Diakoptic Method as a Generalized Form of the Thevenin's Theorem

David Matoušek
Department of Electrical Engineering
University of Pardubice
Pardubice, Czech Republic
david.matousek@upce.cz

Bohumil Brtník
Department of Electrical Engineering
University of Pardubice
Pardubice, Czech Republic
bohumil.brtnik@upce.cz

***Abstract*—This paper deals with the composition of a circuit matrix using the diakoptic method. The diakoptic method is a generalization of Thevenin's theorem. The diakoptic method allows solving complex electrical and electronic circuits. This method divides a complex electrical circuit into simpler circuits. The description of each of these circuits executes by the same or different method. For example, we can describe the first part of a circuit (with current sources) by the node voltage method. After then, we describe the second part of a circuit (with switched capacitors) by the nodal charge equations method, etc.**

Keywords—Thevenin's theorem, current source, voltage source, diakoptic method, matrix description, incidence matrix

I. Introduction

The general principle of solution of more complex electrical circuits exploits the separation of the complex circuit into simpler circuits [1] and more others. Each simpler electrical circuit matches a two-port network. Therefore, the description of this circuit executes by the individual submatrix. Mathematical description of these two-port networks uses simple parameters for matrix formulation. Subsequently, these parameters convert to parameters resulting from the interconnection of the partial two-port networks and are therefore suitable for further calculation.

However, the above method is not the only one to solve complex circuits. Alternative methods exploit the principle of finding the resulting circuit matrix not by a parallel connection of two-port networks but by a series or cascade connection of two-port networks.

The principle of compiling a matrix for circuit description by the diakoptic method, which follows from Thevenin's theorem, is described below. While the available literature commonly describes the interconnection of two-port networks, the algorithm for constructing a matrix using the diakoptic method is not generally known. Thus, we describe this algorithm is in this text.

The designation, which was used under the name diakoptic, was introduced by Gabriel Kron in [1]. This is a general principle that allows simplifying the solution of a complex physical system by dividing it into many simpler parts [1], [2]. It is used in various technical fields, including electrical engineering.

II. Assembly of a Matrix Describing a Electrical Circuit by the Diakoptic Method

In the solution of electrical and electronic circuits [3], [4], [5], [6], [8], [9], [10] etc., a diakoptic method is often used.

This method follows from the generalization of the well-known and easily derivable and therefore easily understood Thevenin's theorem: any electrical circuit from aspect to its two terminals A and B can replace by an equivalent voltage source V_0 with internal resistance R_0, as shown in Fig. 1. While the original circuit contains voltage source V, resistor net R_1 to R_4, and load impedance Z. The equivalent circuit consists of the equivalent source V_0 and resistance R_0, and the original load Z.

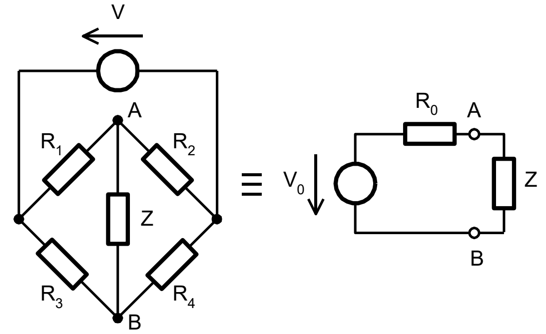

Fig. 1. Thevenin's theorem.

While in Fig. 1 there is a passive impedance Z between the considered terminals. In Fig. 2 there is a voltage source V_L with an internal impedance Z_L at the top between the A, B terminals. Here V_0 is the no-load voltage between A, B terminals and R_0 the resistance between the terminals of the original circuit.

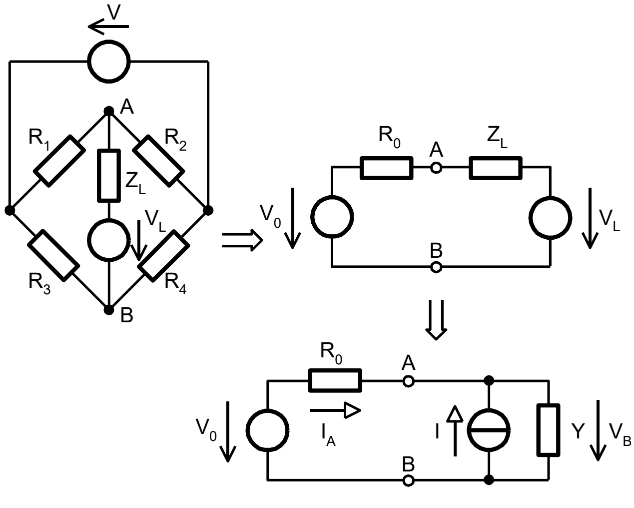

Fig. 2. Circuit modification step by step.

