Intuitionistic Fuzzy Neural Network for Time Series Forecasting - The Case of Metal Prices *

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Abstract. Forecasting time series is an important problem addressed for years. Despite that, it still raises an active interest of researchers. The main issue related to that problem is the inherent uncertainty in data which is hard to be represented in the form of a forecasting model. To solve that issue, a fuzzy model of time series was proposed. Recent developments of that model extend the level of uncertainty involved in data using intuitionistic fuzzy sets. It is, however, worth noting that additional fuzziness exhibits nonlinear behavior. To cope with that issue, we propose a time series model that represents both high uncertainty and non-linearity involved in the data. Specifically, we propose a forecasting model integrating intuitionistic fuzzy sets with neural networks for predicting metal prices. We validate our approach using five financial multivariate time series. The results are compared with those produced by state-of-the-art fuzzy time series models. Thus, we provide solid evidence of high effectiveness of our approach for both one- and five-day-ahead forecasting horizons.

Keywords: Fuzzy time series · Fuzzy neural network · Intuitionistic fuzzy sets

1 Introduction

Despite spectacular achievements in the field, forecasting time series still raises an active interest among researchers. Their efforts' main goal is to design a forecasting model that would be able to capture the uncertainty involved in data and, thanks to that, produce more accurate forecasts.

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One of the most essential approaches models time series using fuzzy sets. The method consists of three main stages. First, time series are partitioned and fuzzified. Then, the prediction model is created using fuzzy logic relations (fuzzy if-then rules). The model is used to predict future values of the fuzzified time series (fuzzy time series). Finally, if the knowledge of their crisp values is required for the considered application, the predictions are defuzzified.

In the literature, for data partitioning, equal-sized intervals were typically used [2–4]. However, if the data distribution were not uniform over the universe of discourse, the equally distributed fuzzy sets did not represent the underlying data effectively. This in turn led to higher forecasting errors [1]. To address this issue, the parameters of fuzzy time series were determined using different methods, including mathematical optimization and clustering-based methods. Due to their robustness and capacity to solve global optimization problems, evolutionary algorithms were frequently used to optimize the parameters of fuzzy sets. In this case, the limitation is the susceptibility to over-fitting. Clustering-based methods, in turn, usually provided a good trade-off between computational demand and forecasting accuracy [5, 6]. It is, however, worth noting that the existing fuzzification methods are not capable of modeling the dynamic behavior of time series. To overcome this problem, we propose in this paper a fuzzification method that assigns membership and non-membership values of fuzzy sets by incorporating variance in the time series.

To perform forecasts, fuzzy logic relations between previous and forecasted values were traditionally produced from the historical time series. The relations were generated and selected using the fuzzy sets with the highest membership values. The weighted and polynomial constructions were introduced for each fuzzy logic relationship to assign larger weights to recent time series observations (compared with the latter) [7] or to those with higher empirical probabilities [8]. The fuzzy trend of the forecasted value was also incorporated into the final forecasts [9, 10]. The main problem with the construction of traditional fuzzy logic relationships is their poor generalization capacity, this is poor out-of-sample forecasts. Moreover, for many observations, no matched fuzzy logic relations are available and, hence, no reliable forecast can be performed [11]. To address these issues, neural networks were employed to learn the relationships using different strategies.

Several studies used neural networks to predict the consequences of the relationships based on the index numbers of input fuzzy sets (antecedents) [12, 13]. Central values of fuzzy sets were used for the same purpose [14]. Alternatively, input and output membership degrees were used to represent the fuzzy sets [2, 15]. Multilayer perceptron neural networks were employed in the above studies. Pi-Sigma neural network used by [16] represents an a higher-order alternative with a fewer number of units, leading to enhancement in convergence speed. However, this neural network model is more susceptible to over-fitting. Support vector regression was applied to produce predictions for unrecognized multivariate fuzzy logic relationships [17]. In an adaptive neuro-fuzzy inference system (ANFIS) [18, 19], hidden units of neural network represent the if-then rules given in advance, and neural network is used to learn the consequent parameters of the rules. Another layer can be used to represent the parameters of fuzzy sets. Interval type-2 (intuitionistic) fuzzy sets were used in ANFIS to represent the additional uncertainty in financial time series [20–23].

