters

Intervals: The train platforming problem, especially if it is considered whole day timetable, cannot be solved easily, due to their NP-hard nature. As noted by dissertation work of Bai, division of timetable to intervals can be useful to tackle such a problem. So, in this dissertation whole timetable are divided into intervals in account of number of trains from timetable for 2016/2017 year. There are approximately **700** arriving and departing trains in a weekday which is randomly selected. In Figure 6.5, intensity of train traffic for a weekday is provided.

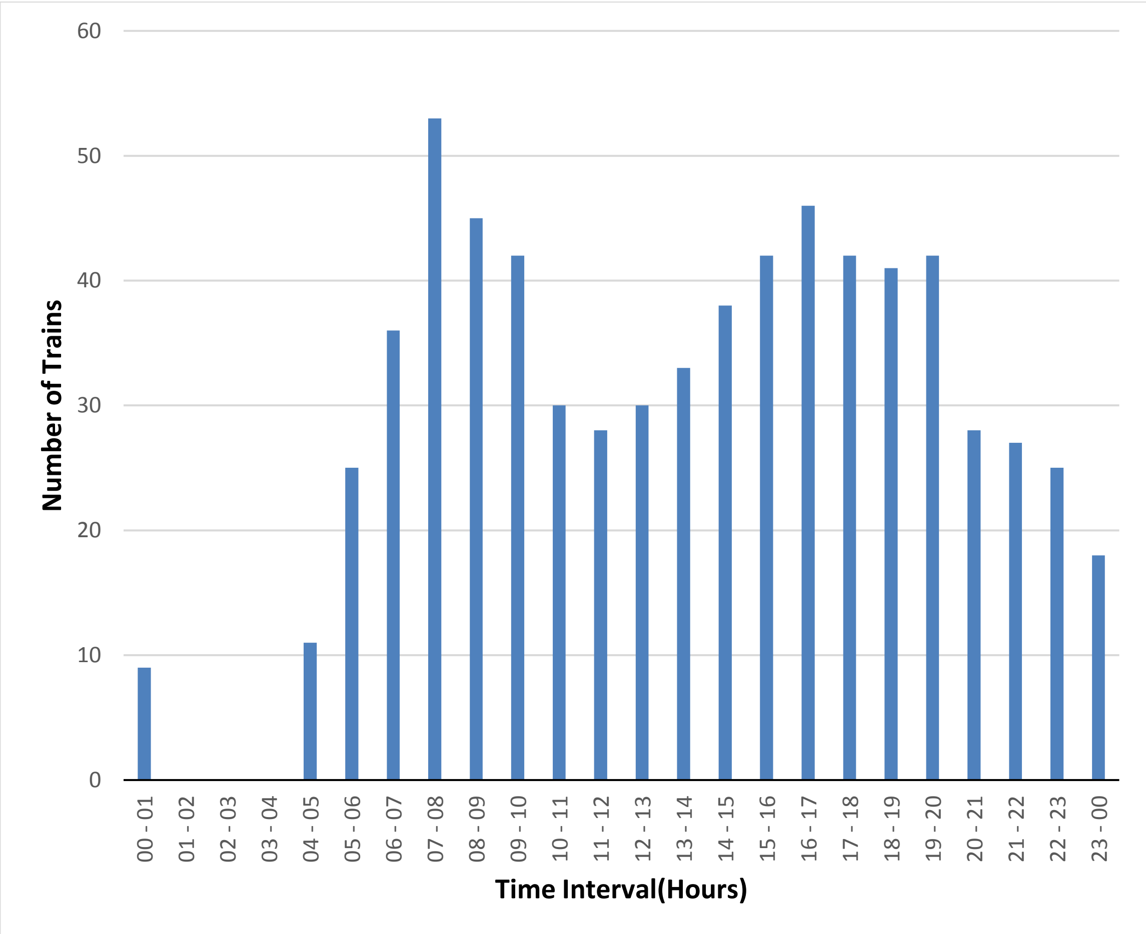

Figure 6-5 Intensity of train traffic for a weekday in Prague station

From the chart above we can see that there is no active train in the railway station between 1 am and 4 am. For this reason, division of time interval is done by taking account of beginning of interval is equal to 3 am($t = 0$). The number of trains in between *t=0* and *t=1260* is 690 and this number divided to total hours (21). Average number of trains in each interval is found 36. The time intervals are determined based on this assumption.

Sojourn time($\boldsymbol{p_j}$)**: S**ojourn time of transit trains is calculated based on difference between arrival and departure time of trains. Besides that, it is not possible to calculate them for originating and terminating trains since we have information about only arrival or departure times. At this point, simplifying assumptions are required in order to provide a reasonable basis for estimating sojourn time and it gets the value interval $[5{,}10]$ on basis of categorization of trains. The international trains should occupy the platform track more longer than regional trains. In Table 6.2, sojourn times of trains are presented.

Importance Degree of Trains($\boldsymbol{w_j}$)**:** Weights of trains are determined based on their importance. As it was expected, international trains have a higher degree of importance. The importance degree of trains is illustrated in Table 6.2.

Table 6-2 Important Degree and Sojourn Times of Trains

Important Degree	Train Type	Sojourn times
6	EC, rj, EN	10
5	IC, SC, LE, RJ	10
4	Ex	10
3	Rx,R	8
2	Sp	8
1	Os	5

Arrival and Departure Lines of Trains ($\boldsymbol{gA_j}, \boldsymbol{gB_j}, \boldsymbol{bA_j}, \boldsymbol{bB_j}$): In Prague, there are 5 arrival lines and 5 departure lines from the station to neighboring stations Vysočany, Vršovice, Holešovice, Smíchov and Libeň and lines are numbered consecutively. Line numbers are set out in Table 6.3.

Table 6-3 Line numbers

Directions	**Arrival**	**Departure**
Smíchov	2	1
Vršovice	4	3
Vršovice	6	5
Vysočany	8	7
Holešovice	8	7
Libeň	10	9

The parameters *ht*, *st* and *pt* are fixed to 2, 1 and 0 for Prague case. All values are in minutes. To better understand of parameters, an example of timetable is presented in Table 6.4.

