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Modeling of Economic Processes Using ARIMA Models

Case study: Applying ARIMA Models to Qatar's GDP

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Zásady pro vypracování

The aim of this work is to map the family of ARIMA models (SARIMA, FARIMA, etc.) in terms of their application to different types of processes. Part of this work will be analysis of selected real process, incl. verification and comparison of proposed models.

Guidelines:
- Objective and methods.
- Literature Review.
- Research Methodology.
- Case study and Results.
- Recommendation and Conclusion.

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I declare:

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In Pardubice on April 21, 2021

Osama Altayyar by own hand

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ANNOTATION

The theoretical part of this thesis titled “Modeling of Economic Processes Using ARIMA Models” focuses on the process of predicting and analyzing future values of an economy using various types of economic models. This part includes a brief introduction to the economics, modeling process and information on various types of models that can be used. Examples of those models: AR, MA, ARMA, ARIMA, SARIMA, ARIMAX, and SARIMAX models. The practical part includes a case study on applying ARIMA models to Qatar's GDP. The objectives of this case study are to provide, analyze, forecast, process, show results, and give recommendations about Qatar’s GDP using ARIMA model processes.

KEYWORDS

Models, Forecasting, ARIMA, Economics, Time series, GDP

NÁZEV

Modelování ekonomických procesů pomocí modelů ARIMA

ANOTACE

Teoretická část diplomové práce Modelování ekonomických procesů pomocí modelů ARIMA se zaměřuje na procesy predikce a analýzy budoucích hodnot ekonomiky pomocí různých typů ekonomických modelů. Tato část zahrnuje úvod do ekonomiky, procesu modelování a poskytování informací o dalších typech modelů, které lze použít. Příklady těchto modelů: modely AR, MA, ARMA, ARIMA, SARIMA, ARIMAX a SARIMAX.

Praktická část obsahuje jednu případovou studii o aplikaci modelů ARIMA na katarský HDP. Cílem této případové studie je poskytnout, analyzovat, předpovědět, zpracovat, představit výsledky a poskytnout doporučení ohledně katarského HDP a seznámit nás s používáním modelů ARIMA.

KLÍČOVÁ SLOVA

modely, predikce, ARIMA, ekonomika, časové řady, HDP

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LIST OF ABBREVIATIONS AND ACRONYMS

AR	Autoregressive
ARMA	Autoregressive moving average
MA	Moving average
GDP	Gross domestic product
GNI	Gross national income
PACF	Partial autocorrelation function
ACF	Autocorrelation function
BIC	Bayesian information criterion
ARIMA	Autoregressive integrated moving average
SARMA	A seasonal autoregressive average
SARIMA	A seasonal autoregressive integrated moving average
ARIMAX	Autoregressive integrated moving average with exogenous variables.
ARMAX	Autoregressive moving average with exogenous variables.
SARIMAX	A seasonal autoregressive integrated moving average with exogenous

INTRODUCTION

In many areas such as quality control and economics, there is an interest in forecast future outcomes and as well as modeling data overtime. Since prediction plays an important role in decision-making policies and monetary policy analysis of the countries, these countries seek to develop the forecasting methods that are used in the economic forecasting process. There are many ways to perform and modeling data over time either with one or several predictor variables. The (ARIMA) autoregressive integrated moving average and exponential smoothing models are considered as univariate methods that are used in forecasting future outcomes.

Time series analysis is the set of recorded measurements arranged according to their time of occurrence of one or more variables. The analysis of time series can be used to examine how the changes associated with the shifts in other variables compare to chosen data point over the same time. The time series analysis can be used in other applications such as sales forecasting, workload process, quality control projections, economic forecasting, yield projections, inventory studies, stock market analysis, and budgetary analysis.

(ARIMA) autoregressive integrated moving average models: in theory, ARIMA models mean the most general models for forecasting a time series which time series should be stationary which means over time it should show similar behavior in form of the constant, autocovariance, expected value, and in conjunction with nonlinear transformations such as deflating or logging. In the ARIMA models, we called a random variable is stationary in the series time if statistical properties are constant over time and stationary time series has no trend. By allowing the differencing of the time series the ARMA model extends to nonstationary times series and the result we can have ARIMA models.