Hesitant fuzzy sets and probabilistic fuzzy sets represent other extensions of fuzzy sets used for fuzzy time series forecasting [4, 24, 25]. To avoid the overfitting problem of single prediction models, several studies introduced combinations of fuzzy neural networks utilizing both interval type-2 fuzzy sets [26] and intuitionistic fuzzy sets [27]. The main drawback of the presented interval type-2 (intuitionistic) fuzzy neural networks is their use of static membership functions (lower, upper, or non-membership functions). This is, the degree of hesitancy does not consider volatility in the time series data.

We propose an intuitionistic fuzzy neural network that incorporates this concept. In addition, it utilizes intuitionistic fuzzy operators to calculate the firing weights of if-then rules and a defuzzification method designed for intuitionistic fuzzy sets to aggregate the outcomes of the rules. Gradient descent is used as a training algorithm for the intuitionistic fuzzy neural network. The proposed model is also highly computationally efficient because only consequent parameters of the rules are adapted while the parameters of fuzzy sets and rule antecedents are generated using a clustering algorithm.

As far as we know, this is the first extension of a fuzzy neural network that considers the volatility in the time series to assign the degree of hesitancy to observations in the data partitioning stage. For the first time, a generalization of a neuro-fuzzy system is used for predicting metal prices.

The rest of this paper is organized in the following way. Section 2 outlines the proposed intuitionistic fuzzy neural network for time series forecasting. Section 3 presents the metal price datasets. Section 3 shows the results of the experiments and comparisons with existing time series methods. Section 4 concludes this paper and discusses future research.

2 Intuitionistic Fuzzy Neural Network for Time Series Forecasting

Let us remind at first the definition of an intuitionistic fuzzy set A which is [28]:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\,\}\,, \tag{1}$$

where $\mu_A(x)$ and $\nu_A(x)$ respectively is the membership and a non-membership degree of element x to the set A, X is the universe of discourse. It holds that $0 \le \mu_A(x) \le 1$, $0 \le \nu_A(x) \le 1$ and $0 \le \mu_A(x)+\nu_A(x) \le 1$. The hesitation degree $\pi_A(x)$ denotes an additional degree of uncertainty, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Here we propose an intuitionistic fuzzy neural network which is based on that outlined in [29] and consists of six layers (Fig. 1).

Input layer: The first layer is used to forward the crisp inputs $x_1^t, x_2^t, \ldots, x_n^t$ to the next layer.

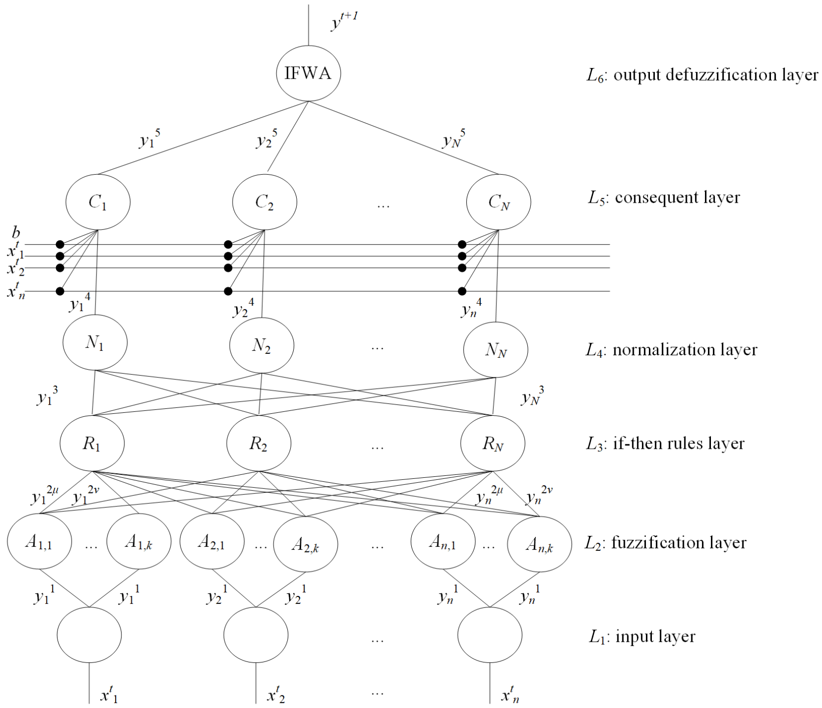

Fig. 1. Intuitionistic fuzzy neural network for time series forecasting.