Table 6-4 An example of timetable

Train no	*Train*	*Arrival Direction*	*Departure Direction*	*Arrival Time*	*Departure Time*
1	Os 2505	**Vysočany**	Vršovice		04:36
2	Rx 751	Smíchov	**Libeň**	05:41	
3	rj 580	Libeň	**Smíchov**	07:06	
4	R 1148	Vršovice	Vysočany	07:18	07:25
5	EC 176	**Vršovice**	Holešovice		08:28
6	SC 505	Smíchov	Libeň	09:16	09:37

In Table 6.4, directions which are marked as bold are parking stations of associated trains. So, Os 2505 and EC 176 are originating trains. Rx 751 and rj 580 are terminating trains. The parameter $delay_2$ and $delay_3$ are equal to one and their waiting times at the platform track does not affect objective function. The sojourn time of trains and importance degree of trains are as follows: $p_j = \{5,8,10,7,10,21\}$ and $wt_j = \{1,3,6,3,6,5\}$.

6.2.3 Validation of Mathematical Model

The case problems for each interval were solved by means of the proposed mathematical model (M1). The running time of the Cplex solver of GAMS is limited to 5000 seconds for all the tests.

The results of mathematical model are presented in Table 6.5. The first three columns summarize the properties of intervals such as intervals' number (*no*), length of time interval in minutes, and number of trains that are in considered time interval. The fourth column shows the number of delayed trains with respect to obtained results. The fifth column indicates number of transferred trains to the next interval. The sixth, seventh and eighth columns represent waiting times and total delay in minutes. The timetable delays that are caused by parameter assumptions are showed in ninth column. The obtained objective value and solution time in seconds using the GAMS/Cplex solver.

Table 6-5 The results of Mathematical Model

Interval No	Length of Time Interval	Number of Trains	Number of Delayed Trains	Number of Transferred Trains	$w1_j$	$w2_j$	Total Delay	Timetable Delay	z	t
1	36	9	0	0	0	0	0	0	0	1.56
2	127	36	1	9	1	0	1	0	*1	5000
3	60	36	30	-	-	-	1069	14	*2794	5000
4	44	36	-	-	-	-	-	-	-	-
5	42	36	-	-	-	-	-	-	-	-
6	55	36	-	-	-	-	-	-	-	-
7	54	36	-	-	-	-	-	-	-	-
8	74	36	-	-	-	-	-	-	-	-
9	79	36	-	-	-	-	-	-	-	-
10	69	36	-	-	-	-	-	-	-	-
11	59	36	-	-	-	-	-	-	-	-
12	51	36	-	-	-	-	-	-	-	-
13	41	36	-	-	-	-	-	-	-	-
14	53	36	-	-	-	-	-	-	-	-

*not optimal, just feasible solution could be obtained in 5000 second.

As can be seen from the table (above), we could not obtain optimal solutions for time interval 1 and 2 until Cplex reaches the time limit 5000 seconds. We certainly sure that optimal solution for each interval is not smaller than zero. The GAMS solution of Interval 2 is very close to optimal solution even if it is not optimal. So, the trains that are also included in both Interval 2 and Interval 3 can be transferred to Interval 3. However, the objective function of Interval 3 which are obtained in 5000 seconds is definitely far from optimal solution and it is not seems to be a reasonable solution quality. By the reason the fact that solution of each interval excluding Interval 1 and its next interval are interdependent, we could not run GAMS/Cplex solver for the other intervals.

6.2.4 Validation of Proposed Matheuristic

Verification of matheuristic algorithm may be not effective since it is designed for complex station topologies. In this section, it is explained in detail using consecutive real case intervals. So, starting point is Interval 2 because interval 1 is completely independent from other intervals. In Table 6.6, required parameters of Interval 2 are set out.

Firstly, Interval 2 has been solved using assignment algorithm and decision model with following parameters: headway and switch interval is 2, the parameter *dt1* and *dt2* is 4. The objective function value for initial solution is equal to 7 that was calculated using GAMS with Cplex as a solver and the problem solved within 17 seconds. Figure 6.6 sets out trains and allocated platform tracks for initial solution.

As can be seen in Figure 6.6, platform track 3 is not occupied by any train. It is dead-end track. Arrival and departure directions of trains which are assigned to these tracks should be Vysočany, Libeň, Holešovice in the northern part of station. In considered time period, only Fast trains (R) have these arrival or departure directions. However, it is not possible to assign fast trains to these tracks from the point of platform length. Dead-end tracks are generally used for regional trains (Os) and Leo Express trains(LE), as these trains satisfied the platform length restrictions and if the both-directional trains (electrical units) are used. In Figure 6.6, as soon as R 616 does not depart, Os 9401 arrives to the station and platform track 16 is allocated by these trains. Although occupation times of trains are overlapped, they satisfy all restrictions from the point of safety since capacity of platform track 16 is two trains.

In Table 6.7, waiting times of trains that are different from zero for Interval 2 are represented. The waiting time of trains in this table except Train *30* are included to the objective function. Besides that, waiting time of train 30 that is 3 minutes is not considered since the parameter $delay_{30}$ is equal to one. When improvement algorithm runs, it is neglected.

