Neural Intuitionistic Fuzzy System with Justified Granularity

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Abstract

Fuzzy systems are commonly used to model uncertainties occurring in data. They have been intensively investigated and extended to be used for the construction of time series forecasting models. In particular, intuitionistic fuzzy sets have been used to capture higher levels of uncertainty occurring in the modeled data. Neural networks were also used to reflect non-linearity relationships frequently observed in time series. Following this line of research, in this paper, we propose a new hybrid fuzzy system merging the most promising data representation methods with an advanced optimization technique, which is the principle of justified granularity. In this technique, we construct an innovative time series forecasting model. In the experimental part of the paper, we demonstrate the advantages arising from applying the proposed approach to metal price forecasting. We provide solid evidence that the proposed model is competitive with other state-of-the-art models known from the literature for forecasting horizons of one and five days.

Keywords: Fuzzy systems, neural networks, time series forecasting

1 Introduction

Representing uncertainty and non-linearity in time series is an important problem that has been addressed for many years. One of the models applied for this task is a fuzzy system (FS).

The FS-based approach to time series forecasting comprises three stages:

1. Approximation of time series. The collected historical time series are partitioned and fuzzified. The produced fuzzy time series (FTS) approximate the original time series (TS) to represent the uncertainty involved in them.
2. Construction of a fuzzy system. From the obtained FTS, fuzzy if-then rules are inferred. The set of these rules representing temporal relationships existing in the FTS is called a FS. In this way, the FS forms a TS forecasting model which is used to predict the future values of the FTS.
3. Forecasting and defuzzification. The FS is used to forecast the FTS. Then, if necessary, the FTS is defuzzified. As a result, we obtain crisp forecasts.

Note that the presented stages are mutually dependent. The construction of the forecasting model used at Stage 2 depends on how the TS approximation in Stage 1 is made. The partitioning and fuzzification of TS substantially influence the possibility of creating an effective fuzzy system. Besides, the fuzzy system constructed in Stage 2 affects the accuracy of forecasts produced at Stage 3.

It is worth mentioning that the efforts of researchers are usually focused on improving a particular, selected stage of the above procedure. This leads, however, to the necessary adaptation of the other parts of the described process.

This paper proposes an amendment of both the first and second stages of the FS-based forecasting approach. At first, we improve the partitioning and the fuzzification of the TS. We propose to use the principle of justified granularity to optimize the construction of fuzzy sets. Then, on that basis, we construct intuitionistic fuzzy sets. As a result, we obtain intuitionistic fuzzy time series (IFTS) that are later subject to forecasting. Accordingly, instead of using FTS, we apply the optimized IFTS to construct an intuitionistic fuzzy system used in Stage 2 of the considered approach. Moreover, by complementing that system with a neural network, we improve its effectiveness for TS forecasting.

There have been several attempts to incorporate the additional uncertainty into FTS models (Gaxiola et al, 2014; Luo et al, 2019; Eyoh et al, 2018) in order to improve their inference capability and robustness to noise. However, as far as we know, the proposed neural intuitionistic fuzzy system with the justified granularity is the first extension of FTS forecasting models that considers the volatility in the TS data in Stage 1. In addition, intuitionistic fuzzy operators are exploited to obtain the firing weights of if-then rules and a weighted average method intuitionistic fuzzy sets is used to defuzzify the outcome of the proposed neural intuitionistic fuzzy system of Takagi-Sugeno-Kang (TSK) type. In the earlier version of this paper (Hajek et al, 2021), we argued that the main limitation of the used fuzzy clustering approach is that no if-then rules were matched for many observations due to the high volatility in the TS data. To address this issue, here we replace the fuzzy clustering algorithm with fuzzy association rules for rule generation. The gradient descent algorithm is used to train the neural intuitionistic fuzzy system. High accuracy and computational efficiency are achieved by learning both the consequent parameters of the if-then rules and the parameters of fuzzy sets. In summary, the contributions of this study are threefold:

- A novel neural intuitionistic fuzzy system utilizing intuitionistic fuzzy sets and justified granularity is proposed. Different from the existing TS forecasting models, this extension of a neural fuzzy systems preserves the semantics of fuzzy sets while considering the TS characteristics to assign the the membership and non-membership degrees to TS elements.
- For the first time, the architecture of neural intuitionistic fuzzy system for IFTS forecasting is developed. To this end, intuitionistic fuzzy operators are proposed to process the information in terms of membership and non-membership degrees. To ensure a high level of TS coverage and sufficient complexity of the architecture, fuzzy association rules are generated.
- To demonstrate the performance of the neural intuitionistic fuzzy system, several sets of experiments are conducted to forecast precious metal prices, which are known to be highly volatile. As far as we know, this is the first time that a generalization of a neural fuzzy system is applied to metal price forecasting.

The remainder of this paper is organized as follows. Based on the literature review given in Section 2, we motivate the research presented in this paper. In Section 3, which covers the theoretical background, we formalize the addressed problem and present all the elements known from the literature that we later use for the construction of the proposed model. Section 4 outlines the proposed TS forecasting approach. Section 5 provides experimental evidence for the high competitiveness of the proposed model. Section 6 concludes the paper.

2 Related work

As indicated in the introduction, the FS-based approach to forecasting consists of three stages. The literature review presented in this section is focused on two key stages, namely (1) TS approximation and (2) construction of FSs.

2.1 Approximation of time series

The approximation of TS, which is the first stage of the FS-based approach, was addressed in numerous papers. For the partitioning of TS in the time domain, the equal-sized intervals were predominantly used (Yu and Huarng, 2010; Bisht and Kumar, 2016). This assumes however uniform distribution of data over the universe of discourse, which rarely occurs in reality. Consequently, the fuzzy sets constructed over the equal-sized intervals do not represent the underlying data well (Bose and Mali, 2019).

For that reason, to optimize the parameters of fuzzy sets distributed non-uniformly over the universe of discourse, diverse optimization methods were used, including clustering-based methods that provided a good trade-off between computational demand and forecasting accuracy (Roy, 2016; Bose and Mali, 2018; Bougoudis et al, 2018). Evolutionary algorithms were also used for this task due to their capacity to find the optimal global solution (Bas et al, 2018). The disadvantage may be their computational requirements and susceptibility to overfitting.

To handle the uncertainty in the TS data and improve forecasting performance, the TS space can be divided into information granules representing semantically sound entities (Lu et al, 2014). Different granular spaces were used in the literature to produce high forecasting accuracy in multi-factor FTS models (Deng et al, 2016; Singh and Dhiman, 2018). A time-dependent fuzzy information granule was proposed by Yang et al (2017) for effective long-term TS forecasting. However, either evolutionary algorithms are used to optimize the parameters of generated fuzzy sets, or fuzzy systems must be trained to achieve high accuracy.

It should be noted that the above methods are not capable of modeling the dynamic behavior of TS. To address this issue, we propose a novel TS approximation method combining (1) data partitioning based on the concept of justified granularity to produce the parameters of unequal-sized intervals and (2) fuzzification of intuitionistic fuzzy sets by exploiting the variance in the TS data.

2.2 Construction of fuzzy systems

For FTS forecasting, it is necessary to determine fuzzy logic relationships between their previous and forecasted values. This is done using historical TS data (Bose and Mali, 2019). Conventional FTS methods based on fuzzy relationship matrices (Song and Chissom, 1993) have been extended to improve forecasting performance. For example, exponential FTS were introduced to replace conventional FTS in order to assign a larger weight to recent TS observations (Talarposhti et al, 2016). This idea was shown to be useful for forecasting stock market indexes as more recent changes are more important to stock market investors. Fuzzy empirical probabilities were introduced into the FTS model to exploit probabilistic and fuzzy uncertainties in financial TS patterns (de Lima Silva et al, 2019). A trend weighting function was incorporated into the FTS two-factor model to improve the forecasting performance (Kumar et al, 2019). Moreover, for the same purpose, fuzzy sets were extended using hesitant fuzzy sets (Bisht and Kumar, 2016, 2019) and probabilistic fuzzy sets (Gupta and Kumar, 2019) in FTS forecasting models.

The main disadvantage of these fuzzy logic relationship-based models is their poor generalization capacity, often leading to inaccurate out-of-sample forecasts. Moreover, it is difficult to find matched fuzzy logic relationships and, therefore, reliable predictions cannot be performed for these observations (Li and Yu, 2020). To overcome these problems, neural networks were integrated with FTS models to learn the nonlinear relationships from the underlying data.

To obtain the set of fuzzy logic relationships, neural networks were trained to predict the consequences of the relationships by using the index numbers of fuzzy sets of the antecedents in the high-order FTS model (Chen, 2014; Egrioglu et al, 2009). Similarly, index numbers were replaced by the central values of fuzzy sets, and a neural network was used to to defuzzify the FTS data (Singh and Borah, 2013). In other studies, membership values were used to feed neural networks calculating fuzzy logic relationships (Yu and Huarng, 2010; Yolcu et al, 2016). Support vector regression was also used to compute unrecognized high-order fuzzy logic relationships from the stock market TS data (Wu et al, 2021). Alternatively, hybrid neural fuzzy models were used to learn the relationships, using an if-then rule-based inference mechanism to generate defuzzified forecasts. Adaptive neuro-fuzzy inference system (ANFIS) represents the most widely applied hybrid model integrating fuzzy inference systems and neural networks. In ANFIS, the antecedents of if-then rules are given in advance, and a neural network is employed to learn the rule consequents that are represented by fuzzy sets (in the Mamdani fuzzy inference system) (Gaxiola et al, 2014) or by linear functions (in the Takagi-Sugeno-Kang (TSK) fuzzy inference system) (Peng et al, 2015; Su and Cheng, 2016).

