On Radio Wave Forward Scattering on Rain Drops

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Abstract—In this contribution critical size parameter (x_{crit}) was defined and its values were found. It is limitation for utilisation of Rayleigh approximation for forward scattering functions (for $x>x_{crit}$ Mie theory is required for calculation. It is possible only in case of assumption of spherical particles. For meteoradar frequencies 5.5, 10 and 35 GHz the critical values of size parameter (x_{crit}) were calculated, its values are: 0.0168, 0.0199 and 0.0403 respectively. If size parameter is lower than these critical values, the simple Rayleigh approximation to compute complex forward scattering function is possible.

Keywords—forward scattering function, Mie theory of scattering function, Rayleigh approximation, critical size parameter

I. Introduction

Scattering in forward direction ($\theta = \pi - \varphi \; and \; \varphi = \pi$) is the object of the interest and bistatic scattering geometry is shown in Fig.1. It enables to evaluate the effects of the rain medium on the propagating wave [1].

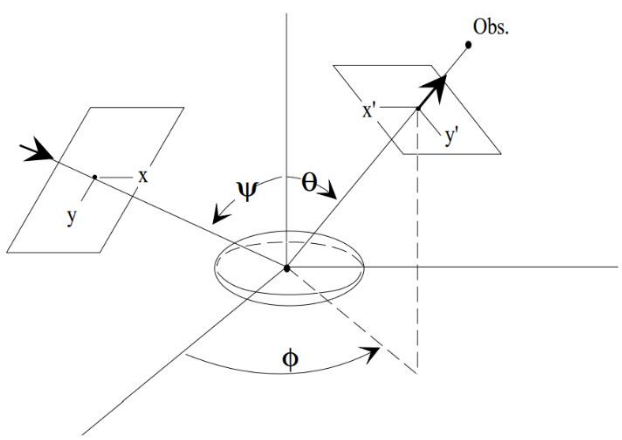

Fig. 1. Single obstacle, scattering geometry [1]

Electromagnetic waves propagating through rain particles are suffering of absorption and scattering properties if rai drops: these effects are depending on the concentration of particles, size distribution, shape and orientation of the particles.

The problem of the forward scattering in contradistinction to back scattering lies in two effects – there are both absorption and scattering. In the case of back scattering (important for radar technique) we do not need to by busy with rain wave absorption. In ideal conditions, which is our interest in this work, we will assume that rain drops have spherical shape. In real conditions we should consider the orientation of the rain drops (the decomposition of the canting angles) and also real shape of drops, which was described by Pruppacher and Pitter [2]. The important fact is that the drop, which sizes is smaller than 1 mm, could be considered as a spherical one and drops bigger than 1 mm have spheroidal shape tending to the ellipsoidal one.

In this paper the Rayleigh approximation for "Rayleigh region" and more general Mie theory will be described.

We present here comparison of both algorithms for calculation of forward scattering functions. We determinate limits for Rayleigh approximation utilisation.

II. Scattering function definitions

In this contribution, we work with two definitions of scattering functions:

A. after Morrison-Cross [3]

$$\hat{\boldsymbol{E}}^{s} = \hat{\boldsymbol{E}}^{\boldsymbol{i}} \cdot \hat{\boldsymbol{S}}(\boldsymbol{K_1}, \boldsymbol{K_2}) \cdot (jk_0 z)^{-1} \cdot \exp(-jk_0 z) \qquad (1)$$

where $\hat{\boldsymbol{E}}^{s}$ is a phasor of scattered electric field intensity [V/m]

$\hat{\boldsymbol{E}}^{i}$ is the phasor of the intensity of the incident electric field [V/m]

$\hat{\boldsymbol{S}}(\boldsymbol{K_1}, \boldsymbol{K_2})$ is the complex scattering (dimensionless) function of the drop for the direction of incident wave $\boldsymbol{K_1}$ and the direction of scattered wave $\boldsymbol{K_2}$

z is the distance of the scattered electric field intensity from the center of the drop

k_0 is the vacuum wave number [m-1]