This research will provide also a brief introduction about the other models that can be used in the times series analysis such as (AR) autoregressive model, differencing process, (MA) moving average model, (ARMA) autoregressive–moving-average model, the Box-Jenkins model, (p,d,q) orders, (SARMA) a seasonal autoregressive moving average, (ARIMAX) autoregressive integrated moving average with exogenous variables, (SARIMAX) seasonal autoregressive integrated moving averages with exogenous regressors, linear regression model and (ARIMA) autoregressive integrated moving average model with their owns equations, processes, stages and some examples.

The main aim of this research to model and forecasting Qatari GDP based on the ARIMA approach based on the given annual data. The stages of the ARIMA approach are conducted to obtain an appropriate ARIMA model for the Qatari GDP, and we will apply this model to forecast the Qatari GDP for the next six years (from 2019 to 2025), and the second aim, to be familiar with time series analysis and forecasting by using the ARIMA models.

The first chapter contains an introduction to economics. Simply, economics is the study that helps persons make better decisions by studying ways to satisfy unlimited desires using limited available resources. The study of economics is divided into two branches: microeconomics and macroeconomics. Microeconomic is defined as a study that focuses and analyses singular factors such as a person and the decisions made by these individual participants in the economy. Macroeconomics on the other hand, analyses the overall performance of the economy.

The second chapter focuses on economic modeling. In this chapter, there is explained how to model topics related to economics. Modeling explains and shows how the different economic concepts work. The modeling process of economic concepts means the process of designing and creating an outcome using a mathematical and conceptual framework to explain specific economic problems or phenomena. The economic model represents the quantitative relationships between a set of logics and a set of variables, one model can have different responses when it has a different exogenous variable. Economists work to make a simple economic model of the phenomena, which provides better understanding and helps to deal with various economic problems by analyzing, theorizing, simplifying investigations, fitting theories, presenting the useful information, testing, evaluating, and selecting the right data and variables for modeling solutions to known or anticipated problems. There are many types of economic models that can be used to explain economic phenomena. Examples include general equilibrium models, flow chart model, graphs, stochastics models or mathematics models.

The third chapter is a brief introduction about (AR) autoregressive model, (MA) moving average model, (ARMA) autoregressive–moving-average model, the Box-Jenkins model, (p, d, q) orders, and (ARIMA) autoregressive integrated moving average model with their equations, processes, and some examples.

The fourth chapter presents a case study focused on applying ARIMA models to Qatar's GDP. This chapter starts with an overview of Qatar as a country with information on location, population, the capital city and more. The country’s economy and the GDP are discussed, and finally, the (ARIMA) autoregressive integrated moving average models are applied to analyze Qatar's GDP, find results, provide recommendations, and a conclusion for the topic.

1 OBJECTIVES AND METHODOLOGY

In this chapter, we will discuss the objectives and the methodology of this research.

1.1 Objectives

The main objective of this thesis is to forecast time-series data in Qatar's Gross domestic product (GDP). We will start by assessing and collecting the data for the Qatar GDP, analyzing those data by applying the framework of autoregressive integrated moving average model (ARIMA) and its process, and forecasting in goals to evaluate how well it works.

The objectives of the thesis paper will seek to answer the following questions:

Q1: What is the ARIMA model and its parameters (p, d, q)?

Q2: What are the processes of applying ARIMA model?

Q3: What are the other types of economic models we can use for forecasting time series data?

Q4: What are the results of the Qatari GDP dataset forecasting?

1.2 Methodology

The method of this research will be used to achieve the objectives.

This method will be collecting data, applying, forecasting, and analyzing Qatar’s GDP by using the ARIMA model as is shown in the following steps:

- Collecting information about the GDP.
- Describe the main features in the time series data.
- Monitoring and discover the time series changing.
- Finding the optimal parameters for the model.
- Applying and build the ARIMA model.
- Make forecasting of the values of the time series.

2 LITERATURE REVIEW

This chapter will provide an overview about economy, modeling in economy and using the ARIMA models in the forecasting and analyzing time series processes.

2.1 Economic

Economics is a Latin word (Oikou-Nomos) meaning (home management), it was difficult to develop a comprehensive definition of economics due to the numerous topics it addresses and its connection to other sciences.