To incorporate the additional uncertainty present in many real-life TS data, several extensions of fuzzy inference systems were developed. A type-2 fuzzy inference system incorporating the footprint of uncertainty was applied by Gaxiola et al (2014) to calculate the type-2 fuzzy weights in a neural network, which significantly enhanced its robustness to TS noise. To reduce the over-fitting problem of single forecasting models, ensembles of neural interval type-2 fuzzy systems were proposed for TS prediction (Soto et al, 2018). By introducing intuitionistic fuzzy sets into the interval type-2 fuzzy inference system, its inference capability was improved by enriching the reasoning information (Luo et al, 2019; Eyoh et al, 2018).

Existing neural fuzzy models for FTS forecasting are compared in Table 1. The main disadvantage of the neural fuzzy models is that only static membership functions have been considered in the partitioning and fuzzification process. In other words, even though the degree of hesitancy was taken into account in the fuzzification process by using type-2 (intuitionistic) fuzzy sets, volatility in the TS data was neglected. In fact, the clustering-based and discretization methods used in previous neural fuzzy models were not designed originally for the estimation of type-2 or intuitionistic fuzzy sets. Moreover, they are based only on data density without preserving semantics of fuzzy sets. To overcome these disadvantages, the principle of justifiable granularity is proposed in this study to reflect both the data coverage and semantics of fuzzy sets by maintaining an acceptable degree of specificity. In addition, unlike existing models, the proposed fuzzification process allows us to effectively model the variance in TS data. Motivated by the applications of intuitionistic fuzzy sets in FTS forecasting, we propose a neural intuitionistic fuzzy system that not only overcomes the above problems but also retains the computational efficiency and interpretability of its rule base by generating it using fuzzy association rules.

Table 1 Comparison of neural fuzzy models for FTS forecasting.

Study	Fuzzy set	NN (learning)	Partitioning and fuzzif.	Defuzzif.	Fuzzy system
Egrioglu et al (2009)	FS	MLP (BP)	equal-sized	WA	FLR
Yu and Huarng (2010)	FS	MLP (BP)	equal-sized	WA	FLR
Singh and Borah (2013)	FS	MLP (BP)	RD	WA	FLR
Chen (2014)	FS	MLP (BP)	EBD	MPI	FLR
Gaxiola et al (2014)	T2FS	MLP (BP)	equal-sized	WA	Mamdani FRBS
Peng et al (2015)	FS	MLP (BP)	increment. clust.	WA	TSK FRBS
Yolcu et al (2016)	FS	MLP (BP)	fuzzy c-means	WA	FLR
Su and Cheng (2016)	FS	MLP (BP)	subtract. clust.	WA	TSK FRBS
Singh (2017)	FS	MLP (BP)	EBD	WA	FLR
Bas et al (2018)	FS	Pi-Sigma (PSO)	equal-sized	MPI	FLR
Soto et al (2018)	T2FS	MLP (BP)	PSO	WA	TSK T2FRBS
Eyoh et al (2018)	T2IFS	MLP (BP)	equal-sized	WA	TSK T2IFRBS
Luo et al (2019)	T2IFS	RNN (EKF)	density clust.	WA	TSK T2IFRBS
this study	IFS	MLP (BP)	PJG	IFWA	TSK IFRBS

Legend: BP - backpropagation algorithm, EBD - entropy-based discretizazion, EKF - extended Kalman filter, FLR - fuzzy logic relationship, FRBS - fuzzy rule-based system, FS - fuzzy set, IFWA - intuitionistic fuzzy weighted average, MLP - multilayer perceptron, MPI - middle point of interval, NN - neural network PJG - principle of justified granularity, PSO - particle swarm optimization, RD - re-partitioning discretization, RNN - recurrent NN, T2IFS - type-2 intuitionistic fuzzy set, TSK - Takagi-Sugeno-Kang, WA - weighted average.

3 Background

As mentioned in the introduction, in this paper, we address a problem of TS forecasting which we formalize as follows.

3.1 Time series forecasting - problem formulation

Let $T = \{0, 1, ..., n\}$ be a discrete time scale of the length $n \in \mathbb{N}$. For each $t \in T$, we define a vector of crisp variables $X^t =< x^t(0), x^t(1), ..., x^t(m) >$, where $m + 1 \in \mathbb{N}$ is the length of that vector. Variables $x^t(i) \in \mathbb{R}$ are called predictors, regressors, independent or explanatory variables (Hyndman and Athanasopoulos, 2018). The set of vectors X^t collected over time forms a multivariate TS $\{X^t\}$.

Assume $y^t \in \mathbb{R}$ is a distinguished crisp variable called a forecast variable, regressand, dependent or explained variable (Hyndman and Athanasopoulos, 2018). The goal of TS forecasting is to find a forecasting model M representing the relationship between X^t and y^t enabling to calculate the forecast $\hat{y}^{t+1} = M(\{X^t\})$. In this case, M is called a regression model.

As the model M is induced from historical data, it is usually not perfect. It means the values of $\hat{y}^{t+1}$ differ from real-world observations y^{t+1}.

The forecasting error measured at time t is calculated as $e^{t+1} = \hat{y}^{t+1} - y^{t+1}$ (Shmueli (2011)). For a longer forecasting horizon $h > 1$, a mean percentage error (MAPE) is calculated by the formula (1).

$$MAPE = \frac{1}{h}\sum_{t=1}^{h} |\frac{e^{t+1}}{y^{t+1}}| \tag{1}$$

The root mean square error (RMSE) is calculated by formula (2).

$$RMSE = \sqrt{\frac{1}{h}\sum_{t=1}^{h}(e^{t+1})^2} \tag{2}$$

3.2 Fuzzy sets and intuitionistic fuzzy sets

Let us assume an element x from the universe of discourse X belongs to a fuzzy set F in the degree μ_F. The fuzzy set is defined by formula (3).

$$F = \{\langle x, \mu_F \rangle x \in X\} \tag{3}$$

For the purpose of this paper, we assume that the fuzzy set is represented by a triangular membership function:

$$\mu(x) = max(min(\frac{x-a}{b-a}, \frac{c-x}{c-b}), 0), \tag{4}$$

where a and c locate the feet of the membership function, and b defines its peak.

An intuitionistic fuzzy set A is complemented by a non-membership degree (Atanassov, 1999) of the element x and is defined by formula (5).

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle x \in X\} \tag{5}$$

It holds that $0 \leq \mu_A(x) \leq 1$, $0 \leq \nu_A(x) \leq 1$ and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The hesitation degree $\pi_A(x)$ is calculated as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

3.3 Principle of justifiable granularity

To optimize the fuzzy membership functions used in the proposed fuzzy system, we use the principle of justified granularity (PJG).

The PJG was originally proposed to optimize the construction of an information granule (Pedrycz and Vukovich, 2001). Let us go into detail about that technique.

Let $x \in [x_{min}, x_{max}]$ be a real-valued variable, where x_{min}, x_{max} are its lower and upper bounds, respectively. We form in the interval $[x_{min}, x_{max}]$ a parametrized triangular fuzzy number F. Its membership function is denoted as $\mu_F(x;\ m, a, b)$ with the parameters a, b standing for its bounds, and m

$(a < m < b)$ as its modal value, i.e., $\mu_F(x;\ m, a, b) = 1$. We assume that m is calculated as the median (Pedrycz and Homenda, 2013).

We fit F to the observations of the variable $x \in [x_{min}, x_{max}]$. For this purpose, we optimize the parameters a and b of $F(x;\ m, a, b)$. First, we decompose $X(x;\ m, a, b)$ to two linear functions. For $a \leq x \leq m$, we have the left-hand side of the membership function:

$$X(x;\ m, a) = \frac{1}{m-a} \cdot x - \frac{a}{m-a}. \tag{6}$$

For $m \leq x \leq b$, the right-hand side of the membership function is accordingly defined:

$$X(x;\ m, b) = \frac{1}{m-b} \cdot x - \frac{b}{m-b} \tag{7}$$

To optimize X two conflict requirements, namely coverage and specificity, are considered.

We define the coverage of X in the interval $[a, b]$ by formulas (8) and (9) for the left-hand and right-hand sides of the membership function, respectively:

$$cov([a, m]) = \sum_{x \in [a,m]} X(x;\ m, a), \tag{8}$$

$$cov([m, b]) = \sum_{x \in [m,b]} X(x;\ m, b). \tag{9}$$

The specificity measures the amount of information contained in a fuzzy subset. The specificity also evaluates the degree to which a fuzzy subset points to one element as its member. An increase in the specificity of information tends to increase the usefulness of the information (Yager, 1998). For example, in the case of a numeric interval, the inverse of the length of this interval can serve as a sound measure of specificity. The shorter the interval, the better the satisfaction of the specificity requirement (Wang et al, 2014; Pedrycz et al, 2014).