Table 6-6 Solution report of Interval 2

j	***w1_j***	***j***	***w2_j***
28	1	**5**	4
36	1	**16**	1
		30	3

Table 6-7 The required parameters of Interval 2

No	Train	Arrival Direction	Departure Direction	wt_j	$delay_j$	p_j	rs_j	d_j	int_j
1	EN 476	Libeň	Holešovice	6	0	21	43	66	1
2	Os 2503	Vysočany	Vršovice	1	0	5	59	66	0
3	Os 8802	Libeň	Smíchov	1	0	5	70	77	0
4	Os 5801	Vršovice	Vysočany	1	0	5	75	82	0
5	Os 2505	Vysočany	Vršovice	1	0	5	89	96	0
6	Ex 571	Vršovice	Libeň	4	0	10	97	109	1
7	Os 9101	Vysočany	Vršovice	1	0	5	101	108	0
8	Os 8801	Smíchov	Smíchov	1	0	11	94	107	0
9	Os 9500	Vršovice	Vysočany	1	0	2	104	108	0
10	Os 2500	Vršovice	Vysočany	1	0	1	110	113	0
11	Os 2507	Vysočany	Vršovice	1	0	10	114	126	0
12	R 941	Vršovice	Vysočany	3	0	8	117	127	0
13	EC 356	Libeň	Smíchov	6	0	10	120	132	1
14	Os 2000	Vršovice	Vršovice	1	0	13	123	138	0
15	Os 8803	Smíchov	Smíchov	1	0	11	124	137	0
16	Os 9521	Vysočany	Vršovice	1	0	3	127	132	0
17	R 616	Vršovice	Holešovice	3	0	8	128	138	0
18	Ex 121	Vršovice	Libeň	4	0	10	130	142	1
19	Os 9401	Vršovice	Vysočany	1	0	5	135	142	0
20	Os 2005	Vysočany	Vršovice	1	0	5	138	145	0
21	Os 2502	Vršovice	Vršovice	1	0	14	140	156	0
22	Os 9900	Libeň	Smíchov	1	0	5	140	147	0
23	Os 9903	Smíchov	Libeň	1	0	5	141	148	0
24	R 701	Holešovice	Vršovice	3	0	8	142	152	0
25	Os 25900	Libeň	Smíchov	1	0	5	145	152	0
26	RJ 1001	Smíchov	Libeň	5	0	10	152	164	1
27	Os 9000	Vršovice	Vršovice	1	0	20	153	175	0
28	Os 8805	Smíchov	Smíchov	1	0	11	154	167	0
29	EC 273	Vršovice	Libeň	6	0	10	157	169	1
30	Rx 751	Smíchov	Libeň	3	1	8	159	169	0
31	Os 9100	Vršovice	Vysočany	1	0	2	160	164	0
32	Os 9502	Vršovice	Vysočany	1	0	2	164	168	0
33	Os 9105	Vysočany	Vršovice	1	0	1	165	168	0
34	R 964	Libeň	Vršovice	3	1	8	169	179	0
35	Os 9905	Smíchov	Libeň	1	0	7	169	178	0
36	Os 2504	Vršovice	Vršovice	1	0	14	170	186	0

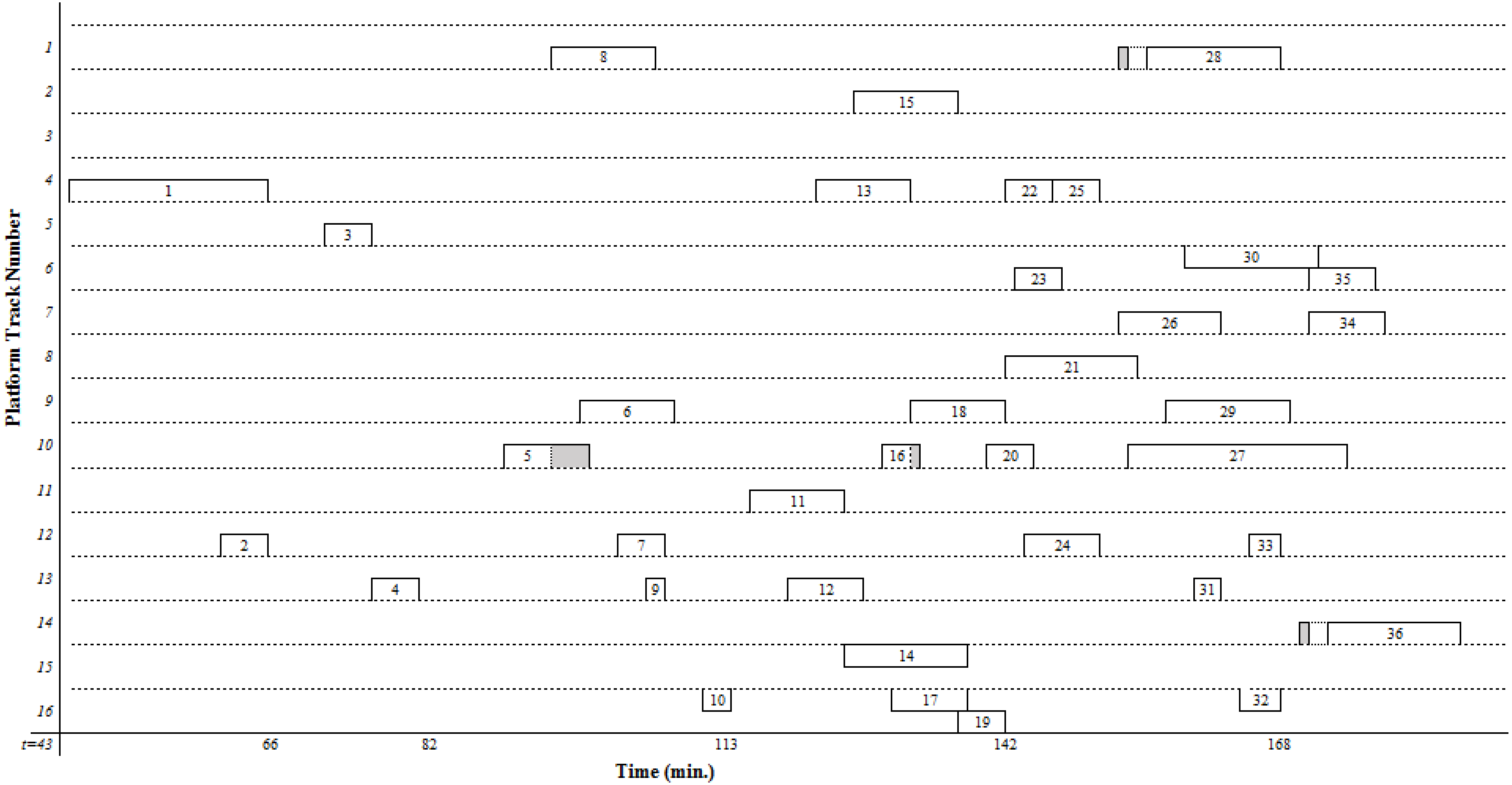

Figure 6-6 Platform track assignment for Interval 2

The steps of improvement algorithm for Interval 2 is given as follows:

- Start with train j which is $\max_j ww1_j$: $(j = 28)$
- Determine the reason of delayed $j = 28, j \in N$
- Find the train q in which $c_q + st = a_j$ and $\sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0$, $q = 25$
- Change the platform of train j. Because of crossing, train j should assign the right side of platform track 4. Crossing is shown in Figure 6.7. The new assignment of train 28 should be platform track 5.