Many economists have tried to find a comprehensive definition of economics according to their analysis and opinions on it, and some differences appeared in their definitions. The most famous of them was Adam Smith, Karl Marx and Thomas Malthus.

Economics is the study and analysis of the human economic activity of an individual, organization, society, etc., providing their needs and satisfying their desires through available and scarce resources, and includes all operations of production, exchange, distribution, and consumption of goods and services, including issues of finance, wealth distribution and the level of well-being, and presenting studies and forecasts about past and future years. The economy is also used to organize the economic sectors, including the industrial sector, the financial sector, and the commercial sector. The economy is divided into two parts: macroeconomics is concerned with studying and analyzing the economy, such as unemployment, taxes, and others, while the other part is microeconomics, which studies and analyzes consumer behavior or organization (Morrow, 2006).

2.1.1 Microeconomics

Microeconomics is a branch of economics that studies and analyzes the economic activities of a person or small organization, such as studying the supply and demand for products, the balance between them, determining the supply and demand in the market, studying the interaction between the seller and the buyer in the market, consumer behavior, product behavior, and providing solutions to their economic problems to reach balance in the market and help them in making decisions (Harper, 2011).

2.1.2 Demand and supply

In this chapter, the information about demand and supply will be provided.

2.1.2.1 Demand

Demand is the sum of planned expenditures or the quantities of goods and services required to buy at a specific price and time. With the constancy of factors affecting demand, its subsets include:

- The existence of alternatives or supplements to the required goods.
- Active market size (population).
- Public and special taste (such as the tendency to a certain type of tea).
- Income distribution among the population.
- Future expectations for a specific situation (such as a severe winter forecast that leads to high demand for winter clothing).
- The demographic composition of the population.

The demand curve is showing the relationship between the price and quantities of goods or services at a specific time.

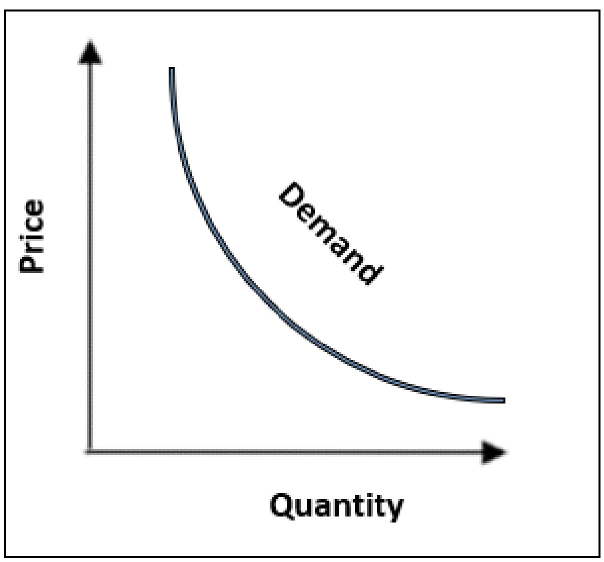

Figure 1 Demand Curve

(Kenton, 2019)

As it is shown in figure (1), the price of a good or service will show on the vertical axis, quantities of good or service will show on the horizontal axis. From the figure (1) the price and goods have inverse relationship as much price increasing as much the quantity decreasing (Nelson, 2014).

2.1.2.2 Supply

Supply means the total commodities produced that the producer wants to offer in the market and sell during a certain period and at a certain price. The supply curve shows the relationship and correlation between the cost of goods or services and the quantity supplied at a specific time.

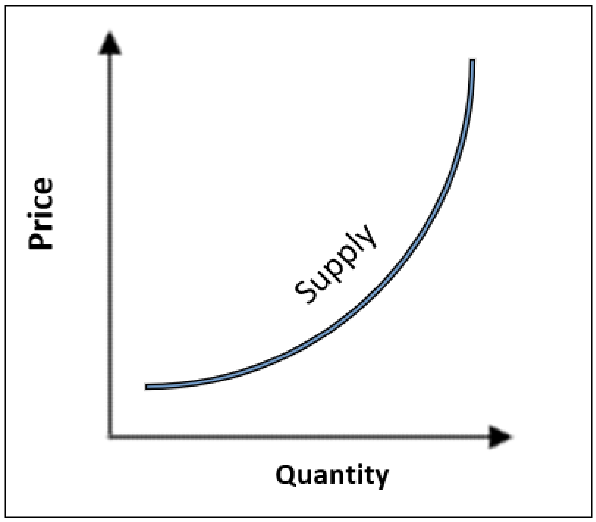

Figure 2 Supply Curve

(Kenton, 2019)

As it is shown in figure (2), the price of a good or service will show on the vertical axis, supplied quantities of good or service will show on the horizontal axis. From the figure (2) the price and goods have proportional relationship as much price increasing as much the quantity increasing (Kenton, 2019).