Fuzzification layer: Neurons in this layer represent antecedent intuitionistic fuzzy sets for the i-th input attribute x_i^t. To fuzzify the crisp values, the input attributes are first compared with the membership functions. Here, we use the Gaussian membership function defined as follows:

$$\mu(x_i^t) = e^{-\frac{(x_i^t - c)^2}{2\sigma^2}}, \tag{2}$$

where c is the center and σ is the width of the membership function. The values of these parameters are obtained automatically using the subtractive clustering algorithm [30]. To obtain the intuitionistic fuzzy sets, the following fuzzification method is used [31]:

$$\mu_A(x_i^t) = \mu(x_i^t) \times (1 - \delta D), \tag{3}$$

$$\nu_A(x_i^t) = 1 - \mu(x_i^t) \times (1 - \delta D) - \delta D, \text{where} \tag{4}$$

$$D = (max(\mu(x_i^t), \mu(x_i^{t-1}), \ldots, \mu(x_i^{t-4})) - min(\mu(x_i^t), \mu(x_i^{t-1}), \ldots, \mu(x_i^{t-4}))) \tag{5}$$

and δ is set to 1 in agreement with [31]. Hence, intuitionistic fuzzy sets are given as $A = \{\langle x_i^t, \mu_A(x_i^t), \nu_A(x_i^t)\rangle \,|x_i^t \in X_i\}$, where X_i is the universe of discourse for the i-th input attribute. Note that the fuzzification parameter D considers the volatility of the last five observations in the time series. Still, this number can be adapted to the needs of the specific forecasting problem.

If-then rules layer: Neurons in this layer represent the if-then rules of the first-order Takagi-Sugeno-Kang type. The j-th rule R_j, $j = 1, 2, \ldots, N$, can be defined as follows:

$$\begin{aligned} R_j : \text{if } x_1^t \text{ is } A_{1,j} \text{ and } x_2^t \text{ is } A_{2,j} \text{ and } \ldots \text{and } x_i^t \text{ is } A_{i,j} \text{ and } \ldots \text{and} \\ x_n^t \text{ is } A_{n,j} \text{then } y_j^{t+h} = a_{0,j} + a_{1,j}x_1^t + \cdots + a_{i,j}x_i^t + \cdots + a_{n,j}x_n^t, \end{aligned} \tag{6}$$

where $A_{i,j}$ is antecedent intuitionistic fuzzy set for the i-th input attribute x_i^t and j-th rule R_j, y_j^{t+h} is the predicted output for the j-th rule, h is the forecasting horizon, and $a_{0,j}, a_{1,j}, \ldots, a_{i,j}, \ldots, a_{n,j}$ are the consequent parameters.

To calculate the firing weight w_j of the j-th rule R_j, Gödel t-norm operators are used as follows:

$$w_j^{\mu} = \min_{j=1,2,\ldots,N} (\mu_A(x_1^t), \mu_A(x_2^t), \ldots, \mu_A(x_n^t)), \tag{7}$$

$$w_j^{\nu} = \max_{j=1,2,\ldots,N} (\nu_A(x_1^t), \nu_A(x_2^t), \ldots, \nu_A(x_n^t)), \tag{8}$$

$$w_j = w_j^{\mu} - w_j^{\nu}, \tag{9}$$

where w_j^{μ} and w_j^{ν} denote the membership and non-membership degrees of the firing weight W_j, respectively. Note that only positive firing weights are considered (with acceptance degree higher than non-acceptance degree) in agreement with [32].

Normalization layer: Neurons in this layer calculate the normalized values of the firing weights w_j^{norm}.

Consequent layer: Consequent parameters $a_{0,j}, a_{1,j}, \ldots, a_{n,j}$ are represented by neurons in this layer by calculating the outputs of the rules as $y_j^{t+h} = a_{0,j} + a_{1,j}x_1^t + \cdots + a_{i,j}x_i^t + \cdots + a_{n,j}x_n^t$.