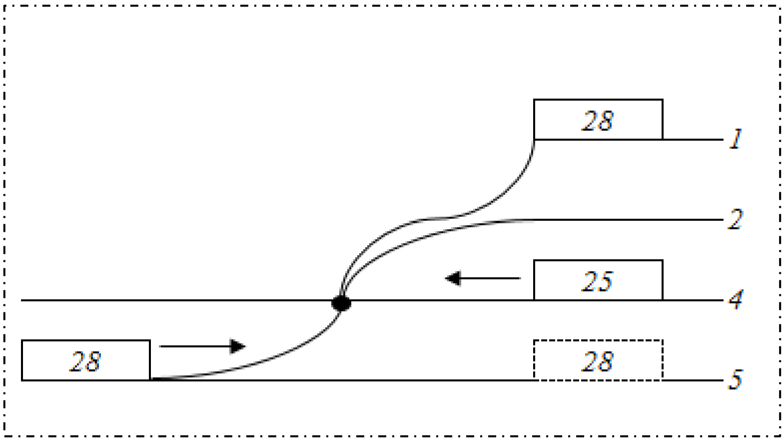

Figure 6-7 Crossing between train 24 and 28

- Solve decision model for determination of decision variables. New objective function is equal to 6.
- Find the train j that is $\max_j WW1 \cup WW2$, $(j = 5)$
- Determine the reason of delayed $j = 5, j \in N$
- Find the train q in which $v_q + st = z_j$ and $\sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0$, $q = 6$
- Change the platform of train j. Because of crossing positions of train j and q, train j should assign the left side of platform track 9. The new assignment of train 5 should be platform track 8.
- Solve decision model and new objective function is equal to 2.
- Find the train j that is $\max_j WW1 \cup WW2$, $(j = 36)$
- Determine the reason of delayed $j = 36, j \in N$
- Find the train q in which $c_q + st = a_j$ and $\sum_{r=1}^{g} res1_{jlr} * res2_{qmr} > 0$, $q = 33$

- Change the platform of train *j*. Because of parallel positions of train *j* and *q*, train *j* should be assigned the right side of platform track 14. Parallel positions of trains are shown in Figure 6.8. The new assignment of train 36 should be platform track 15.

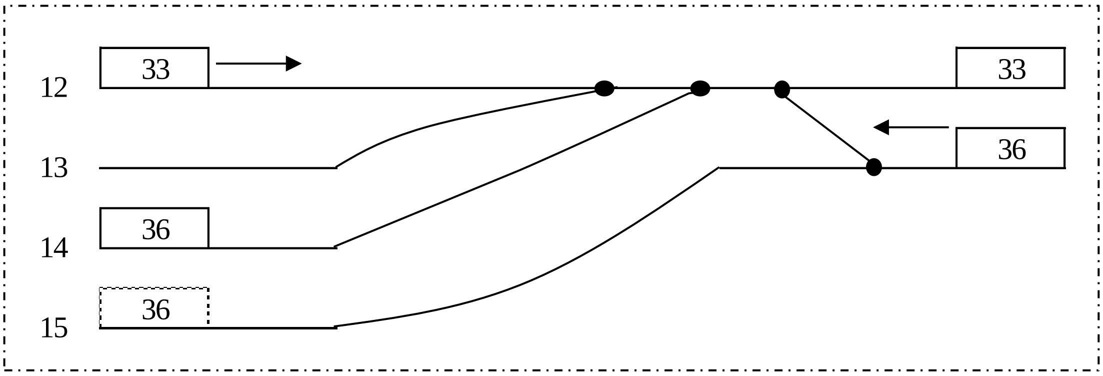

Figure 6-8 Parallel Position between train 33 and 36

- New objective function is equal to 1 and only train *16* are waiting at the platform track. Same procedures are applied to train *16*.
- Determine the reason of delayed $j = 16, j \in N$
- Find the train q in which $v_q + st = z_j \ and \ \sum_{r=1}^{g} res2_{jlr} * res1_{qmr} > 0$, $q = 18$
- Change the platform of train *j*. Because of crossing positions of train *j* and *q*, train *j* should assign the left side of platform track *9*. The new assignment of train *16* should be platform track 8.
- Final objective function is zero. Final schedule for interval 3 after improvement algorithm is given in Figure 6.9.