When it comes to a triangular fuzzy set A, the specificity measure is determined by considering the specificity of a certain a-cut of the fuzzy set and then integrating the corresponding partial results. For $X(x;\ m, a)$ we have:

$$sp([a, m]) = 1 - \frac{0.5 \cdot |m-a|}{|m - x_{min}|}. \tag{10}$$

Accordingly for $X(x;\ m, b)$ we have:

$$sp([m, b]) = 1 - \frac{0.5 \cdot |b-m|}{|x_{max} - m|}. \tag{11}$$

To optimize the coverage and specificity of the fuzzy granule, the performance index Q is defined separately for both sides of the membership function. Formulas (12) and (13) are used for this purpose:

$$Q(a) = cov([a, m]) \cdot sp([a, m)], \tag{12}$$

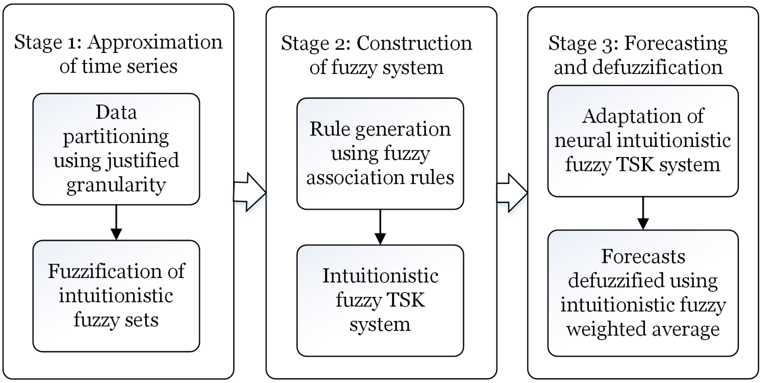

Fig. 1 A conceptual framework of the proposed IFTS forecasting model.

$$Q(b) = cov([m, b]) \cdot sp([m, b)]. \tag{13}$$

The bounds a, b of the granule are optimized to find the trade-off between the coverage and specificity of the granule. The optimization results in the following expressions:

$$a_{opt} = argmax_a \; Q(a), \tag{14}$$

$$b_{opt} = argmax_b \; Q(b). \tag{15}$$

3.4 Takagi-Sugeno-Kang fuzzy inference system

The approach proposed in this paper is based on the fuzzy inference system of TSK type, which consists of if-then rules of the first order. The j-th rule R_j, $j = 1, 2, \ldots, N$, can be defined as follows:

$$\begin{aligned} R_j : \text{if } x_1^t \text{ is } A_{1,j} \text{ and } x_2^t \text{ is } A_{2,j} \text{ and } \ldots \text{and } x_i^t \text{ is } A_{i,j} \text{ and } \ldots \text{and} \\ x_n^t \text{ is } A_{n,j} \text{then } y_j^{t+h} = a_{0,j} + a_{1,j}x_1^t + \cdots + a_{i,j}x_i^t + \cdots + a_{n,j}x_n^t, \end{aligned} \tag{16}$$

where $A_{i,j}$ is antecedent intuitionistic fuzzy set for the i-th input attribute x_i^t and j-th rule R_j, y_j^{t+h} is the predicted output for the j-th rule, h is the forecasting horizon, and $a_{0,j}, a_{1,j}, \ldots, a_{i,j}, \ldots, a_{n,j}$ are the consequent parameters.

4 Neural Intuitionistic Fuzzy System

As depicted in Fig. 1, the concept of the proposed IFTS forecasting model comprises three stages: (1) approximation of TS using intuitionistic fuzzy sets, (2) construction of a neural intuitionistic fuzzy system of TSK type, and (3) defuzzification of forecasted values using the intuitionistic fuzzy weighted average (IFWA) operator.

4.1 Partitioning and fuzzification of time series

In the first stage of the proposed approach, we partition and fuzzify the considered TS. The proposed method for approximating TS includes the following steps:

Step 1. Following previous studies (Bisht and Kumar, 2016; Gupta and Kumar, 2019), the universe of discourse is defined using standard deviation $U = [D_{min} - \sigma, D_{max} + \sigma]$, where D_{min}, D_{max} and σ are minimum, maximum, and standard deviation of the TS data in the training set.

Step 2. According to the Principle of Justified Granularity, we calculate the modal value m, minimum x_{min} and maximum x_{min} of the TS from the k^{th} time interval $t \in [(k-1)\cdot w, k\cdot w]$. Then an interval $[x_{min}, x_{max}]$ within the universe of discourse (amplitude or the change of amplitude) is created. Over this interval, a fuzzy triangular number X_k is constructed with the membership function $X_k(x;\ a, m, b)$, where the parameters a and b are subject to optimization.

Initially, they are assumed as $a = x_{min}$ and $b = x_{max}$, then they are optimized using the function of coverage (8),(9), specificity (10),(11), and optimization indexes (12),(13) given in Section 3.3.

As a result, the parameters of the granule are optimized as $a = a_{opt}$ and $b = b_{opt}$ (see formulas (14) and (15)). Thus, the resulting, optimized triangular membership function is obtained as $X_k(x;\ a_{opt}, m, b_{opt})$.

Step 3. Fuzzify TS data using the optimized triangular membership functions from Step 2. To perform the fuzzification of intuitionistic fuzzy sets, we employ the fuzzification method developed by Hajek et al (2020) and calculate the membership and non-membership degree for the considered TS as follows:

$$\mu_A(x_i^t) = \mu(x_i^t) \times (1 - \delta D), \tag{17}$$

$$\nu_A(x_i^t) = 1 - \mu(x_i^t) \times (1 - \delta D) - \delta D, \text{ where:} \tag{18}$$

$$D = (max(\mu(x_i^t), \mu(x_i^{t-1}), \dots, \mu(x_i^{t-4})) - min(\mu(x_i^t), \mu(x_i^{t-1}), \dots, \mu(x_i^{t-4}))) \tag{19}$$

and δ is set to 1 in agreement with Hajek et al (2020). Hence, intuitionistic fuzzy sets are given as $A = \{\langle x_i^t, \mu_A(x_i^t), \nu_A(x_i^t)\rangle x_i^t \in X_i\}$, where X_i is the universe of discourse for the i-th input attribute. Note that in our case, the fuzzification parameter D considers the volatility of the last five observations in the TS $(x_i^t, x_i^{t-1}, \dots, x_i^{t-4})$ but its calculation can be tailored to the specific forecasting problem.

4.2 Construction of the fuzzy system

To generate the if-then rules for the fuzzy inference system of TSK type defined by formula (16), we extract fuzzy association rules using the extended Apriori algorithm (Chen and Wei, 2002). To model the linguistic labels in the fuzzy association rules, we use the triangular membership functions obtained using the Principle of Justified Granularity in Step 2 of the TS approximation process. The if-then rules are generated from the frequent itemsets based on the pre-defined minimum support and confidence. Only those rules are considered in which the forecasted attribute under study represented the rule consequent.

4.3 Forecasting and defuzzification

In this step, the parameters of membership functions a_{opt}, m, b_{opt} and consequent parameters $a_{0,j}, a_{1,j}, \ldots, a_{n,j}$ of the if-then rules are adapted using the gradient descent algorithm due to its stable convergence (Eyoh et al, 2018; Hajek and Olej, 2017). The algorithm for updating the above parameters can be defined as follows:

$$w_{i+1} = w_i - \eta \nabla_\theta J\left(w_i;\ x^{(t)};\ y^{(t+h)}\right), \tag{20}$$

where w is the updated parameter of the neural intuitionistic fuzzy system $w = \{a_{opt}, m, b_{opt}, a_{0,j}, a_{1,j}, \ldots, a_{n,j}\}$, η denotes learning rate, i is the iteration index, J is the objective function (root mean square error (RMSE)), and $x^{(t)}$ and $y^{(t+h)}$ represent the input and output for the t-th TS observation, respectively.

Recall that the outputs of the if-then rules are calculated as $y_j^{t+h} = a_{0,j} + a_{1,j}x_1^t + \cdots + a_{i,j}x_i^t + \cdots + a_{n,j}x_n^t$. To obtain the defuzzified forecast y_{IFWA}^{t+h}, the weighted average of the outputs is calculated in the following steps. First, the firing weight w_j is obtained for the j-th rule R_j using the Gödel t-norm operators defined in the following way (Angelov, 1995):

$$w_j^\mu = \min_{j=1,2,\ldots,N} (\mu_A(x_1^t), \mu_A(x_2^t), \ldots, \mu_A(x_n^t)), \tag{21}$$

$$w_j^\nu = \max_{j=1,2,\ldots,N} (\nu_A(x_1^t), \nu_A(x_2^t), \ldots, \nu_A(x_n^t)), \tag{22}$$

$$w_j = w_j^\mu - w_j^\nu, \tag{23}$$

where w_j^μ and w_j^ν are the membership and non-membership degrees of the firing weight w_j, respectively. It is worth noting that only those rules are considered for which the firing weights are positive. Their acceptance degree is higher than their non-acceptance degree. The firing weights w_j are transformed to the normalized values w_j^{norm}.