B. after Uzunoglu [4]:

$$\hat{\boldsymbol{E}}^{s} = \hat{\boldsymbol{E}}^{i} \cdot \hat{\boldsymbol{f}}(\boldsymbol{K_1}, \boldsymbol{K_2}) \cdot (z)^{-1} \cdot \exp(jk_0 z) \qquad (2)$$

$\hat{\boldsymbol{f}}(\boldsymbol{K_1}, \boldsymbol{K_2})$ - is the complex scattering function of the drop ([m], usual dimension is also [cm]) for the direction of incident $\boldsymbol{K_1}$ and scattered waves $\boldsymbol{K_2}$

Special cases: if $\boldsymbol{K_1} = \boldsymbol{K_2}$, it is the forward scattering

The relationship for the conversion from $\hat{\boldsymbol{S}}$ to $\hat{\boldsymbol{f}}$ scattering function definition is:

$$\hat{f}_{forward} = -j \cdot \frac{\lambda}{2\pi} \cdot \hat{S}^* \quad (3)$$

III. Utilization of Forward Scattering Functions

In our study we are interested in imaginary part of scattering functions, which correspond to radiowave attenuation by rain particles. The formula to compute the specific rain attenuation is as follows [3]:

$$A = 8.6859 \cdot 10^3 \cdot \frac{2\pi}{k} \cdot Imag \int_0^\infty \hat{f}_{h,v} N(D) dD \quad (4)$$

Where $\hat{f}_{h,v}$ – complex scattering function

k is wave number

N(D) - is the drop size spectrum, see [5,6]

IV. Rayleigh Approximation of Forward Scattering Function

In the Rayleigh scattering each particles is considered as a dipole. To use Rayleigh approximation each homogeneous particle (in our contribution rain drop) has to be much smaller than the wavelength of incident radiation. Or the equivalent droplet diameter should be much smaller than the wavelength or, more precisely, the size parameter x should be much smaller than 1 [7].

In this work we consider more precise conditions for Rayleigh approximation application. Therefore we define the „critical size parameter." The equivalent droplet diameter D should be much smaller than λ. More precisely:

$$x \ll 1 \quad (5)$$

While size parameter is defined as follows:

$$x = \frac{2 \cdot \pi \cdot a}{\lambda} \quad (6)$$

where a - is equivolumetric drop radius [m] ($D=2a$),

λ is the wave length in vacuum[m].

At the same time, there should be a small phase shift after the passage of the electromagnetic wave through rain drop, so next condition is important as well:

$$2\pi \cdot |\hat{m}| \cdot a/\lambda_0 \ll 1.$$

In order to precise condition (5) we have suggested to replace it by this condition:

$$x < x_{crit} \quad (7)$$

Next expression of Rayleigh approximation was found in [8] and is used to calculate approximation of forward scattering function under condition (7):

$$\hat{f}_{fRay} \approx \frac{-j \cdot \lambda}{2\pi} \cdot \left[j \cdot \left(\frac{\pi}{\lambda}\right)^3 8 \cdot \hat{K} \cdot a^3 + \frac{128}{3} \left(\frac{\pi}{\lambda}\right)^6 \cdot \hat{K}^2 \cdot a^6 \right]^* \quad (8)$$

where $\hat{\boldsymbol{K}}$ is an auxiliary parameter:

$$\hat{\boldsymbol{K}} = \frac{\hat{\boldsymbol{m}}^2 - 1}{\hat{\boldsymbol{m}}^2 + 2} \quad (9)$$

where $\hat{\boldsymbol{m}}$ is complex refractive index of water of rain drop

Note that scattering functions are complex ones.

In Fig. 2 there are shown courses of imaginary parts of scattering functions (proportional to rain attenuation) calculated by Rayleigh theory for chosen frequencies. These frequencies are often used the radar meteorology.