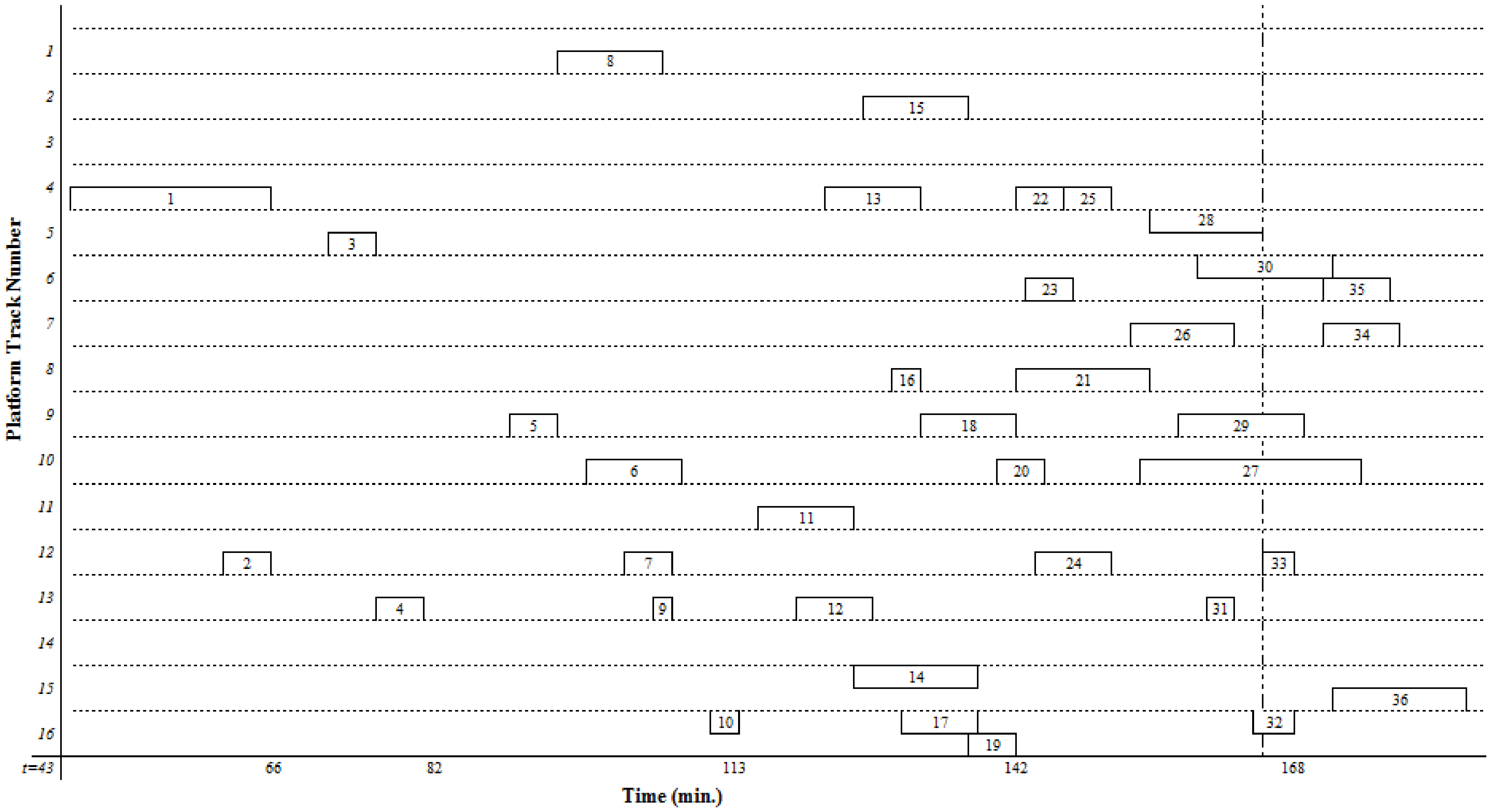

Figure 6-9 Final Schedule for Interval 2 After Improvement Algorithm

To solve Interval 3, we need to know which trains should be transferred to Interval 3 from Interval 2. Transferred trains are detected according to arrival time to the homes signal of first train in Interval 3. In Table 6.8, required parameters of Interval 3 are set out.

Table 6-8 The required parameters of Interval 3

No	Train	Arrival Direction	Departure Direction	wt_j	$delay_j$	p_j	rs_j	d_j	int_j
1	Ex 1541	Holešovice	Vršovice	4	0	10	170	182	1
2	Os 9902	Libeň	Smíchov	1	0	2	173	177	0
3	R 975	Vršovice	Libeň	3	0	8	174	184	0
4	RJ 1020	Libeň	Smíchov	5	1	10	176	188	1
5	R 921	Vršovice	Vysočany	3	0	8	177	187	0
6	Rx 876	Libeň	Vršovice	3	1	8	181	191	0
7	Rx 766	Libeň	Smíchov	3	0	8	182	192	0
8	Os 8807	Smíchov	Smíchov	1	0	11	184	197	0
9	Os 9501	Vysočany	Vršovice	1	0	3	187	192	0
10	Os 9102	Vršovice	Vysočany	1	1	5	190	197	0
11	EC 113	Vršovice	Libeň	6	0	10	190	202	1
12	Os 9907	Smíchov	Smíchov	1	0	16	194	212	0
13	Os 9107	Vysočany	Vršovice	1	0	1	195	198	0
14	EC 178	Vršovice	Holešovice	6	0	10	196	208	1
15	Os 2002	Vršovice	Vysočany	1	1	5	197	204	0
16	R 732	Vršovice	Holešovice	3	0	8	197	207	0
17	Os 2506	Vršovice	Vršovice	1	0	14	200	216	0
18	R 703	Holešovice	Vršovice	3	0	8	202	212	0
19	Os 9904	Libeň	Smíchov	1	0	2	203	207	0
20	Os 9909	Smíchov	Libeň	1	0	2	204	208	0
21	Sp 1941	Vysočany	Vršovice	2	0	8	205	215	0
22	Os 25901	Smíchov	Libeň	1	1	5	209	216	0
23	Sp 1500	Vršovice	Vysočany	2	1	8	210	220	0
24	EN 444	Libeň	Vršovice	6	1	10	211	223	1
25	Os 8809	Smíchov	Smíchov	1	0	11	214	227	0
26	Os 9522	Vršovice	Vysočany	1	0	2	216	220	0
27	Ex 524	Libeň	Vršovice	4	1	10	217	229	1
28	rj 71	Vršovice	Libeň	6	0	10	218	230	1
29	Rx 771	Smíchov	Libeň	3	1	8	219	229	0
30	Os 9104	Vršovice	Vysočany	1	0	2	220	224	0
31	Os 9109	Vysočany	Vršovice	1	0	4	222	228	0
32	Rx 885	Smíchov	Libeň	3	0	8	223	233	0
33	Os 9911	Smíchov	Libeň	1	1	5	224	231	0
34	R 952	Vysočany	Vršovice	3	1	8	226	236	0
35	Os 2508	Vršovice	Vršovice	1	0	14	230	246	0
36	Os 9523	Vysočany	Vršovice	1	0	3	230	235	0