Defuzzification layer: One output neuron in this layer calculates the weighted average of outputs from the preceding layer to obtain the defuzzified forecast as follows:

$$y_{IFWA}^{t+h} = \frac{\sum_{j=1}^{N} y_j^{t+h} w_j^{norm}}{\sum_{j=1}^{N} w_j^{norm}}. \tag{10}$$

To train the synapse weights of the intuitionistic fuzzy neural network, the gradient descent algorithm was used due to its stable convergence reported in earlier studies [23, 29]. The algorithm can be defined as follows:

$$w_{i+1} = w_i - \eta \nabla_{\theta} J \left(w_i; x^{(t)}; y^{(t+h)} \right), \tag{11}$$

where w is synapse weight, η is learning rate, i denotes the iteration index, J is the objective function (root mean square error (RMSE)), $x^{(t)}$ and $y^{(t+h)}$ represent the input and output for the t-th observation in the time series.

3 Model Validation

In this section, we validate the proposed intuitionistic fuzzy neural network for time series forecasting (IFNN-TS).

3.1 Experimental Setup

The data used in this study are the closing prices of five major metals, namely gold, silver, palladium, platinum, and rhodium. The dataset covers the period from 2007 to 2017, including 3,949 trading days. Specifically, daily spot prices in USD per ounce were collected from the Kitco database. The prices are depicted in Fig. 2. To evaluate the proposed forecasting model's robustness, two datasets were generated for each metal price, the one-day-ahead (daily) and five-day-ahead (weekly) forecasting. Sequential validation was used by partitioning data into the training set immediately followed by the testing set in ratio 9:1 following earlier relevant studies [33]. This is, the first 3,554 samples represented training data, and the following 395 samples were used as testing data.

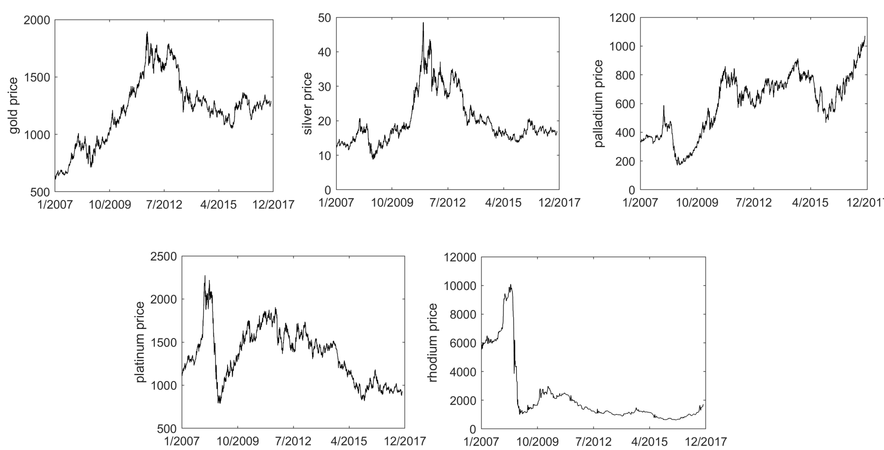

Fig. 2. Metal price data.

We followed earlier research [34–36] and consider the technical indicators of respective metal prices, previous oil price (Brent crude oil price, BRN), exchange rate (US Dollar to Chinese Yuan, USDCNY), and news sentiment indicators as input attributes. More precisely, 20-day technical indicators were calculated, including exponential moving average (EMA, trend-type indicator), relative strength index (RSI, oscillator-type indicator), and rate of change (ROC, volatility-type indicator):

$$EMA_t = \frac{2}{21}(SMA_t - EMA_{t-1}) + EMA_{t-1}, \tag{12}$$

$$RSI_t = 100 - \frac{100}{1 + RS}, ROC_t = \frac{P_t - P_{t-20}}{P_{t-20}} \times 100, \tag{13}$$

where SMA_t is 20-day simple moving average, RS is the ratio of smoothed average of 20-day upward / downward ROC, and P_t is the metal price at day t.