In the next step, we convert the voltage source V_L with internal impedance Z_L to the current source I with internal admittance Y in a known manner. The resulting circuit diagram is in Fig. 2 below.

The first part on the left side of the schematic diagram now contains a voltage source V_0 with an internal resistance R_0 in series, and we can easily describe this part by the loop current method. The second part on the right side of the schematic diagram now contains a parallel combination of current source I with internal admittance Y, and we can easily describe this part by the node voltage method. Notice key circuit quantities are marked in this figure only.

Previously presented circuit substitution is one of more possible variants. The second variant of circuit substitution is rest in the exchange of the left part of the circuit to a circuit with the current source. After that, we convert the right part of the circuit to a circuit with the voltage source. This solution is depicted in Fig. 3. We can see that both electrical circuits from Fig. 2 and Fig. 3 are equivalent.

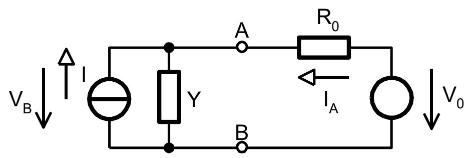

Fig. 3. The second variant of the circuit modification.

Circuit from Fig. 3 is described as follows by (1), where I_A is current from voltage source V_0 and Y is equivalent admittance of impedance Z_L:

$$\begin{aligned} V_0 &= R_0 \cdot I_A + V_B \\ I &= Y \cdot V_B - I_A \end{aligned} \tag{1}$$

After rewritten we get (2).

$$\begin{aligned} V_0 &= R_0 \cdot I_A + 1 \cdot V_B \\ I &= -1 \cdot I_A + Y \cdot V_B \end{aligned} \tag{2}$$

In general, therefore, we can divide the circuit into two parts. One of them, containing circuit elements G, Y, and input and output currents I_1, I_2 and nodal voltages V_1, V_2, can generally be described by the node voltage method. The second part, containing the elements R, Z, and input and output voltages V_3, V_4 and loop currents I_3, I_4, can then be generally described by the method of loop currents, as shown in Fig. 4.

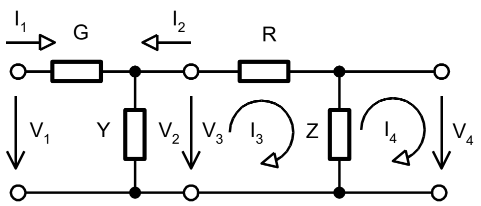

Fig. 4. The description of the first part of the circuit is by the node voltage method, the second one by the loop current method.

Therefore, we can generalize the schematic diagram from Fig. 4 to the scheme in Fig. 5.

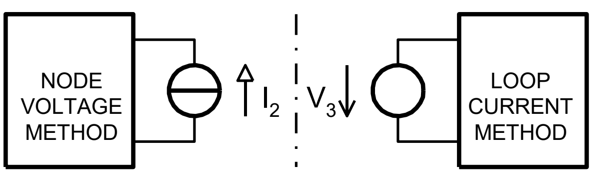

Fig. 5. Generalized circuit diagram from Fig. 4.

Circuit from Fig. 4 is described as follows by (3).

$$\begin{aligned} I_1 &= G \cdot (V_1 - V_2) \\ I_2 &= G \cdot (V_2 - V_1) + Y \cdot V_2 \\ V_3 &= R \cdot I_3 + Z \cdot (I_3 - I_4) \\ -V_4 &= Z \cdot (I_4 - I_3) \end{aligned} \tag{3}$$

In the first step, after renumbering of (3), we get the result in the form (4).

$$\begin{aligned} I_1 &= G \cdot V_1 - G \cdot V_2 \\ I_2 &= G \cdot V_2 - G \cdot V_1 + Y \cdot V_2 \\ V_3 &= R \cdot I_3 + Z \cdot I_3 - Z \cdot I_4 \\ -V_4 &= Z \cdot I_4 - Z \cdot I_3 \end{aligned} \tag{4}$$

In the second step, we can rewrite (4) into (5).

$$\begin{aligned} I_1 &= G \cdot V_1 - G \cdot V_2 \\ I_2 &= -G \cdot V_1 + Y \cdot V_2 + G \cdot V_2 \\ V_3 &= R \cdot I_3 + Z \cdot I_3 - Z \cdot I_4 \\ -V_4 &= Z \cdot I_4 - Z \cdot I_3 \end{aligned} \tag{5}$$

In the last step, it is necessary to extend (5) by the equations which describe the circuit quantities at the interface between the parts described according to Kirchhoff's circuit laws, i.e. $V_2 = V_3$ and $I_2 = -I_3$ into form (6).