Then, the IFWA operator is applied to calculate the defuzzified output of the neural intuitionistic fuzzy system as follows:

$$y_{IFWA}^{t+h} = \frac{\sum_{j=1}^N y_j^{t+h} w_j^{norm}}{\sum_{j=1}^N w_j^{norm}}. \tag{24}$$

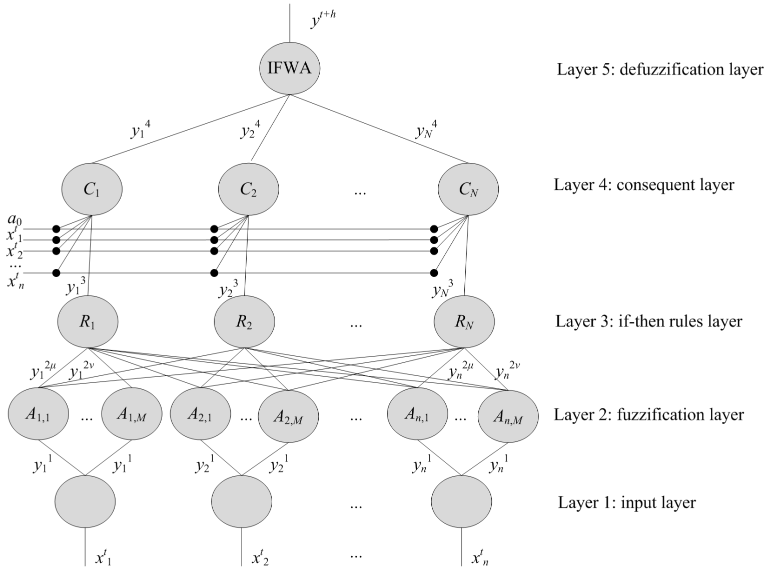

Fig. 2 Architecture of neural intuitionistic fuzzy system for IFTS forecasting.

4.4 Architecture of neural intuitionistic fuzzy system for time series forecasting

The proposed five-layered architecture of neural intuitionistic fuzzy system for IFTS forecasting is outlined in Fig. 2. Detailed operation of each layer is as follows.

Input layer: Layer 1 is designed to pass the crisp input attributes $x_1^t, x_2^t, \ldots, x_n^t$ to the fuzzification layer.

Fuzzification layer: Each neuron in Layer 2 represents an antecedent intuitionistic fuzzy set. For the i-th input attribute x_i^t, M intuitionistic fuzzy sets are constructed $A_{i,1}, A_{i,2}, \ldots, A_{i,M}$. Specifically, the PJG is first used to optimize information granules in terms of triangular fuzzy sets. Then, intutionistic fuzzy sets are obtained using the fuzzification method in formulas (17) and (18).

Rule layer: For each rule $R_1, R_2, \ldots, R_N$ (defined in formula (16)), there is a neuron in Layer 3 whose output is the firing weight w_j calculated using formula (23).

Consequent layer: In Layer 4, neurons represent the outputs of the rules y_j^{t+h}, which are calculated as the weighted sums of the crisp input attributes $x_1^t, x_2^t, \ldots, x_n^t$ with consequent parameters $a_{0,j}, a_{1,j}, \ldots, a_{n,j}$ used as the weights.

Defuzzification layer: The neuron in Layer 5 represents the final defuzzified output y^{t+h} obtained using the IFWA operator in formula (24).

5 Model Validation

In this section, we validate the proposed neural intuitionistic fuzzy system for TS forecasting of precious metal prices. Precious metals are not only considered an investment and a store of value, but they are also increasingly used as industrial commodities. For this reason, price fluctuations in the precious metal market have been increasingly volatile, which is generally considered harmful, as it entails uncertainty about future prices and translates into the volatility of material costs. This is also why the forecasting of precious metals has attracted broad interest in recent years (Du et al, 2020; Salisu et al, 2020).

5.1 An illustrative example of univariate time series forecasting

To illustrate each step of the proposed neural intuitionistic fuzzy system, the forecasting model is used to predict the daily gold price in January 2017.

Step 1. The universe of discourse is given as $U = [1162.39, 1230.26]$, where $D_{min} = 1175.85$, $D_{max} = 1216.80$ and $\sigma = 13.46$.

Step 2. First, a sequence of three information granules is produced $\{X_k\} = \{X_1, X_2, X_3\}$, as shown in Table 2. Then, the parameters a and b of the fuzzy triangular numbers $X_k(x;\ a_{opt}, m, b_{opt})$ are obtained using formulas (14) and (15) (Table 3), and linguistic terms are assigned to the fuzzy triangular numbers.

Table 2 Sequence of granules for gold price time series.

t	granule	gold price	t	granule	gold price	t	granule	gold price
1	1	1175.85	6	2	1190.35	11	3	1200.55
2	1	1178.50	7	2	1203.00	12	3	1212.85
3	1	1189.50	8	2	1216.05	13	3	1260.80
4	1	1178.55	9	2	1214.75	14	3	1195.00
5	1	1205.05	10	2	1196.05	15	3	1189.70

Table 3 Parameters of fuzzy triangular numbers.

fuzzy number	a_{opt}	m	b_{opt}	linguistic term
X_1	1178.50	1178.55	1205.05	low
X_2	1190.35	1203.00	1214.75	high
X_3	1189.70	1200.55	1212.85	medium

Step 3. Pairs of membership and non-membership degrees $[\mu_A(x^t),\ \nu_A(x^t)]$ are obtained as the result of fuzzification of intuitionistic fuzzy sets (formulas

(17) and (18)), as shown in Table 4. For example, $[\mu_{low}(x^9), \nu_{low}(x^9)] = [0.34 \times (1-(0.55-0.00)), 1-0.34\times(1-(0.55-0.00))-(0.55-0.00)] = [0.15, 0.29]$.

Table 4 Intuitionistic fuzzy sets.

t	low	medium	high	t	low	medium	high
5	[0.00,0.00]	[0.02,0.34]	[0.00,0.17]	11	[0.00,0.66]	[0.00,0.00]	[0.03,0.16]
6	[0.00,0.00]	[0.16,0.04]	[0.00,0.00]	12	[0.00,0.66]	[0.00,0.00]	[0.00,0.19]
7	[0.00,0.00]	[0.00,0.20]	[0.00,0.00]	13	[0.24,0.39]	[0.00,0.00]	[0.07,0.12]
8	[0.00,0.45]	[0.00,0.20]	[0.00,0.00]	14	[0.24,0.18]	[0.00,0.00]	[0.00,0.19]
9	[0.15,0.29]	[0.12,0.08]	[0.00,0.00]	15	[0.18,0.06]	[0.00,0.51]	[0.00,0.63]
10	[0.11,0.55]	[0.00,0.00]	[0.00,0.00]				

Step 4. Three rules can be defined as follows:

$$R_1 : \text{if } x_1^t \text{ is } \textit{low} \text{ then } y_1^{t+1} = a_{0,1} + a_{1,1}x_1^t, \tag{25}$$

$$R_2 : \text{if } x_1^t \text{ is } \textit{medium} \text{ then } y_2^{t+1} = a_{0,2} + a_{1,2}x_1^t, \tag{26}$$

$$R_3 : \text{if } x_1^t \text{ is } \textit{high} \text{ then } y_3^{t+1} = a_{0,3} + a_{1,3}x_1^t. \tag{27}$$

Step 5. Neural network is trained for one-day-ahead forecasting. For the given data, there is one neuron in the input layer and one neuron in the output defuzzification layer. The number of neurons in the fuzzification layer is set to three (i.e., the number of intuitionistic fuzzy sets), three neurons in the if-then rules layer represent the three rules, and three neurons in the consequent layer represent the consequents of the three rules. The trained consequent parameters are $a_{0,1} = 0.00083, a_{1,1} = 0.996, a_{0,2} = 0.00085, a_{1,2} = 0.997, a_{0,3} = 0.00084, a_{1,3} = 1.005$. The defuzzified outputs are presented in Table 5, and the RMSE for the given training data is 11.82.

Table 5 Defuzzified forecasts.

t	y_1^{t+1}	y_2^{t+1}	y_3^{t+1}	y_{IFWA}^{t+1}	t	y_1^{t+1}	y_2^{t+1}	y_3^{t+1}	y_{IFWA}^{t+1}
5	1199.92	1211.54	1201.86	1204.44	11	1195.44	1207.01	1197.37	1199.94
6	1185.28	1196.76	1187.20	1187.20	12	1207.69	1219.38	1209.64	1212.23
7	1197.88	1209.47	1199.81	1202.39	13	1211.62	1223.35	1213.58	1216.18
8	1210.87	1222.59	1212.83	1215.43	14	1189.91	1201.43	1191.83	1189.91
9	1209.58	1221.29	1211.53	1211.53	15	1184.63	1196.10	1186.55	1184.63
10	1190.96	1202.49	1192.88	1195.44					

5.2 Experimental setup for multivariate forecasting of precious metal prices

Accurate forecasting models are critical to the decision-making of investors and mining companies and are important to the efficient functioning of commodity markets. Forecasting real-world precious metal prices is a complex task

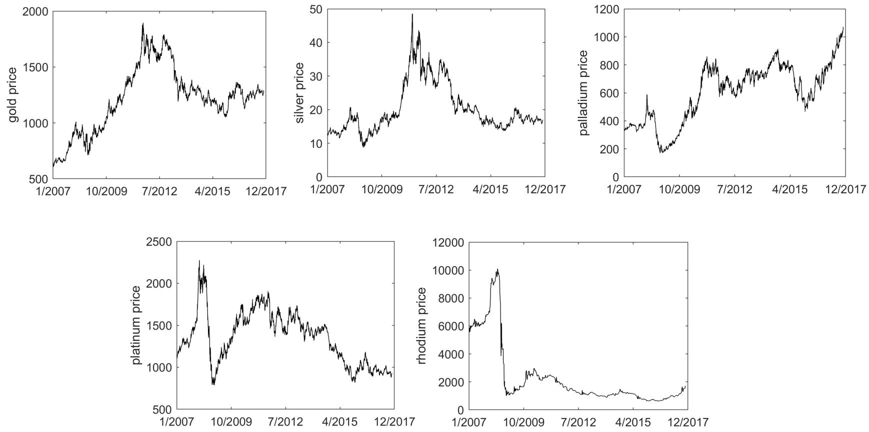

Fig. 3 The daily metal prices from January 1, 2007 to December 31, 2017.

where various factors need to be accounted for. The prices are often influenced by changes in the global economy, oil price, current exchange rates and other factors, so they often fluctuate and vary considerably (Liu et al, 2020; Hajek and Novotny, 2022). Consistent with this stream of research, multivariate forecasting of precious metal prices was investigated in this study.