The previous day's closing prices were used for BRN and USDCNY, and the data for these inputs were obtained from the MarketWatch database. To consider the information effects on metal prices, we calculated the intensity of positive and negative news sentiment using SentiWordNet publicly available at https://github.com/aesuli/SentiWordNet. The Thomson Reuters newswire service was used to obtain the news related to metals for the respective period. In total, 266,165 news articles were collected, and the mean values of SentiWordNet sentiment indicators were calculated for each day.

The following state-of-the-art models were considered for comparison:

- ANFIS-GA [37], initialized using the subtractive clustering algorithm with the same settings as in IFNN-TS and trained using the genetic algorithm with the parameters adopted from [37].
- INFN-PSO [29], an intuitionistic neuro-fuzzy network trained using particle swarm optimization. Again, we used the subtractive clustering algorithm to initialize the parameters of the model and trained it in agreement with the settings recommended in [29].
- IT2FLS-EKM [38], the interval type-2 fuzzy logic system with the enhanced Karnik-Mendel algorithm, generated in fuzzy logic toolbox as the interval type-2 Sugeno FIS and tuned using the gradient descent algorithm.
- ES (exponential smoothing) [39], represented by triple ES (Holt-Winters) model with smoothing factors of 0.2.
- ARIMA [40], adopting the ARIMA(1,1,0) model found by [40] using the Hyndman and Khandakar algorithm.
- RF [41], random forest trained using 100 random trees.
- MLP [40], multilayer perceptron NN with the settings adopted from [40] as follows: one hidden layer of 24 sigmoidal neurons, the momentum of 0.5, and the learning rate of 0.001.
- LSTM [42], long short-term memory NN with an LSTM layer of 200 neurons followed by a dense layer of 32 neurons (the structure was adopted from [42]) trained using stochastic gradient descent.

Forecasting performance was evaluated using RMSE and mean absolute error (MAE) on the testing data separately for the 1-day-ahead and 5-day-ahead forecasting horizon. In addition, we present the mean directional accuracy (MDA) to evaluate the proposed system's capacity to predict the correct forecast direction (upward or downward) and investigate the financial performance of the constructed precious metals portfolio in terms of its return and risk. All the experiments were carried out in the Matlab Fuzzy Logic Toolbox in Matlab R2020a.

3.2 Experimental Results

First, we used the subtractive clustering algorithm to generate the antecedent intuitionistic fuzzy sets and the rule base. Note that the use of the subtractive clustering algorithm to generate the if-then rules enabled us to substantially reduce the complexity of the rule base. Experiments were performed for different values of the radius of influence (resulting in different numbers of antecedent intuitionistic fuzzy sets and rules) to prevent under-fitting and over-fitting. More precisely, we examined three settings with $N = \{3, 5, 7\}$ rules and antecedent intuitionistic fuzzy sets. Due to space limitations, we only show the performance of IFNN-TS for N=3 and N=5 rules (Table 1) because the performance for N=7 deteriorated substantially. Obviously, N=3 was a preferable setting in terms of both forecasting accuracy and interpretability at the rule base / fuzzy partition level. In all experiments, we used the gradient descent algorithm (with 100 iterations and the learning rate $\eta = 0.01$) to train the IFNN-TS. For the example of the gold price, the obtained rule base is as follows:

R_1 : if EMA_t is *medium* and RSI_t is *medium* and ROC_t is *high* and BRN_t is *low* and $USDCNY_t$ is *medium* and POS_t is *low* and NEG_t is *high* then $y_1^{t+1} = 0.11 + 0.97 \times EMA_t \ - 0.09 \times RSI_t + 6.18 \times ROC_t - 0.17 \times BRN_t + 6.82 \times USDCNY_t + 0.0006 \times POS_t - 0.04 \times NEG_t$,

R_2 : if EMA_t is *high* and RSI_t is *low* and ROC_t is *low* and BRN_t is *high* and $USDCNY_t$ is *low* and POS_t is *high* and NEG_t is *low* then $y_1^{t+1} = 0.06 + 1.06 \times EMA_t \ - 0.002 \times RSI_t + 11.72 \times ROC_t - 1.18 \times BRN_t + 1.08 \times USDCNY_t + 0.003 \times POS_t + 0.04 \times NEG_t$,

R_3 : if EMA_t is *low* and RSI_t is *high* and ROC_t is *medium* and BRN_t is *medium* and $USDCNY_t$ is *high* and POS_t is *medium* and NEG_t is *medium* then $y_1^{t+1} = -0.14 + 0.92 \times EMA_t \ + 0.12 \times RSI_t - 6.83 \times ROC_t + 1.38 \times BRN_t - 8.02 \times USDCNY_t - 0.03 \times POS_t + 0.04 \times NEG_t$.