$$\begin{aligned} I_1 &= G \cdot V_1 - G \cdot V_2 \\ I_2 &= -G \cdot V_1 + V_2 \cdot (Y + G) \\ V_3 &= I_3 \cdot (R + Z) - Z \cdot I_4 \\ -V_4 &= Z \cdot I_4 - Z \cdot I_3 \\ V_2 &= V_3 \\ I_2 &= -I_3 \end{aligned} \tag{6}$$

We can simplify this system of the six equations (6) by including the last two after the first four equations. This set of equations (7) is already suitable for rewriting into matrix form.

$$\begin{aligned} I_1 &= G \cdot V_1 - G \cdot V_2 + 0 \cdot I_3 + 0 \cdot I_4 \\ I_2 &= -G \cdot V_1 + V_2 \cdot (Y + G) + \\ &\quad + (-1) \cdot I_3 + 0 \cdot I_4 \\ V_3 &= 0 \cdot V_1 + 1 \cdot V_2 + I_3 \cdot (R + Z) - Z \cdot I_4 \\ -V_4 &= 0 \cdot V_1 + 0 \cdot V_2 + Z \cdot I_4 - Z \cdot I_3 \end{aligned} \tag{7}$$

Thus, (7) we can rewrite this set into a square matrix form. In this matrix form is the system matrix of the circuit (8).

The matrix (8) consists of four submatrices. The individual submatrices have the following meaning.

$$\begin{array}{r|c|c|c|c|} & V_1: & V_2: & I_3: & I_4: \\ \hline I_1: & G & -G & 0 & 0 \\ \hline I_2: & -G & Y+G & -1 & 0 \\ \hline V_3: & 0 & 1 & R+Z & -Z \\ \hline V_4: & 0 & 0 & -Z & Z \\ \hline \end{array} \tag{8}$$

The first matrix (9) is the admittance matrix describing the left part of the circuit by the node voltage method.

$$\begin{array}{|c|c|} \hline G & -G \\ \hline -G & Y+G \\ \hline \end{array} \tag{9}$$

The second submatrix (10) is the impedance matrix describing the right part of the circuit by the loop current method.

$$\begin{array}{|c|c|} \hline R+Z & -Z \\ \hline -Z & Z \\ \hline \end{array} \tag{10}$$

The voltages at the interface between the two subcircuits are the same. The currents at the interface between the two subcircuits have the same value, differing only in their directions. We qualify this fact by a negative sign between currents I_2 and I_3 (11).

$$\begin{aligned} V_2 &= V_3 \\ I_2 &= -I_3 \end{aligned} \tag{11}$$

The last two submatrices (11), (12) are incidence matrices because from the circuit diagram (see Fig. 3) is evident the relation between voltages and currents in the circuit (11).

Therefore, after rewriting (11) into a matrix form, incidence matrices are (12), (13).

$$\begin{array}{|c|c|} \hline 0 & 0 \\ \hline -1 & 0 \\ \hline \end{array} \tag{12}$$

Zero elements in (12) indicate no interconnection between the currents I_1, I_3, and I_1, I_4 and I_2, I_5. Analogically, zero elements in (13) indicate no interrelationship between the following voltages V_3, V_1, V_4, V_1, and V_4, V_2.

$$\begin{array}{|c|c|} \hline 0 & 1 \\ \hline 0 & 0 \\ \hline \end{array} \tag{13}$$

It follows from the structure of matrices (12), (13) having interchanged rows and columns and elements replaced by additions that these are matrices transposed to each other. Then (4) transcribed to (8), we can write for not one, but for N nodes and loops in the general matrix form (14) or (15). Here, simple elements are replaced by matrices.

From the comparison of matrices (13) and (14), we can see that we have two possible shapes of the resulting matrix. The first of these is (14).

$$\begin{array}{|c|c|} \hline \boldsymbol{Y} & \boldsymbol{M} \\ \hline -\boldsymbol{M}^T & \boldsymbol{Z} \\ \hline \end{array} \tag{14}$$

And/or second one is (15).

$$\begin{array}{|c|c|} \hline \boldsymbol{Y} & -\boldsymbol{M}^T \\ \hline \boldsymbol{M} & \boldsymbol{Z} \\ \hline \end{array} \tag{15}$$

Where $\boldsymbol{Y}$ is the admittance matrix, $\boldsymbol{Z}$ is the impedance matrix, $\boldsymbol{M}$ is the incidence matrix and $\boldsymbol{M}^T$ is the transposed incidence matrix.

Now, we can express this matrix by the general scheme according to Fig. 6. The diagram in Fig. 6 shows two circuits meeting in N two-ports.

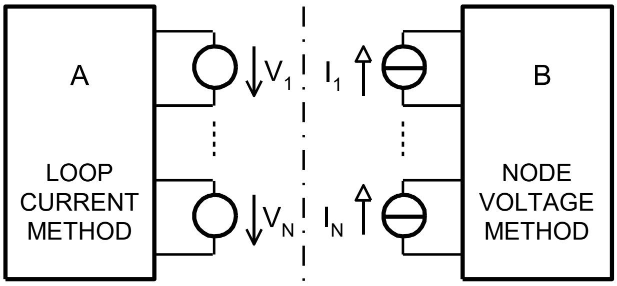

Fig. 6. The general circuit diagram for more nodes and loops.