The closing prices of five major precious metals were used in this study, namely gold, silver, palladium, platinum, and rhodium. The daily metal prices TS data were collected from January 1, 2007 to December 31, 2017, covering 3,949 trading days (daily observations). More precisely, daily spot prices in USD per ounce were obtained from the Kitco database[1]. The price TS of gold, silver, palladium, platinum, and rhodium are illustrated in Fig. 3.

To evaluate the performance of the proposed neural intuitionistic fuzzy system, two forecasting horizons were considered for the metal prices, leading to two datasets, one for the one-day-ahead (daily) forecasting, and the other for the five-day-ahead (weekly) forecasting. Sequential validation was applied by partitioning the data into the training data immediately followed by the testing data in ratio 9:1 in agreement with previous studies (Wang et al, 2019; Livieris et al, 2020). This is, the first 3,554 observations were used as training data, and the following 395 observations were used for model testing.

In this set of experiments, we adopted the multivariate approach used in earlier studies (Kristjanpoller and Hernández, 2017; Jabeur et al, 2021) and considered the technical indicators of respective metal prices, the exchange rate of US Dollar to Chinese Yuan (USDCNY), previous Brent crude oil price (BRN), and news sentiment indicators as input attributes. More precisely, 20-day technical indicators were calculated, and three types of indicators were included: (1) trend-type indicator (an exponential moving average (EMA)),

[1] https://www.kitco.com/

(2) oscillator-type indicator (relative strength index (RSI)), and volatility-type indicator (rate of change (ROC)). These indicators are defined as follows:

$$EMA_t = \frac{2}{21}(SMA_t - EMA_{t-1}) + EMA_{t-1}, \tag{28}$$

$$RSI_t = 100 - \frac{100}{1+RS}, ROC_t = \frac{P_t - P_{t-20}}{P_{t-20}} \times 100, \tag{29}$$

where SMA_t is 20-day simple moving average at day t, RS is the ratio of the smoothed average of 20-day upward / downward ROC, and P_t is the metal price at day t. Earlier research has indeed uncovered some compelling evidence that trading strategies based on technical indicators can be used to achieve abnormal returns in metal commodity markets (Narayan et al, 2015).

The previous day's closing prices of BRN and USDCNY were used and their respective data were collected from the MarketWatch database. Hence, the effects of other commodity (oil) market and foreign exchange market were considered. To incorporate the information effects on precious metal prices, the intensity of positive and negative news sentiment was obtained using SentiWordNet (publicly available at https://github.com/aesuli/SentiWordNet). The headlines of news articles related to metals for the respective period 2007-2017 were collected using the Thomson Reuters newswire service. In total, 266,165 headlines were collected, and the mean values of SentiWordNet sentiment indicators were calculated for each day. The detailed statistical description of the experimental datasets is presented in Table 6.

Table 6 Descriptive statistics on attributes.

Attribute	Mean	Min.	Max.	Std.Dev.
gold price	1,213.9	608.4	1,895.0	287.9
EMA^{gold}	1,213.8	612.7	1,821.4	290.9
RSI^{gold}	54.9	0.0	100.0	33.2
ROC^{gold}	0.43	-19.41	19.41	4.23
silver price	20.26	8.79	48.58	7.34
EMA^{silver}	20.26	9.34	43.19	7.33
RSI^{silver}	51.6	0.0	100.0	33.1
ROC^{silver}	0.31	-100.00	24.92	8.20
palladium price	601.2	170.0	1,071.0	205.6
$EMA^{palladium}$	599.9	174.7	1,032.9	204.3
$RSI^{palladium}$	56.1	0.0	100.0	32.6
$ROC^{palladium}$	0.73	-100.00	36.74	8.15
platinum price	1,354.2	787.2	2,276.1	311.7
$EMA^{platinum}$	1,354.9	808.1	2,102.9	311.7
$RSI^{platinum}$	52.1	0.0	100.0	34.4
$ROC^{platinum}$	-0.04	-100.00	24.52	6.60
rhodium price	2,279.3	625.0	10,100.0	2,224.3
$EMA^{rhodium}$	2,284.0	625.5	9,826.2	2,205.9
$RSI^{rhodium}$	44.9	0.0	100.0	38.6
$ROC^{rhodium}$	-0.28	-100.00	44.40	9.28
BRN	82.0	27.8	146.1	26.8
USDCNY	6.62	6.04	7.81	0.42
$SentiWordNet_{pos}$	0.23	0.00	1.83	0.10
$SentiWordNet_{neg}$	-0.22	-1.46	0.00	0.11

To demonstrate the forecasting efficiency of the proposed neural intuitionistic fuzzy system, the following state-of-the-art neural fuzzy models were considered:

- IFNN-TS (intuitionistic fuzzy neural network for TS forecasting) (Hajek et al, 2021), an earlier version of the neural intuitionistic fuzzy system. The parameters of the membership functions and rule antecedents of the IFNN-TS system were set using the subtractive clustering algorithm and the consequent parameters of the if-then rules were adapted using the gradient descent algorithm.
- ANFIS-GA (ANFIS trained using a genetic algorithm) (Alameer et al, 2019), with the membership functions and rule base initialized using the subtractive clustering algorithm with the same settings as in IFNN-TS and trained using the GA with the parameters adopted from Alameer et al (2019).
- INFN-PSO (intuitionistic neuro-fuzzy network trained using particle swarm optimization) (Hajek and Olej, 2017), with the parameters of the model initialized using the subtractive clustering algorithm and trained following the settings recommended in Hajek and Olej (2017).
- IT2FLS-EKM (interval type-2 fuzzy logic system with the enhanced Karnik-Mendel algorithm) (Wu and Mendel, 2009), generated in the Matlab Fuzzy Logic Toolbox as the interval type-2 TSK fuzzy inference system and tuned using the gradient descent algorithm.

For all the above methods, three settings were examined with $N = \{3, 5, 7\}$ rules and antecedent fuzzy sets / intuitionistic fuzzy sets / interval type-2 fuzzy sets. All the experiments with these methods were carried out in the Matlab Fuzzy Logic Toolbox in Matlab R2020a. In all experiments, we used the gradient descent algorithm (with 100 iterations and the learning rate $\eta = 0.01$) to train the neural intuitionistic fuzzy system and other neural fuzzy models.

We also compared the performance of the proposed model with several state-of-the-art models used in previous studies for metal price forecasting:

- ES (exponential smoothing) (Hassani et al, 2015), adopting the triple ES (Holt-Winters) model with smoothing factors of 0.2.
- ARIMA (autoregressive integrated moving average) (Lasheras et al, 2015), using the ARIMA(1,1,0) model found by Lasheras et al (2015) using the Hyndman and Khandakar algorithm.
- RF (random forest) (Liu and Li, 2017), trained using 100 random trees and 500 iterations.
- XGBoost (extreme gradient boosting) (Jabeur et al, 2021), using the XGBoost regressor algorithm with gbtree booster, the learning rate of 0.05, the number of estimators of 100, and a maximum depth of 5.
- MLP (multilayer perceptron) (Lasheras et al, 2015), a shallow NN model with the settings adopted from Lasheras et al (2015) as follows: one hidden layer of 24 sigmoidal neurons, the momentum of 0.5, and the learning rate of 0.001.

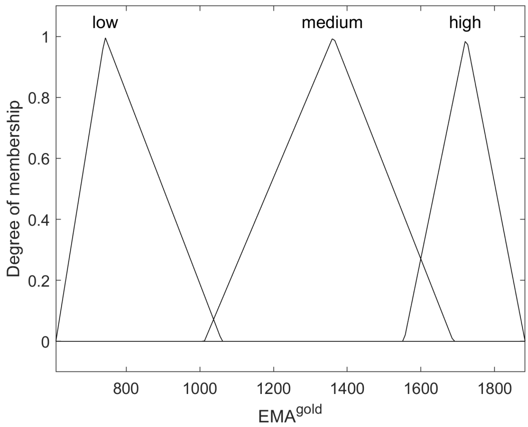

Fig. 4 Example of membership functions obtained using justified granularity.

- LSTM (long short-term memory) (Livieris et al, 2020), a deep NN model with an LSTM layer of 200 neurons followed by a dense layer of 32 neurons (the structure was adopted from Livieris et al (2020)) trained using the stochastic gradient descent algorithm.