Experimental results in Table 1 show the effectiveness of the proposed IFNN-TS by comparison with three models of fuzzy neural networks and five other benchmark forecasting models used previously for metal price prediction. The results of the comparisons show that IFNN-TS was highly competitive regarding all metal prices in terms of both forecasting horizons. Best performance in terms of RMSE was achieved for one-day-ahead forecasting of gold, silver and platinum prices. A non-parametric Friedman test was used to compare the results of the models statistically. The average ranks of the IFNN-TS models were as follows: 2.2 for MAE and one-day-ahead forecast, 2.0 for RMSE and one-day-ahead forecast, 2.0 for MAE and five-day-ahead forecast, and 2.4 for RMSE and five-day-ahead forecast. Significant differences were observed for the average ranks of the compared methods at $p < 0.05$, indicating significantly different performance across the error measures and forecasting horizons. In the next step, the Holm–Bonferroni posthoc procedure was used to compare the performance between the best forecasting model and the other models. For the one-day-ahead forecasting, IFNN-TS significantly outperformed ARIMA, ES, RF, MLP, and

LSTM at $p < 0.05$. For the five-day-ahead forecasting, IFNN-TS performed significantly better than RF, MLP, LSTM, and ANFIS-GA at $p < 0.05$. These results were consistent for MAE and RMSE.

Table 1. Results of metal price forecasting (the best result is in bold).

		IFNN-TS		IFNN-TS		ANFIS-GA		INFN-PSO		IT2FLS-EKM	
		N=3 rules		N=5 rules		N=3 rules		N=3 rules		N=3 rules	
Metal	Forecast	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
gold	1-day	6.687	**9.689**	10.248	13.426	7.045	9.994	**6.684**	9.764	9.731	13.067
gold	5-day	15.408	19.680	18.022	22.962	16.980	21.335	16.058	20.497	18.011	22.820
silver	1-day	**0.155**	**0.223**	0.268	0.340	0.175	0.243	0.177	0.246	0.194	0.257
silver	5-day	**0.364**	0.474	0.435	1.453	0.461	0.602	0.369	0.483	0.379	0.502
palladium	1-day	7.843	10.978	11.617	15.291	7.976	10.997	**7.792**	**10.811**	7.993	10.917
palladium	5-day	16.873	21.681	19.973	25.187	19.306	24.697	17.611	22.643	17.365	22.480
platinum	1-day	8.658	**11.550**	11.830	14.973	**8.635**	11.579	9.670	12.536	9.698	12.800
platinum	5-day	**18.968**	25.000	19.973	25.187	21.537	27.881	19.340	25.186	19.468	25.430
rhodium	1-day	16.766	23.574	10.814	21.852	**12.605**	**18.007**	13.460	21.996	15.723	22.537
rhodium	5-day	24.100	38.365	22.623	42.414	33.144	47.137	22.204	36.562	24.543	39.459
		ES		ARIMA		RF		MLP		LSTM	
Metal	Forecast	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE
gold	1-day	16.092	20.665	9.949	12.538	11.434	14.933	10.936	13.697	23.845	26.618
gold	5-day	17.066	21.485	**10.747**	**13.431**	18.630	23.104	11.851	14.659	40.757	44.271
silver	1-day	0.376	0.471	0.305	0.377	0.239	0.311	0.422	0.484	0.331	0.386
silver	5-day	0.389	0.499	0.567	0.683	0.369	**0.471**	0.607	0.674	0.514	0.581
palladium	1-day	19.008	24.491	8.117	10.997	28.084	44.407	13.240	16.733	23.173	28.449
palladium	5-day	18.628	23.833	**11.789**	**15.408**	39.000	52.530	19.293	23.892	32.040	40.103
platinum	1-day	19.954	25.342	15.398	18.626	15.986	21.408	18.385	21.249	19.900	25.642
platinum	5-day	20.185	26.426	19.387	**23.135**	23.957	31.448	24.599	27.542	27.569	35.163
rhodium	1-day	19.948	32.646	35.700	46.425	100.64	126.57	56.930	67.615	65.726	87.358
rhodium	5-day	**19.532**	**35.204**	125.17	178.98	104.28	127.36	103.62	120.43	95.874	130.39