We can use the diakoptic method for solving a more complex electrical circuit. For example, an electrical circuit which one part consists of the switched capacitor circuit as is depicted in Fig. 7 [4]. This circuit can be described by the matrix in form (16).

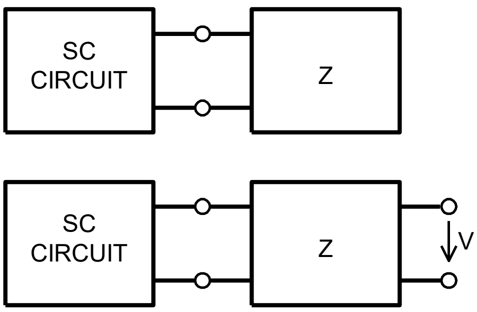

Fig. 7. Example of the combined circuit general diagram. The left part is described by the capacitance matrix. The right part is described by the loop current method. The left part is an SC circuit that is connected by the right part into another circuit in port V.

In this case, the switched capacitor part of the circuit is described in (16) by the capacitance matrix and another part by the loop current method by the $\boldsymbol{Z}$ matrix. The second part can be used for connection into another circuit in port V as is depicted in Fig. 7 belove. In this case, the SC circuit matrix is generally (for example) n-order, $\boldsymbol{Z}$ matrix $n+m$-order and is connected into another circuit in m-nodes.

$$\begin{array}{|cc|c|} \hline \mathbf{C}_{EE} & -z^{-\frac{1}{2}}\mathbf{C}_{EO} & \\ -z^{-\frac{1}{2}}\mathbf{C}_{OE} & \mathbf{C}_{OO} & \mathbf{M} \\ \hline & -\mathbf{M}^T & \mathbf{Z} \\ \hline \end{array} \qquad (16)$$

CONCLUSION

We described a close relation between Thevenin's theorem and the diakoptic method. We can use both to simplify the electrical circuit. Thevenin's theorem converts the original circuit to the circuit that contains a single loop and a single node. Meanwhile, the diakoptic method converts the original circuit to the circuit that contains multiple loops and nodes.

As we mentioned above, we can divide the circuit into subcircuits. We can describe each of these circuits by the most suitable methods according to the given configuration. We can then combine the above description by the node voltage method and the loop current method, the node voltage method and the nodal charge equations method, not only. Also, we describe each part of the circuit with different two-port parameters [4], [7].

We will also appreciate the diakoptic method in the symbolic analysis of circuits. If we solve a more complex electrical circuit, the relationships for the resulting electrical circuit are extensive and confusing and thus practically unusable. On the other hand, if we divide the original electrical circuit into subcircuits, then the symbolic results of individual electrical circuits are simplified.

REFERENCES

[1] H. H. Happ, Diakoptics and Networks. New York: Academic Press, 1971, pp. 312.

[2] J. Schmidtmayer, Matrix calculus and its use in electrical engineering, Praha: SNTL, 1974 (in Czech).

[3] A. I. Rybin, J. K. Trochimenko, M. V. Radionova, “Analysis of irregular networks using diakoptic modification of admittance matrix inversion,” Radioengineering, vol. 2, No. 4, 1993, pp. 9-12.

[4] J. Brzobohatý, J. Pospíšil, “General transformation attribute of linear circuits,” Electronics Horizon, vol. 48, No. 6, 1987, pp. 1-12.

[5] T. Dostál, A. I. Rybin, “Nodal Analysis of Circuits Containing Current Conveyors,” Radioengineering vol. 10, No. 3, 2001, pp. 1-4.

[6] T. Dostál, J. Pospíšil, Electronic network and systems I., printed lectures, Brno: VUTIUM, 1991 (in Czech).

[7] B. Brtník, D. Matoušek, ”Solving BJT Circuits by Nodal Voltage method with Application of Diakoptical Method,” Proceedings of 21st International Conference, Applied Electronics, Pilsen 2016, pp. 35-38.

[8] O. Yevsyeyveva, ”Diakoptical Approach in Telecommunication Engineering,” TCSET 2010, Lviv 2010, pp. 29.

[9] G. N. Stenbakken, J. A. Starzyk: “Diakoptic and large change sensitivity analysis,” IEEE Proceedings – G, vol. 139, No. 1, pp. 114-118, 1992.

[10] T. Dostál, A. I. Rybin, J. K. Trochimenko, “Calculation of Poles of Network Functions Diakoptical Modification of Complex Circuits,” Electronics Horizon, special issue, 1994, pp. 45-47.