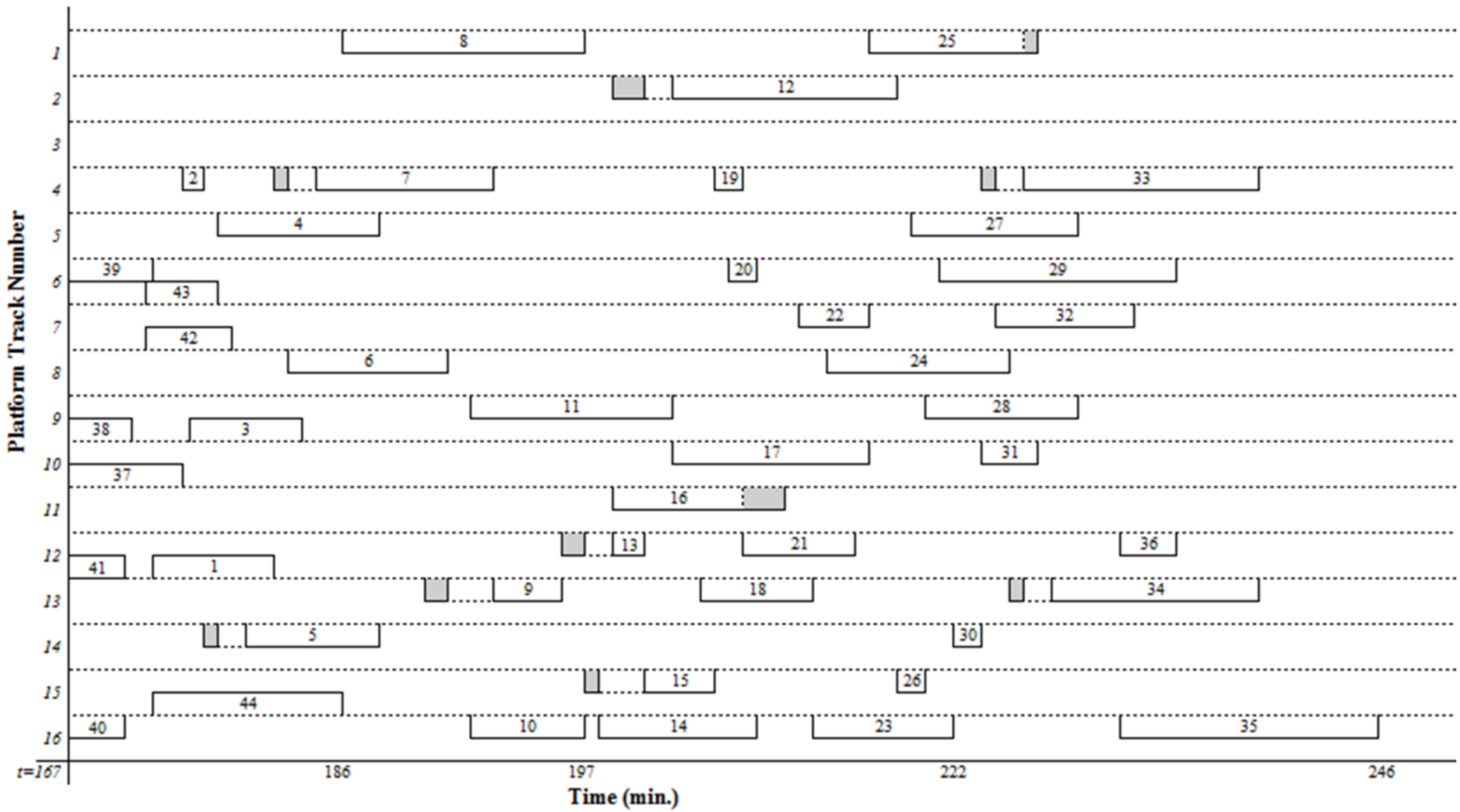

Figure 6-10 Initial Schedule for Interval 3

The parameter rs_1 is equal to 170. So, the trains from Interval 2 which of arrival time to the exit signals is higher than 169 ($c_q + st = rs_j$) are transferred trains. The beginning point of Interval 3 in Interval 2 is shown in Figure 6.9 with dotted line and the list of transferred trains is given in Table 6.9.

Table 6-9 The List of Transferred Train from Interval 2 to Interval 3

No	Train	Arrival Direction	Departure Direction	New No	Platform Track
27	Os 9000	Vršovice	Vršovice	**37**	10
29	EC 273	Vršovice	Libeň	**38**	9
30	Rx 751	Smíchov	Libeň	**39**	6
32	Os 9502	Vršovice	Vysočany	**40**	16
33	Os 9105	Vysočany	Vršovice	**41**	12
34	R 964	Libeň	Vršovice	**42**	7
35	Os 9905	Smíchov	Libeň	**43**	6
36	Os 2504	Vršovice	Vršovice	**44**	15

The number of trains is now 45 in Interval 3 and however, transferred trains have already assigned to the tracks. So, assignment algorithm runs for 36 trains and decision model determines all decision variables using GAMS/Cplex solver. The objective function of Interval 3 is equal to 33 and it is obtained in 40 seconds. The detailed solution report for Interval 3 is given in Table 6.10 and initial schedule for Interval 3 are given in Figure 6-10.

Table 6-10 The solution Report for Interval 3

j	$w1_j$	J	$w2_j$
5	1	**16**	3
7	1	**23**	2
9	4	**24**	3
12	6	**25**	1
13	2	**27**	1
15	1	**29**	6
33	1	**33**	5
34	1	**40**	3