For the experiments with the remaining forecasting methods, we used the implementations of machine learning methods in the Python library Scikit-Learn 0.23.0.

In agreement with earlier related research (Liu et al, 2020; Du et al, 2020), forecasting performance was evaluated using RMSE and mean absolute error (MAE) on the testing data separately for the 1-day-ahead and 5-day-ahead forecasting horizon. In addition, mean directional accuracy (MDA) was used to evaluate the proposed system's capacity to predict the correct forecast direction (upward or downward) and investigate the financial performance of the constructed precious metals portfolio in terms of its return and risk.

5.3 Experimental results

In the first step, we defined the universe of discourse and partitioned it using the Principle of Justified Granularity. Fig. 4 illustrates this process for attribute EMA^{gold}. In our experiments, we examined three settings of the number of triangular membership functions $M = \{3, 5, 7\}$. We only present the results for M=3 and M=5 membership functions because the performance for M=7 deteriorated due to overfitting.

In the next step, the set of if-then rules was constructed using fuzzy association rules. Table 7 shows the number of fuzzy association rules produced together with Ant (the average number of conditions in the antecedents of the fuzzy association rules) and $Conf$ (the average confidence of the fuzzy association rules). The minimum confidence level was set to $Conf$=0.9. Regarding the rule base interpretability, a higher number of rules were required only for rhodium and silver, indicating higher complexity of these datasets. Nevertheless, the number of rule antecedents remained low across all precious metals.

For illustration, the example of the constructed rule base for the gold price (for M=3) was as follows:

$$
\begin{aligned}
&R_1 : \text{if } EMA^t_{gold} \text{ is } \textit{medium} \text{ and } USDCNY^t \text{ is } \textit{low} \text{ then } y_1^{t+1} \text{ is } \textit{medium},\\
&R_2 : \text{if } EMA^t_{gold} \text{ is } \textit{low} \text{ and } USDCNY^t \text{ is } \textit{high} \text{ then } y_2^{t+1} \text{ is } \textit{low},\\
&\ldots\\
&R_{12} : \text{if } EMA^t_{gold} \text{ is } \textit{medium} \text{ and } ROC^t_{gold} \text{ is } \textit{low} \text{ then } y_{12}^{t+1} \text{ is } \textit{medium}.
\end{aligned}
$$

Table 7 Generated fuzzy association rules.

	M= 3 membership functions			M=5 membership functions		
Metal	# rules N	Ant	$Conf$	# rules N	Ant	$Conf$
gold	12	1.75	0.95	19	1.89	0.95
silver	19	1.42	0.95	43	2.09	0.96
palladium	11	1.64	0.98	8	1.38	0.99
platinum	15	2.00	0.98	14	1.64	0.97
rhodium	76	2.20	0.96	23	2.13	0.97

Experimental results in Table 8 show the effectiveness of the proposed neural intuitionistic fuzzy system with justified granularity (NIFS-JG) by comparing its performance against four neural fuzzy models and five other benchmark TS forecasting models used previously for metal price prediction. The results of the comparisons show that NIFS-JG was highly competitive regarding all metal prices in terms of both forecasting horizons. Best performance in terms of RMSE was achieved for one-day-ahead forecasting of gold, and palladium prices, while the proposed model was superior for silver and palladium in case of five-day-ahead forecasting. For most precious metals, the NIFS-JG model also outperformed its earlier version IFNN-TS. The proposed approach to data partitioning and construction of fuzzy systems also appears to be effective compared with the traditional clustering method used in the remaining neural fuzzy models. To be fair, we have to admit that the number of rules produced by the other models was lower than for NIFS-JG, as $M = N$ for the subtractive clustering algorithm. The results also show that the proposed model was highly competitive compared with the state-of-the-art models for metal price forecasting.

Obviously, the results in Fig. 5 also suggest that M=3 membership functions was a preferable setting in terms of both forecasting accuracy and interpretability at the fuzzy partition and rule base level (as compared with NIFS-JG with M=5).

To illustrate the high accuracy achieved using the proposed model, Fig. 6 shows the actual vs. predicted metal prices on the testing data for one-day-ahead forecasts.

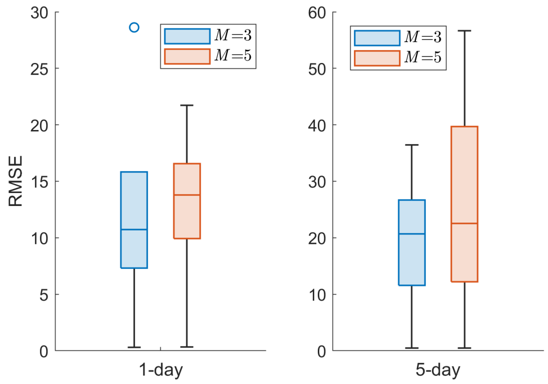

Fig. 5 Average RMSE for different numbers of membership functions.

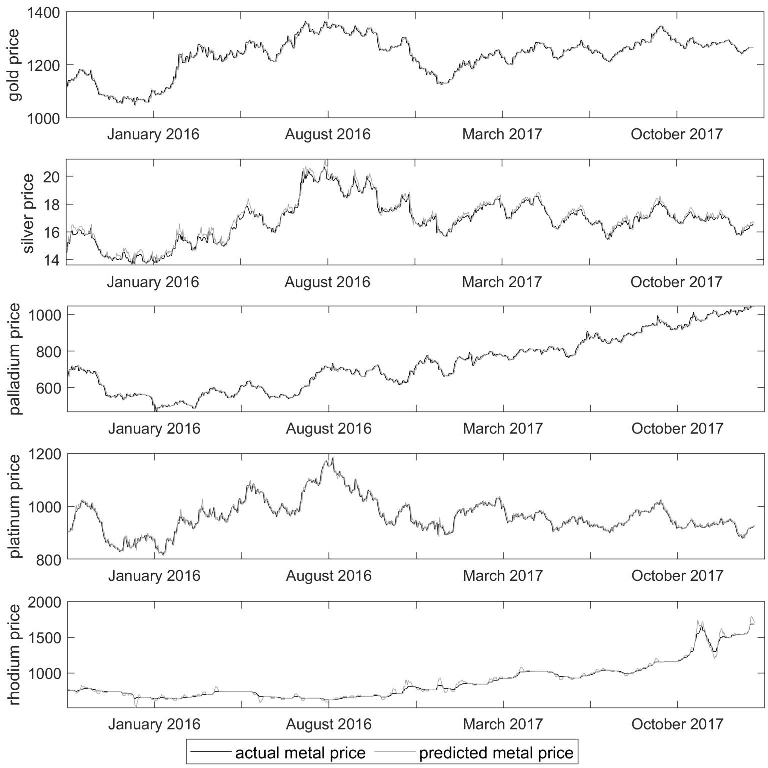

Fig. 6 Actual vs. predicted metal prices for one-day-ahead forecasting.

Table 8 Results of metal price forecasting (the best result is in bold).

		NIFS-JG		NIFS-JG		IFNN-TS	
		M=3		M=5		M=3	
Metal	Forecast	MAE	RMSE	MAE	RMSE	MAE	RMSE
gold	1-day	**6.656**	**9.671**	10.127	13.127	6.687	9.689
gold	5-day	16.068	20.677	17.993	22.551	15.408	19.680
silver	1-day	0.222	0.298	0.259	0.329	**0.155**	**0.223**
silver	5-day	0.380	**0.471**	0.422	0.509	**0.364**	0.474
palladium	1-day	**7.620**	**10.733**	10.411	13.803	7.843	10.978
palladium	5-day	17.551	**15.277**	18.765	16.106	16.873	21.681
platinum	1-day	8.644	11.585	11.685	14.835	8.658	**11.550**
platinum	5-day	19.071	23.398	21.295	34.075	18.968	25.000
rhodium	1-day	16.856	28.621	10.808	21.737	16.766	23.574
rhodium	5-day	22.245	36.460	22.474	56.630	24.100	38.365
		ANFIS-GA		INFN-PSO		IT2FLS-EKM	
		M=3		M=3		M=3	
Metal	Forecast	MAE	RMSE	MAE	RMSE	MAE	RMSE
gold	1-day	7.045	9.994	6.684	9.764	9.731	13.067
gold	5-day	16.980	21.335	16.058	20.497	18.011	22.820
silver	1-day	0.175	0.243	0.177	0.246	0.194	0.257
silver	5-day	0.461	0.602	0.369	0.483	0.379	0.502
palladium	1-day	7.976	10.997	7.792	10.811	7.993	10.917
palladium	5-day	19.306	24.697	17.611	22.643	17.365	22.480
platinum	1-day	**8.635**	11.579	9.670	12.536	9.698	12.800
platinum	5-day	21.537	27.881	19.340	25.186	19.468	25.430
rhodium	1-day	**12.605**	**18.007**	13.460	21.996	15.723	22.537
rhodium	5-day	33.144	47.137	22.204	36.562	24.543	39.459
		ES		ARIMA		RF	
Metal	Forecast	MAE	RMSE	MAE	RMSE	MAE	RMSE
gold	1-day	16.092	20.665	9.949	12.538	11.434	14.933
gold	5-day	17.066	21.485	**10.747**	**13.431**	18.630	23.104
silver	1-day	0.376	0.471	0.305	0.377	0.239	0.311
silver	5-day	0.389	0.499	0.567	0.683	0.369	**0.471**
palladium	1-day	19.008	24.491	8.117	10.997	28.084	44.407
palladium	5-day	18.628	23.833	**11.789**	15.408	39.000	52.530
platinum	1-day	19.954	25.342	15.398	18.626	15.986	21.408
platinum	5-day	20.185	26.426	19.387	**23.135**	23.957	31.448
rhodium	1-day	19.948	32.646	35.700	46.425	100.64	126.57
rhodium	5-day	**19.532**	**35.204**	125.17	178.98	104.28	127.36
		XGBoost		MLP		LSTM	
Metal	Forecast	MAE	RMSE	MAE	RMSE	MAE	RMSE
gold	1-day	9.890	12.651	10.936	13.697	23.845	26.618
gold	5-day	20.034	25.991	11.851	14.659	40.757	44.271
silver	1-day	0.175	0.238	0.422	0.484	0.331	0.386
silver	5-day	0.401	0.512	0.607	0.674	0.514	0.581
palladium	1-day	43.012	67.012	13.240	16.733	23.173	28.449
palladium	5-day	66.324	87.890	19.293	23.892	32.040	40.103
platinum	1-day	13.763	17.322	18.385	21.249	19.900	25.642
platinum	5-day	32.737	43.584	24.599	27.542	27.569	35.163
rhodium	1-day	22.660	28.273	56.930	67.615	65.726	87.358
rhodium	5-day	58.262	75.672	103.62	120.43	95.874	130.39