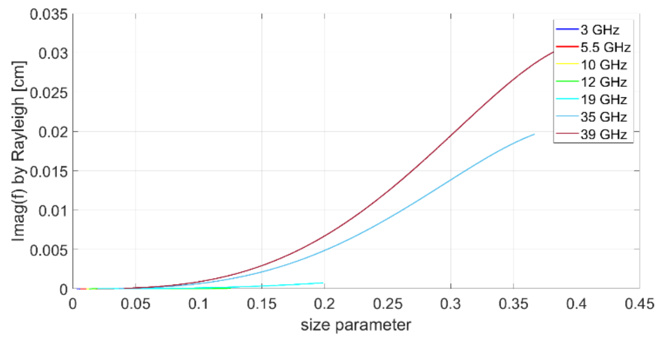

Fig. 2. Imaginary part of forward scattering functions after Rayleigh approximation for different frequencies.

With increasing frequency, the value of attenuation is also increasing within studied frequency bands, cf. (4).

V. Mie Forward Scattering

Mie scattering applies only to spherical objects. which is true for small raindrops as it was mentioned above. On the other hand, Mie scattering is valid for any size parameter (x) value. This given approximation allows to study the dependence of rain attenuation on frequency and temperature, but never depolarization phenomena or scattering dependence on polarization, which applies only to non-spherical (larger) drops.

The Mie algorithm represents an infinite series with the help of the Mie coefficients $\hat{a}_n$ and $\hat{b}_n$. For scattering on a spherical object, we do not need to take into account the polarization due to the rotational symmetry of the sphere [9].

The Morisson-Cross relationship applies to forward scattering [3], which was published, for instance, in [9, 12]:

$$\hat{S}_{forward\_Mie} = \frac{-j \cdot \lambda^3}{\pi^3 \cdot D^2} \left[\sum_{n-1}^{\infty} (2n+1) \cdot (a_n + b_n) \right]^* \quad (10)$$

where D - is the equivolumetric drop diameter (D = 2a)

λ – is the wave length

Theoretically, N_{max} should be infinity (i.e. very large number). But in such case some numerical problems during computer evaluation can occur so N_{max} must be carefully estimated. For N_{max} next recommendation is usually used [10]:

N_{max} = x+4x1/3 +1 for 0.02 < x < 8,

for N_{max} it holds then 2 < N_{max} < 17

N_{max} = x+4.05 x1/3 +2 for 8 < x < 4 200,

for N_{max} it holds then 18 < N_{max} < 4 267

N_{max} = x+4x1/3 +2 for 4200 < x < 20 000,

for N_{max} it holds then 4267 < N_{max} < 20111

In my diploma thesis [11] I found that the developed code according to the given relations works for wavelengths shorter than 1.2 cm (frequencies above 25 GHz), which corresponds to the size parameter x≈0.1, which is the border for the utilisation of the Rayleigh scattering formula. Note that Mie algorithm could be used in Rayleigh region, but not conversely.

In the Fig.3 imaginary parts of forward scattering function after Mie are calculated.

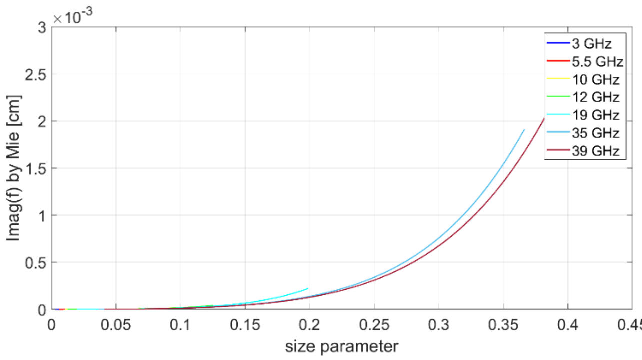

Fig. 3. Imaginary part of forward scattering functions by Mie algorithm for different frequencies

The graphs show that the waveforms at the lower frequencies are steeper than at the higher frequencies.

VI. Comparing Rayleigh and Mie approximation for calculation of forward scattering functions

In the next Figs. 4-6 the imaginary parts of forward scattering functions are found for meteoradar frequencies 5.5, 10 and 35 GHz. These calculations were produced for drop sizes below 1 mm for better demonstration of differences between Rayleigh and Mie algorithms as on higher frequencies the difference is enormous.