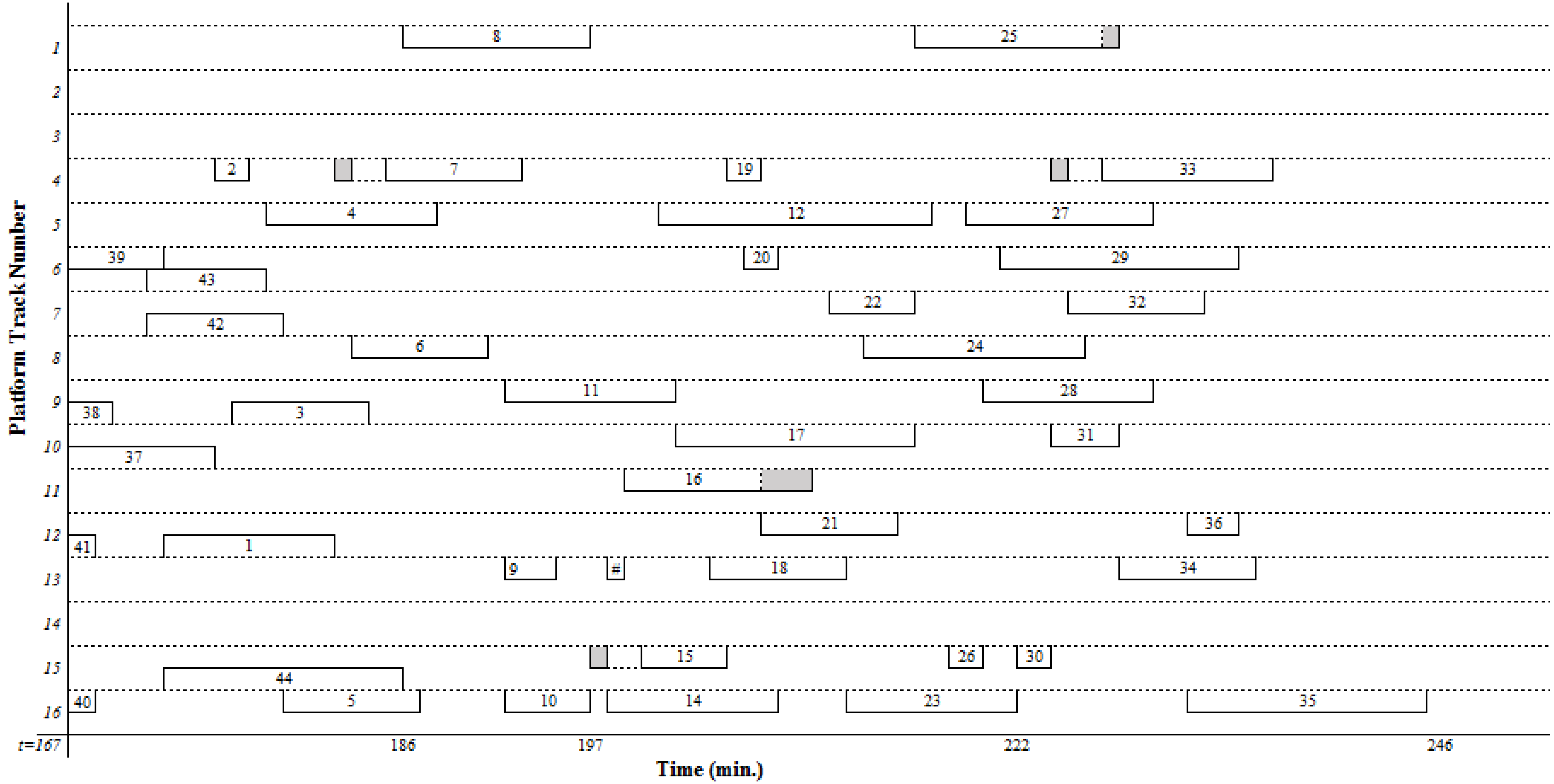

Figure 6-11 Final Schedule for Interval 3 After Improvement Algorithm

In Table 6.10, waiting time of trains are presented and however it cannot be called as "actual delay". Before applying of improvement algorithm, it should be sort out not actual delays. First group of them is the trains that parameter $delay_j$ is equal to 1 and $w2_j$ is bigger than zero. Waiting time at the platform track of train 23, 24, 27, 29, 33 and 40 should be neglected since it is not affect to objective function. The other group of delays is caused by timetable. In Table 6.8, timetable delays can be traced. Train 6 and 7 are arriving from Libeň and time difference between arrival time to the home signal does not meet headway interval condition. So, train 7 should wait for one minute at home signal. In a similar way, train 15 should stay for one minute because of train 14. Train 33 should also wait for one minute at home signals. Another timetable delay is waiting of train 16 at the platform track since headway interval should be ensured when they are leaving from the same line. Totally, timetable is caused to 4 minutes' delay and it is not possible that value of objective function smaller than 14. After that, we can run improvement algorithm and steps of algorithm is given in Table 6.11.

Table 6-11 The steps of algorithm for Interval 3

Steps of Algorithm	**Delayed Train *j***	**Caused Train *q***	**Reason of Delay**	**Total Weighted Timetable Delay**	**Z**	***t(sec.)***
1	12	8	$c_8 + st = a_{12}$	14	27	38
2	9	5	$c_5 + st = a_5$	14	20	36
3	34	30	$c_{30} + st = a_{34}$	14	17	36
4	13	16	$v_{16} + st = z_{13}$	14	15	34
5	25	12	$c_{12} + st = a_{25}$	14	15	-

As can be seen in Table 6.11, first four steps of algorithm improve the objective function remarkably. Besides, fifth step does not change the objective function. It means that new platform track assignment of train 25 cannot be accepted by the algorithm.

The rest intervals of one day timetable are provided in Table 6.12. The first four columns summarize the properties of intervals, such as number of intervals (*ts*), length of time interval, number of trains in considered intervals and number of transferred trains to the next interval. As mentioned before, Interval 1 is independent from other intervals. So, there is no any transferred trains from Interval 1 to Interval 2. The minimum length of time intervals is in

Interval 4, 5 and 13. In Prague main railway station, corresponding hours to these intervals are morning and evening peak time since they have high number of trains.

The fifth column shows timetable delay which are caused by simplification of real operation data. The sixth, seventh and eighth columns show the obtained objective value and solution time in seconds using the GAMS/Cplex solver. However, feasible and acceptable solution is reached only for Interval 2 and we do not have a solution for the other intervals with mathematical model. To demonstrate the performance of matheuristic, we should compare a lower bound for each interval and total weighted timetable delays can be used for lower bound. It can be shown how far obtained solutions with matheuristic is from being optimal solutions.

The last six columns describe the obtained results with the matheuristic algorithm. The first two columns show the results of first stage of matheuristic algorithm: initial solution of platform track assignment algorithm and its solution time in seconds. The following four columns represent the results of improvement algorithm: total weighted delay, number of delayed trains according to this solution, total delay of these trains and solution time of whole matheuristic in minutes. In Table 6.12, the total weighted timetable delay (lower bound) and solutions of matheuristic algorithm are marked as bold for each interval. We are certainly sure that the GAMS/Cplex solver with matheuristic finds optimal solutions for Interval 2 and 10. On the other hand, obtained solutions for rest of intervals is not far from lower bound and optimum solutions may be equal to obtained solutions.

As can be seen in Table 6.12, Interval 4, 5 and 13 have minimum length of time intervals. On the contrary, initial solutions of these intervals are bigger than solution of other intervals. It means that density of trains increased in these intervals and it causes to conflicts. The maximum total delay is eight minutes and maximum number of delayed trains is four in associated interval. Regarding that, solution time of Interval 4 is noticeably larger.

A significant benefit of matheuristic algorithm is not only to find good quality solutions but also to solve in a relative short time. It can be clearly seen that mathematical model could not obtain a solution for most of intervals in 5000 seconds. However, matheuristic algorithm is obtained feasible solutions in average 6.5 minutes.