In addition to the error measures, we evaluated the performance of the model in predicting MDA. Fig. 3 shows that IFNN-TS consistently exceeded 55% across metal prices. Correct forecast of upward/downward direction is important for generating ‘buy’, ‘hold’ and ‘sell’ signals. Therefore, we further investigated the financial performance (return and risk) of the precious metals portfolio constructed based on signals generated using the IFNN-TS-based trading strategy (‘buy’ (‘hold’) signal for upward price forecast, and ‘sell’ signal for downward price forecast). The closing metal prices were used for trading, and the weights of the five metals in the portfolio were equal. We obtained an average return of 54.63% (for one-day-ahead forecasting) and 87.91% (for five-day-ahead forecasting) for the testing period. The forecasting-based trading strategy was more profitable than the traditional buy-and-hold strategy (with an average daily return of 41.29% and weekly return 41.17%). However, it should be noted that our trading strategy was associated with a higher portfolio risk. The standard deviation of returns was used to calculate the risk, obtaining $\sigma = 6.32\%$ and $\sigma = 7.22\%$ for the one-day-ahead and five-day-ahead IFNN-TS forecasting strategies, hence exceeding those for the buy-and-hold strategy ($\sigma = 4.36\%$ and $\sigma = 6.44\%$).

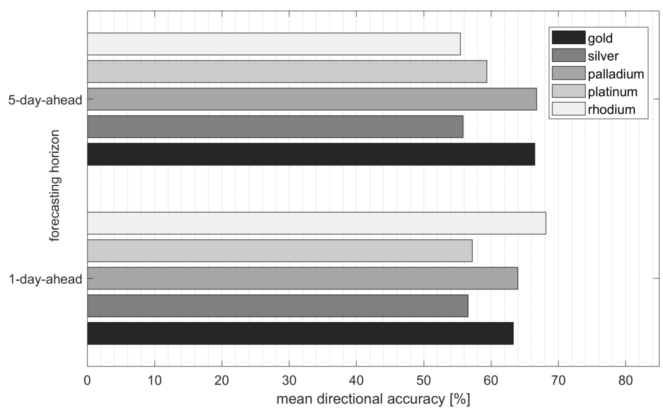

Fig. 3. Mean directional accuracy of IFNN-TS.

4 Conclusion

In this study, we proposed an efficient forecasting model that incorporates intuitionistic fuzzy sets to consider uncertainty present in the time series volatility in the fuzzification process. To defuzzify the forecast, an intuitionistic fuzzy weighted averaging operator was proposed. In the learning process, the model utilizes the capability of neural networks to minimize forecasting error. Good interpretability at the fuzzy partition and rule base level is ensured using a clustering algorithm in the initialization process.

We validated the proposed model using five time series of major metal prices. The proposed model outperformed existing fuzzy neural networks, but it was also competitive compared to existing models used for forecasting metal prices. In addition to low forecasting errors, the model provides investors with an interpretable set of trading rules. Compared with the buy-and-hold strategy, the trading strategy based on our model directional forecasts achieved a higher average return (and risk) of the metals portfolio.

A limitation of the proposed model is that a high degree of hesitancy (caused by high volatility in the time series data) may negatively affect the rules' firing weights. Consequently, no matched if-then rules are available in the rule base. Future research should investigate alternative approaches to generate the rule base to overcome this limitation, such as evolutionary rule selection. The parameters of membership functions and fuzzification could also be adapted in further investigation. Further research might also explore comparisons with recent time series forecasting methods such as Bi-LSTM.

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