To evaluate the error performance of the models statistically, we carried out a non-parametric Friedman test. The NIFS-JG model ranked on average as follows: 3.0 and 3.4 for MAE (one- and five-day-ahead forecasts), and 3.2 and 2.4 for RMSE. Significant differences were observed for the average ranks of the compared methods at $p < 0.05$, indicating significantly different performance across the error measures and forecasting horizons. Therefore, we performed the Holm–Bonferroni posthoc procedure to compare the performance between the best model and the other compared models. For the one-day-ahead forecasting, NIFS-JG performed significantly better than

ES, ARIMA, RF, XGBoost, MLP, and LSTM at $p < 0.05$. For the five-day-ahead forecasting, NIFS-JG significantly outperformed ANFIS-GA, RF, XGBoost, MLP, and LSTM at $p < 0.05$. These results were consistent for both performance measures.

In addition to MAE and RMSE, the performance of the NIFS-JG model was evaluated in terms of MDA. Table 9 shows that NIFS-JG consistently exceeded the baseline naïve benchmark (obtained using the majority class voting). The naïve benchmark indicates the class imbalance of downward and upward movements in the data. Interestingly, the results suggest that the proposed model is particularly effective in the case of highly unbalanced rhodium datasets, implying its successful deployment also in periods of long-term metal price increases/decreases. However, the naïve baseline was also surpassed for all other precious metals, regardless of their upward or downward price volatility.

Table 9 Mean directional accuracy of metal price forecasting.

Metal	Forecast	NIFS-JG	Naïve baseline
gold	1-day	65.13	64.05
gold	5-day	65.74	63.54
silver	1-day	60.41	57.97
silver	5-day	58.63	57.22
palladium	1-day	64.72	62.03
palladium	5-day	64.85	61.52
platinum	1-day	58.38	57.97
platinum	5-day	58.88	57.22
rhodium	1-day	95.81	81.27
rhodium	5-day	87.82	81.01

Correct forecast of upward/downward direction is important for generating 'buy', 'hold' and 'sell' signals. Therefore, we further investigated the financial performance (return and risk) of the precious metals portfolio constructed based on the signals generated using the NIFS-JG-based trading strategy ('buy' ('hold') signal for upward price forecast, and 'sell' signal for downward price forecast). The closing metal prices were used for trading, and the weights of the five metals in the portfolio were equal. Returns for individual precious metal are presented in Fig. 7, showing that the trading strategy based on the NIFS-JG forecasting signals outperformed the buy-and-hold (B&H) strategy particularly in five-day-ahead forecasting. For the portfolio of the five precious metals, we obtained an average return of 59.35% (for one-day-ahead forecasting) and 68.78% (for five-day-ahead forecasting) for the testing period. The forecasting-based trading strategy was more profitable than the traditional buy-and-hold strategy (with an average daily return of 30.92% and a weekly return 33.30%). However, it should be noted that our trading strategy was associated with a higher portfolio risk (portfolio standard deviation). The standard deviation of returns was used to calculate the risk, obtaining $\sigma = 7.52\%$ and $\sigma = 7.80\%$ for the one-day-ahead and five-day-ahead NIFS-JG forecasting

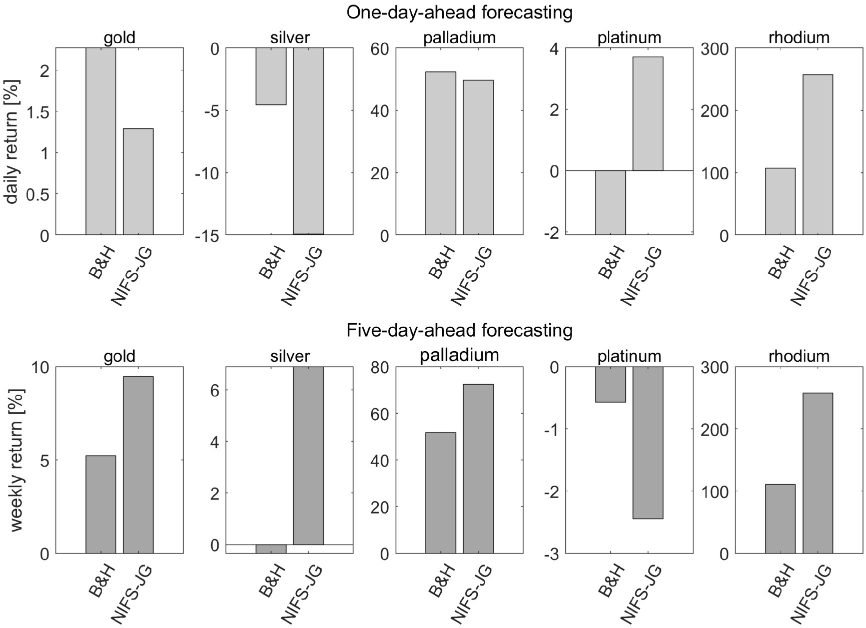

Fig. 7 Daily and weekly returns of buy-and-hold (B&H) and NIFS-JG-based trading strategies.

strategies, hence exceeding those for the buy-and-hold strategy ($\sigma = 4.25\%$ and $\sigma = 5.12\%$).

6 Conclusion

This study has developed an efficient forecasting model that incorporates intuitionistic fuzzy sets to consider uncertainty present in the TS volatility. First, we granulated TS into meaningful entities with high coverage and specificity. We constructed the rule base using fuzzy association rules to make the forecasting model both interpretable at the rule base and fuzzy partition level and comprehensive in terms of coverage with matching rules. Finally, we performed the forecasting using an optimized neural intuitionistic fuzzy TSK system and defuzzified the forecasts using an intuitionistic fuzzy weighted averaging operator. In the learning process, the fuzzy TSK model exploits the capability of neural networks to minimize forecasting error.

We validated the proposed forecasting model using five TS of major precious metal prices in the experimental evaluation. The proposed model not only outperformed existing neural fuzzy systems but was also highly competitive compared to other state-of-the-art metal price forecasting models. In addition to achieving solid predictive performance, the proposed model provides investors with interpretable sets of trading if-then rules, which is superior to the traditional buy-and-hold strategy in terms of portfolio return.

A number of important limitations need to be noted regarding the proposed forecasting model. First, the process of partitioning and fuzzification was based mainly on the TS volatility, while other TS characteristics were not taken into account. Therefore, future research should investigate alternative approaches to generate fuzzy granules to enhance the data partitioning stage with enriched information, such as trend information. Furthermore, the rule base was fixed, which can be a concern if the model needs to be retrained on new data with different patterns. Adaptation of the rule base during the neural learning process is therefore another possible direction for future research. Future studies might also enhance the proposed model by replacing the neural intuitionistic fuzzy TSK system in the forecasting stage with recently proposed extensions of recurrent neural fuzzy systems (Tang et al, 2021; Ding et al, 2021) and by extending the model to allow for forecasting interval-valued TS (Maciel et al, 2021). For the latter, a neural interval type-2 intuitionistic fuzzy system is suggested, in which both the fuzzification and defuzzification of the interval-valued TS should be carried out separately for its lower and upper bounds.

Conflict of Interest

The authors have no conflicts of interest to declare that are relevant to the content of this article.