Table 6-12 The solution of Intervals in One Day Timetable

Intervals					Mathematical Model			Matheuristic					
Interval No	Length of Time Interval	Number of Trains	Number of Transferred Trains	Timetable Delay	*z*	Total Delay	*t(sec.)*	Initial Solution	*t(sec.)*	Improvement Algorithm	Number of Delayed Trains	Total Delay	*t(min.)*
1	225	9	0	**0**	**0**	0	1.56	0	0.23	**0**	0	0	0.23
2	127	36	9	**0**	**1**	1	5000	7	17	**0**	0	0	2.27
3	60	45	10	**14**	**2794**	1069	5000	33	40	**15**	1	1	4.54
4	44	46	9	**30**	**-**	-	-	124	180	**40**	4	8	26.1
5	42	45	10	**28**	**-**	-	-	122	53	**35**	2	3	9.2
6	55	46	12	**28**	**-**	-	-	76	52	**35**	3	5	4.5
7	54	48	10	**15**	**-**	-	-	67	108	**27**	4	5	8.5
8	74	46	7	**29**	**-**	-	-	49	67	**33**	2	4	5.83
9	79	43	6	**32**	**-**	-	-	42	28	**38**	1	2	2.1
10	69	42	5	**0**	**-**	-	-	17	26	**0**	0	0	1.5
11	59	41	10	**23**	**-**	-	-	46	34	**27**	1	4	2.3
12	51	46	9	**14**	**-**	-	-	126	51	**15**	1	1	3.6
13	41	45	13	**68**	**-**	-	-	204	64	**84**	2	5	9.3
14	53	49	-	**23**	**-**	-	-	128	56	**29**	2	2	2.9

7 CONTRIBUTIONS OF DISSERTATION

Contributions of the dissertation can be described in two ways: literature and practical operation. They are given as below:

- *From view point of literature*: Much of the current literature on railway optimization problems pays particular attention to different planning level: strategic, tactical and operational level. In this dissertation, a mixed integer model and matheuristic algorithm are developed to solve the problem in tactical planning level. Since the benchmark come from Prague main station, railway station topology of Prague is seriously considered. There are two significant constraint sets except regular constraints: eligibility restrictions and capacity of platform tracks. There is a relatively small body of literature that is concerned with considering train platforming problem detailedly and analyzing train platforming problem like in this study. With regard to solution methods, to best our knowledge, this research serves as a base for future studies since matheuristic algorithms are reported for the first time in train platforming problem literature.

- *From view point of practical operation*: As mentioned above, train platforming problem described in the dissertation have been dealt with special version of real case problem. A mixed integer programming model is proposed for assignment schedule for trains. Objective of assignment model is to minimize total weighted delays of trains. The assignment model may have two practical applications. The first is the basic form. If the objective is zero without conflicts, railway planner decides that the schedule can be realized from the point of operational restrictions. Second form of utilization is support of decision making processed realized by train dispatchers. Railway operation is not deterministic in everyday operation (some trains are coming with delay etc.). Train dispatchers solve different situation every day due to this fact. In mathematical expression – planned arrivals and departures of trains are replaced by estimated arrivals and departures. In general point of view, basic presumption – timetable with no conflicts – is not strictly fulfilled in estimated (real) situation. Design of this model with objective function minimizing the sum of delays allows that the model can be applied also in everyday decision making process with the aim ensure as smooth operation as possible. Assignment of platform tracks to trains is an integral part of this process.

8 CONCLUSIONS

This dissertation thesis addresses train platforming problem, which is the problem of assigning the routes of the incoming and outgoing trains in a large railway station, so as to satisfy several constraints based on railway system. Inputs of problem are the daily timetable and the structural constraints of the railway station. Daily timetable shows arrival and departure times of trains to/from the railway station per day. Structural constraints are directly related to railway station topology that includes several special constraints about platform tracks, lines and directions. One of important restrictions is about assignment of platform tracks to the trains in railway station. There are three significant considerations: (i) railway junctions between platform tracks and entry/exit lines, (ii) eligibility between train length and platform length, (iii) existence of one direction platforms. The other one is with regard to platform capacity. It is possible to assign two trains to the same platform at a time. However, the direction of trains and classification of train must be considered. There are also several additional operational constraints about dealing with railway station topology.

A considerable amount of literature has been published on train platforming problem. In view of all of studies, one may suppose that it is complex problem. These studies proposed different solving methods to obtain efficient results in reasonable time. In this dissertation, from the point of solution methods, a mixed integer mathematical model for this problem is firstly developed and the objective function of model is to minimize the total weighted delay of trains. It is coded in GAMS/Cplex Solver.

Train platforming problem is relatively easy to solve for small railway stations with very few trains and platform tracks and it can be easily solved with mixed integer mathematical model. However, it becomes a difficult optimization problem when applied to complex railway station topologies, such as those associated with the main European stations, having hundreds of trains and tens of platform tracks. The mathematical model will not be adequate for solving of real life platform track assignment problem.

Matheuristic algorithms are powerful algorithmic approaches which have been applied with great success to many difficult combinatorial optimization problems in recent years. Consequently, a matheuristic algorithm is adopted in order to increase the quality of solution for large size problems. The algorithm consists of three-stages: (i) platform track assignment algorithm, (ii) M2 model for calculation of total weighted delay, (iii) improvement algorithm.

In the proposed algorithm, the algorithms and the sub-problem (M2) work cooperatively. Thanks to the first algorithm, it is determined platform track assignment of each train and then the decision variables x_{jl} are parameters. M2 model that is reduced version of M1 calculates all decision variables. Finally, improvement algorithm enhanced the quality of solutions in each step. The matheuristic algorithm is also coded in GAMS/Cplex Solver.

The railway station topology and timetable in our benchmark come from Prague main railway station with some simplifications. These simplifications were accepted because some data about real operation are not available. On the other hand these simplifications have no impact on evaluation of proposed models from the methodological point of view. In the case of practical utilization of these models, inputs and following are able to be corrected in the case that inputs will be accessible in more detailed form.

To prove efficiency of proposed models, a whole weekday timetable belongs to Prague main railway station which is randomly chosen is divided into time intervals on basis of number of trains. Each time interval except Interval 1 relates to next time interval. Proposed two methods are tested in each interval. Mixed integer model could not reach optimal solution in 5000 seconds. In addition to this, a feasible solution is found for only Interval 2. On the contrary, matheuristic algorithm obtains feasible solutions in a reasonable time for each interval. At this point, it is needed a lower bound to demonstrate efficiency of solution since mathematical model could not success to obtain a feasible solution.

As it is pointed out above, because of simplifications about Prague station, it is possible to find delay in given whole day timetable. Timetable delays are caused by these simplifications and determination for parameters. They are easily detected in timetable and it can be very useful from the point view of lower bound. Results of matheuristic algorithm may be compare timetable delays.

In further work, train platforming problem can be taken into consideration with train timetabling problems in strategic planning level. In addition, simplified operational and infrastructure effects are also providing a space for future research. Consideration of shunting movements is one of the first effects prepared for research and implementation into model.

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