2.1.3 Macroeconomics

Macroeconomics is a branch of economics that studies the economy in general. Macroeconomics focuses on outcomes and events accompanying the economy and measures the performance of the economy as a whole with the aim of understanding it and providing solutions to economic problems.

Macroeconomics primarily focuses on the total changes in the economy such as gross domestic product GDP, national income, unemployment rates, investment, inflation, growth rate and others (Ahuja, 2010).

2.2 Modeling in economics

The economic model is a simplified model and theoretical construction of the real economic operations that allow economists mainly to check economic behavior, understand, explain, and make predictions about economic problems and issues and find preconceived solutions to those problems. The economic model consists of a set of mathematical equations, variables, and logical or quantitative relationships between them. There are many ways to represent the economic model, including words or graphs, charts, or mathematical equations (as it is the main tool to do diagrams) and algebra. Algebra used by analysts because it gives more exact solutions to the work of the economic model (Granger, 1992).

The model could be simple or complex, such as the simple model, the inverse relationship between the demand for bananas and their price if other influences are constant, the higher price of bananas, the less demand for them. As for the complex model, for example, when economic analysts try to predict the true level of economic production, economics analysts used many complex formulas in that, such as nonlinear and interdependent differential equations (Kaewsuwan, 2002).

2.2.1 Type of Models

Models classified into many types according to all variable types or data representing them:

Considering all the variables are deterministic, the economics models classify to:

- Stochastic Models:

It comes from a Greek word στόχος (stókhos) which means 'aim, guess'. It is a mathematical object that contain a family of random variables, each random variable in the stochastic process is uniquely associated with an element in the set and economically model observable values over time and represented by time series.

- Non-stochastic models:

Non-stochastic model is a non-random and deterministic variable that it is a value known on time or in the past time such as temperature yesterday or month dates and could be purely qualitative or quantitative variables (Yates, 2018).

Considering all the variables are quantitative, the economics models classify to:

- Discrete model

It is analog of continuous model and it is formula suits the discrete data in the variables and data change only at a countable set of values in time for example integers and recurrence relations.

- Continuous modeling

It is a mathematical model which data uninterrupted in time; without cessation and data has divisibility of attributes and potentially infinite number such as differential equations and converse to discrete modeling (Barto, 1991).

Considering the model's intended purpose/function:

- Quantitative model

Quantitative model is data or information based on quantities and numbers, to obtain data or information by using a quantifiable measurement process. Such as numerical data, quantitative research, quantitative analysis, statistics, quantitative versus, quantification (science).

- Qualitative model

Qualitative model is data, which is subjective, descriptive, records qualities, and regards phenomenon which could be observed but difficult to measured, such as language (Britt, 2014).

Considering the model's ambit

- General equilibrium models

General equilibrium: it is economic model, which explains the behavior of supply, demand, and prices in the economy in the market to prove the interaction between supply, demand, and prices will result in equilibrium and those models show how the economy reacts to changes in technology and economic policy and external factors (Shoven, 1992).

- Partial equilibrium model

Partial equilibrium model is a type of equilibrium that takes and focuses only on a part of the market and represents the analysis of the given policy that affect directly in the market and does not account for the interactions between various markets in the economy (Adland, 2007).

Considering the economic agent's characteristics

- Rational agent models

It is an agent model sensible, reasonable, and having a good judgment which depends on performance measures, prior knowledge about environment percept sequence, actions, and its results in the aim to get useful information (Gonçalves, 2019).

- Representative agent models

Representative agent to refer to the typical decision-maker and it is a device used in a macroeconomic to find a good solution for specified microeconomic problems and show the relations between the variables in the models them such as the typical firm or the typical consumer (Jackson, 1985).