References

Alameer Z, Abd Elaziz M, Ewees AA, et al (2019) Forecasting copper prices using hybrid adaptive neuro-fuzzy inference system and genetic algorithms. Natural Resources Research 28(4):1385–1401

Angelov P (1995) Crispification: defuzzification of intuitionistic fuzzy sets. BUSEFAL 64:51–55

Atanassov KT (1999) Intuitionistic Fuzzy Sets, Physica-Verlag HD, Heidelberg, pp 1–137

Bas E, Grosan C, Egrioglu E, et al (2018) High order fuzzy time series method based on pi-sigma neural network. Engineering Applications of Artificial Intelligence 72:350–356

Bisht K, Kumar S (2016) Fuzzy time series forecasting method based on hesitant fuzzy sets. Expert Systems with Applications 64:557–568

Bisht K, Kumar S (2019) Hesitant fuzzy set based computational method for financial time series forecasting. Granular Computing 4(4):655–669

Bose M, Mali K (2018) A novel data partitioning and rule selection technique for modeling high-order fuzzy time series. Applied Soft Computing 63:87–96

Bose M, Mali K (2019) Designing fuzzy time series forecasting models: A survey. International Journal of Approximate Reasoning 111:78–99

Bougoudis I, Demertzis K, Iliadis L, et al (2018) Fussffra, a fuzzy semi-supervised forecasting framework: the case of the air pollution in athens. Neural Computing and Applications 29(7):375–388

Chen G, Wei Q (2002) Fuzzy association rules and the extended mining algorithms. Information Sciences 147(1-4):201–228

Chen MY (2014) A high-order fuzzy time series forecasting model for internet stock trading. Future Generation Computer Systems 37:461–467

Deng W, Wang G, Zhang X, et al (2016) A multi-granularity combined prediction model based on fuzzy trend forecasting and particle swarm techniques. Neurocomputing 173:1671–1682

Ding H, Li W, Qiao J (2021) A self-organizing recurrent fuzzy neural network based on multivariate time series analysis. Neural Computing and Applications 33(10):5089–5109

Du P, Wang J, Yang W, et al (2020) Point and interval forecasting for metal prices based on variational mode decomposition and an optimized outlier-robust extreme learning machine. Resources Policy 69:101,881

Egrioglu E, Aladag CH, Yolcu U, et al (2009) A new approach based on artificial neural networks for high order multivariate fuzzy time series. Expert Systems with Applications 36(7):10,589–10,594

Eyoh I, John R, De Maere G, et al (2018) Hybrid learning for interval type-2 intuitionistic fuzzy logic systems as applied to identification and prediction problems. IEEE Transactions on Fuzzy Systems 26(5):2672–2685

Gaxiola F, Melin P, Valdez F, et al (2014) Interval type-2 fuzzy weight adjustment for backpropagation neural networks with application in time series prediction. Information Sciences 260:1–14

Gupta KK, Kumar S (2019) A novel high-order fuzzy time series forecasting method based on probabilistic fuzzy sets. Granular Computing 4(4):699–713

Hajek P, Novotny J (2022) Fuzzy rule-based prediction of gold prices using news affect. Expert Systems with Applications p 116487

Hajek P, Olej V (2017) Intuitionistic neuro-fuzzy network with evolutionary adaptation. Evolving Systems 8(1):35–47

Hajek P, Froelich W, Prochazka O (2020) Intuitionistic fuzzy grey cognitive maps for forecasting interval-valued time series. Neurocomputing 400:173–185

Hajek P, Olej V, Froelich W, et al (2021) Intuitionistic fuzzy neural network for time series forecasting - the case of metal prices. In: IFIP International Conference on Artificial Intelligence Applications and Innovations, Springer, pp 411–422

Hassani H, Silva ES, Gupta R, et al (2015) Forecasting the price of gold. Applied Economics 47(39):4141–4152

Hyndman R, Athanasopoulos G (2018) Forecasting: Principles and Practice. OTexts

Jabeur SB, Mefteh-Wali S, Viviani JL (2021) Forecasting gold price with the xgboost algorithm and shap interaction values. Annals of Operations Research pp 1–21

Kristjanpoller W, Hernández E (2017) Volatility of main metals forecasted by a hybrid ann-garch model with regressors. Expert Systems with Applications 84:290–300

Kumar S, et al (2019) A modified weighted fuzzy time series model for forecasting based on two-factors logical relationship. International Journal of Fuzzy Systems 21(5):1403–1417

Lasheras FS, de Cos Juez FJ, Sánchez AS, et al (2015) Forecasting the comex copper spot price by means of neural networks and arima models. Resources Policy 45:37–43

Li F, Yu F (2020) Multi-factor one-order cross-association fuzzy logical relationships based forecasting models of time series. Information Sciences 508:309–328

de Lima Silva PC, Sadaei HJ, Ballini R, et al (2019) Probabilistic forecasting with fuzzy time series. IEEE Transactions on Fuzzy Systems 28(8):1771–1784

Liu D, Li Z (2017) Gold price forecasting and related influence factors analysis based on random forest. In: Proceedings of the Tenth International Conference on Management Science and Engineering Management, Springer, pp 711–723

Liu Y, Yang C, Huang K, et al (2020) Non-ferrous metals price forecasting based on variational mode decomposition and lstm network. Knowledge-Based Systems 188:105,006

Livieris IE, Pintelas E, Pintelas P (2020) A cnn–lstm model for gold price time-series forecasting. Neural Computing and Applications 32(23):17,351–17,360

Lu W, Pedrycz W, Liu X, et al (2014) The modeling of time series based on fuzzy information granules. Expert Systems with Applications 41(8):3799–3808

Luo C, Tan C, Wang X, et al (2019) An evolving recurrent interval type-2 intuitionistic fuzzy neural network for online learning and time series prediction. Applied Soft Computing 78:150–163

Maciel L, Ballini R, Gomide F (2021) Adaptive fuzzy modeling of interval-valued stream data and application in cryptocurrencies prediction. Neural Computing and Applications pp 1–11

Narayan PK, Ahmed HA, Narayan S (2015) Do momentum-based trading strategies work in the commodity futures markets? Journal of Futures Markets 35(9):868–891

Pedrycz W, Homenda W (2013) Building the fundamentals of granular computing: A principle of justifiable granularity. Appl Soft Comput 13(10):4209–4218

Pedrycz W, Vukovich G (2001) Abstraction and specialization of information granules. IEEE transactions on systems, man, and cybernetics Part B, Cybernetics : a publication of the IEEE Systems, Man, and Cybernetics Society 31:106–11

Pedrycz W, Lu W, Liu X, et al (2014) Human-centric analysis and interpretation of time series: a perspective of granular computing. Soft Computing 18(12):2397–2411

Peng HW, Wu SF, Wei CC, et al (2015) Time series forecasting with a neuro-fuzzy modeling scheme. Applied Soft Computing 32:481–493

Roy A (2016) A novel multivariate fuzzy time series based forecasting algorithm incorporating the effect of clustering on prediction. Soft Computing 20(5):1991–2019

Salisu AA, Ogbonna AE, Adewuyi A (2020) Google trends and the predictability of precious metals. Resources Policy 65:101,542

Shmueli G (2011) Practical Time Series Forecasting, 2nd edn. Statistics.com LLC

Singh P (2017) High-order fuzzy-neuro-entropy integration-based expert system for time series forecasting. Neural Computing and Applications 28(12):3851–3868

Singh P, Borah B (2013) High-order fuzzy-neuro expert system for time series forecasting. Knowledge-Based Systems 46:12–21

Singh P, Dhiman G (2018) A hybrid fuzzy time series forecasting model based on granular computing and bio-inspired optimization approaches. Journal of Computational Science 27:370–385

Song Q, Chissom BS (1993) Fuzzy time series and its models. Fuzzy Sets and Systems 54(3):269–277

Soto J, Melin P, Castillo O (2018) A new approach for time series prediction using ensembles of it2fnn models with optimization of fuzzy integrators. International Journal of Fuzzy Systems 20(3):701–728

Su CH, Cheng CH (2016) A hybrid fuzzy time series model based on anfis and integrated nonlinear feature selection method for forecasting stock. Neurocomputing 205:264–273

Talarposhti FM, Sadaei HJ, Enayatifar R, et al (2016) Stock market forecasting by using a hybrid model of exponential fuzzy time series. International Journal of Approximate Reasoning 70:79–98

Tang Y, Yu F, Pedrycz W, et al (2021) Building trend fuzzy granulation based lstm recurrent neural network for long-term time series forecasting. IEEE Transactions on Fuzzy Systems

Wang C, Zhang X, Wang M, et al (2019) Predictive analytics of the copper spot price by utilizing complex network and artificial neural network techniques. Resources Policy 63:101,414

Wang L, Liu X, Pedrycz W, et al (2014) Determination of temporal information granules to improve forecasting in fuzzy time series. Expert Systems with Applications 41(6):3134 – 3142

Wu D, Mendel JM (2009) Enhanced karnik–mendel algorithms. IEEE Transactions on Fuzzy Systems 17(4):923–934

Wu H, Long H, Wang Y, et al (2021) Stock index forecasting: A new fuzzy time series forecasting method. Journal of Forecasting 40(4):653–666

Yager RR (1998) Computational Intelligence: Soft Computing and Fuzzy-Neuro Integration with Applications, Springer Berlin Heidelberg, Berlin, Heidelberg, chap Measures of Specificity, pp 94–113

Yang X, Yu F, Pedrycz W (2017) Long-term forecasting of time series based on linear fuzzy information granules and fuzzy inference system. International Journal of Approximate Reasoning 81:1–27

Yolcu OC, Yolcu U, Egrioglu E, et al (2016) High order fuzzy time series forecasting method based on an intersection operation. Applied Mathematical Modelling 40(19-20):8750–8765

Yu THK, Huarng KH (2010) A neural network-based fuzzy time series model to improve forecasting. Expert Systems with Applications 37(4):3366–3372