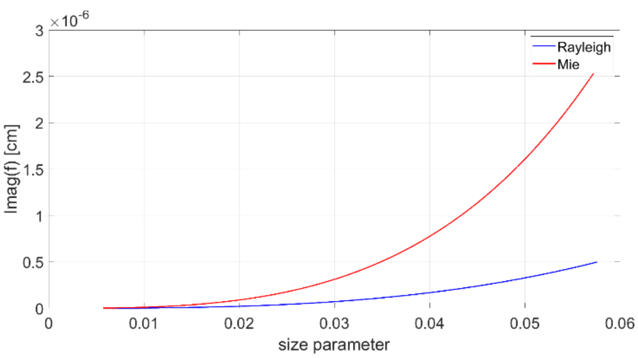

Fig. 4 Comparison of Rayleigh and Mie algorithm computing forward scattering function on rain drops for 5.5 GHz

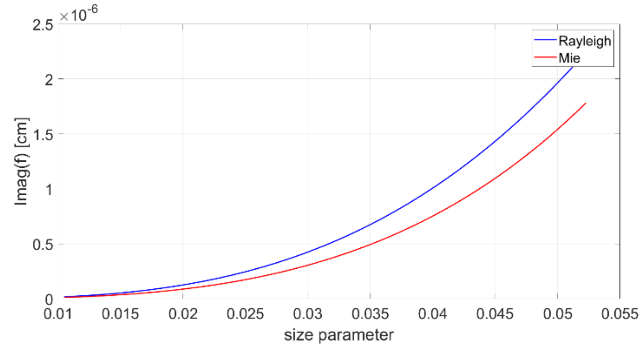

Fig. 5. Comparison of Rayleigh and Mie algorithm computing forward scattering function on rain drops for 10 GHz

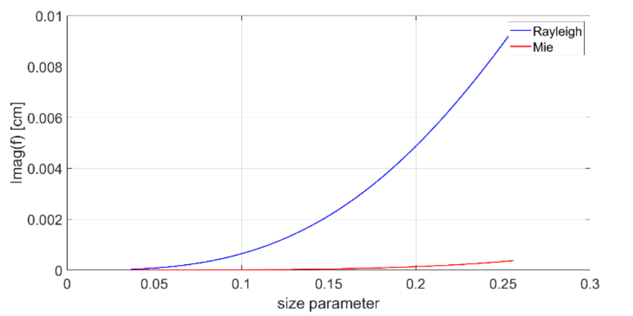

Fig. 6. Comparison of Rayleigh and Mie algorithm computing forward scattering function on rain drops for 35 GHz

The scattering functions in Figs. 4-6 prove that the Mie and Rayleigh scattering above the critical value x_{crit} are no longer of similar values. Therefore, for the accuracy of the calculation, we should pay attention to the observance of the conditions of the validity of the Rayleigh scattering, otherwise we will have to perform more complicated calculations according to Mie's theory on good computer.

In this contribution we define critical size parameter x_{crit}, which responds to 2 % difference between Rayleigh and Mie approximation of forward scattering. It means that below this critical value of size parameter *x* the Rayleigh scattering function might be used. In Table 1 there are shown x-crititcal values ($x_{crit.}$) for different frequencies.

Table I. Critical size parameters depending on frequency

Frequency, GHz	xcrit
1	0.0151
5	0.0168
10	0.0199
15	0.0236
20	0.0272
25	0.0314
35	0.0403
45	0.0471
55	0.0518
65	0.0613

75	0.0628
85	0.0712
94	0.0787
120	0.0880

VII. Conclusion

We focused on forward scattering of radiowaves on rain drops. These functions are important to derive the rain attenuation of micro- and mm- waves. We compared the more general Mie scattering with the Rayleigh approximation. The conditions for Rayleigh scattering were shown.

In this paper we determined the critical values of the size parameter x (these values are the values at which the difference between Rayleigh and Mie scattering is just 2%). For higher values of the size parameter ($x > x_{crit}$) the difference between Rayleigh and Mie scattering is already above 2% and therefore Rayleigh's approximation can be no longer applied, but only Mie's scattering is acceptable.

It will be possible now to determine more precisely the conditions under which it is more advantageous to use easier and simpler Rayleigh scattering approximation than the complicated Mie scattering formulas.

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