Considering the purpose, factors, represent, type and using of data

- Flow chart

Flow chart is a diagram or graphical representation that explained how the economy planned, organized, connected in the system, and shows the relationships between participants in the different stages in the system.

As it is shown in figure (3) the circular flow diagrams are a graphical model show how the economics factors (firms, households, factors markets, product markets) interact with each other. The products market is the place where the firms put and sell their produced goods and services that buying by householders in the market in exchange of money.

The factors market is the place where the firms buy their production processes needs from householders in exchange of money (Jan, 2018).

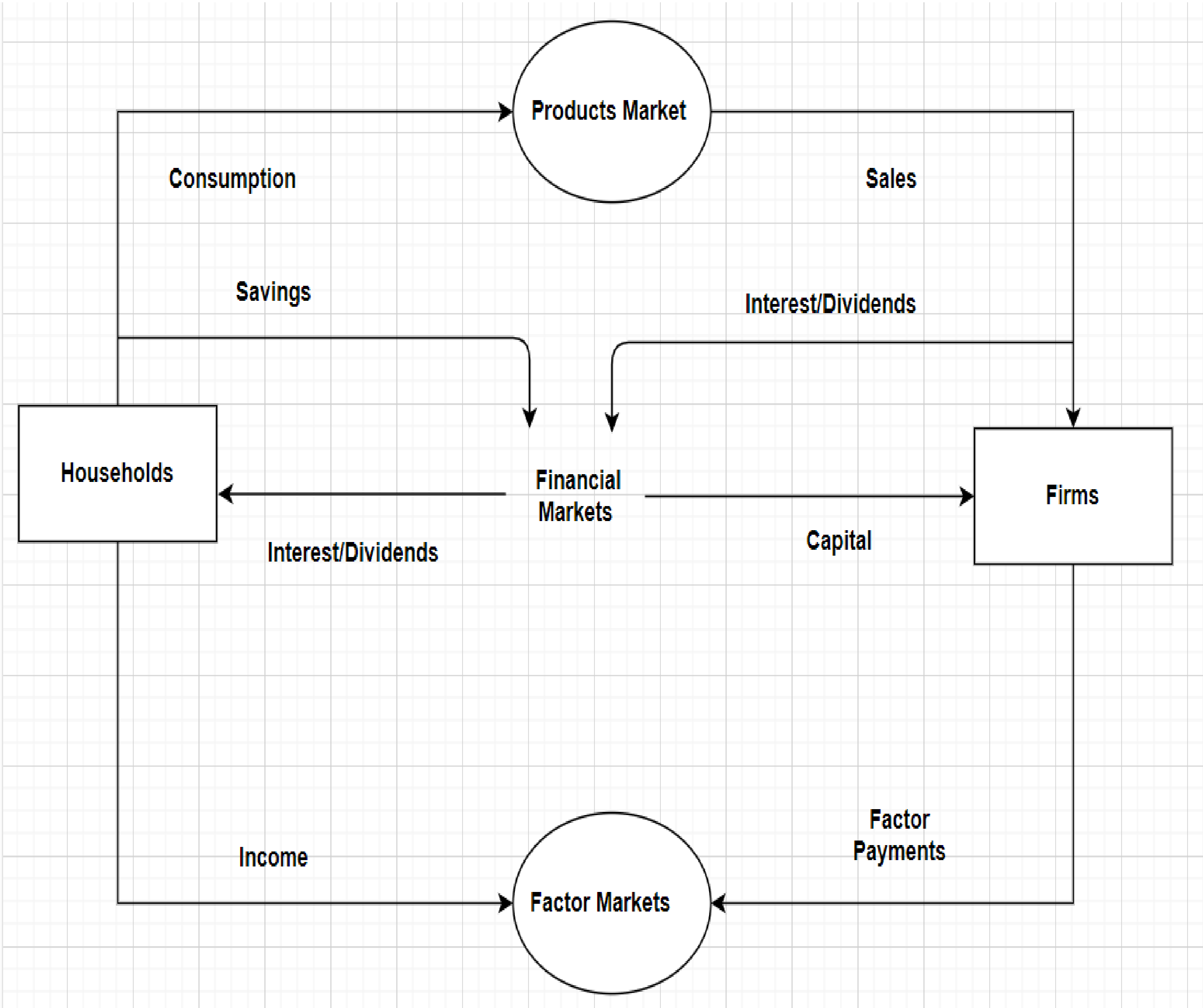

Figure 3 Circular-flow diagram

(Hosny, 2013)

- Graph

Graph is a planned drawing or diagram consisting of lines or sets of numbers which show how they relate to each other and the relationships between them and it has many types such as a single variable, time-series graphs, bar graphs, and others.

As it is shown in figure (4), the price of apple stocks is shown on the vertical axis and time series of goods are shown on the horizontal. From the graph we noticed that prices of the apple stocks change more towards an increase during the time period (Dawson, 2016).

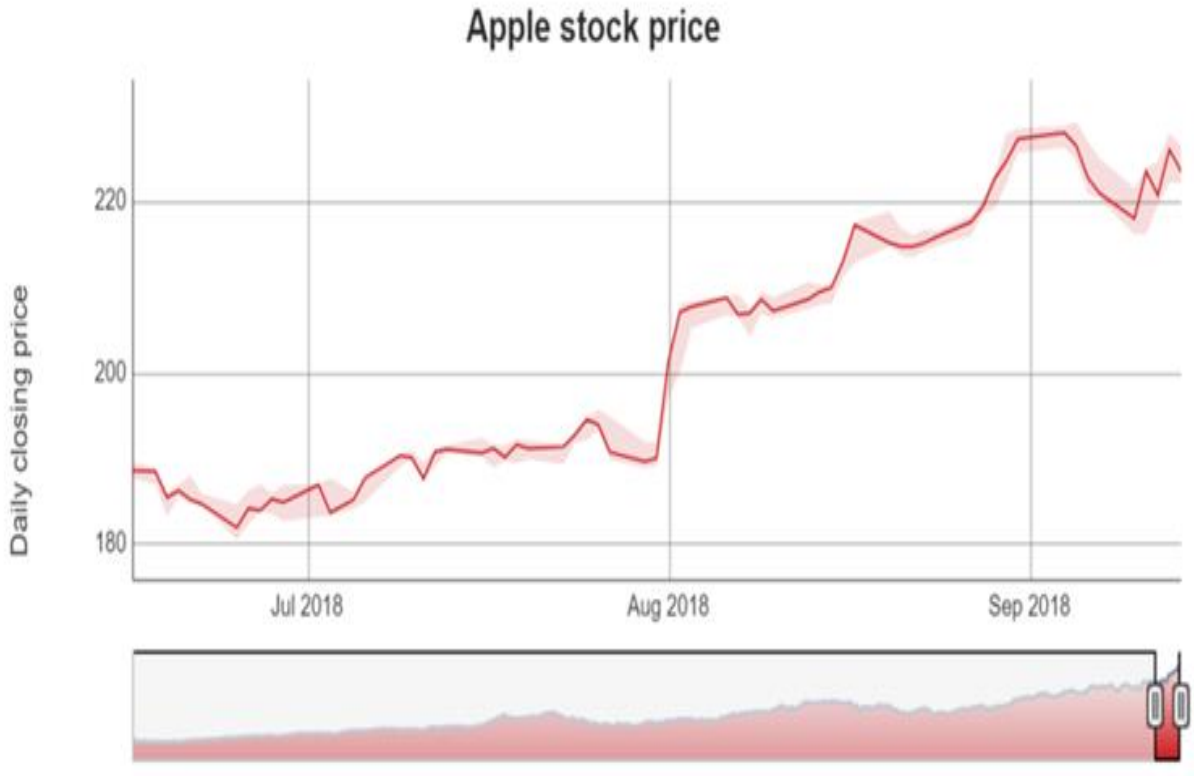

Figure 4 Time series

(Chan, 2018)

- Mathematical model

Mathematical model is a description of the behavior of the system by using mathematical language, formulation, equation, and mathematical concepts.

As is shown in the figure (5), the mathematical models provide answers, insight, and useful guidance for primary application by representing the problems in mathematical formulation from which it will be analyzed to numerical and theoretical analysis to help to solve the problems.

The model represents two ways of communication activities in the flow of the information and show the interacting between sources of information and the nodes.

The models conation of 6 source of information: reports, problem statement, numerical methods, programs, theories and mathematical model (Christof Eck, 2017).

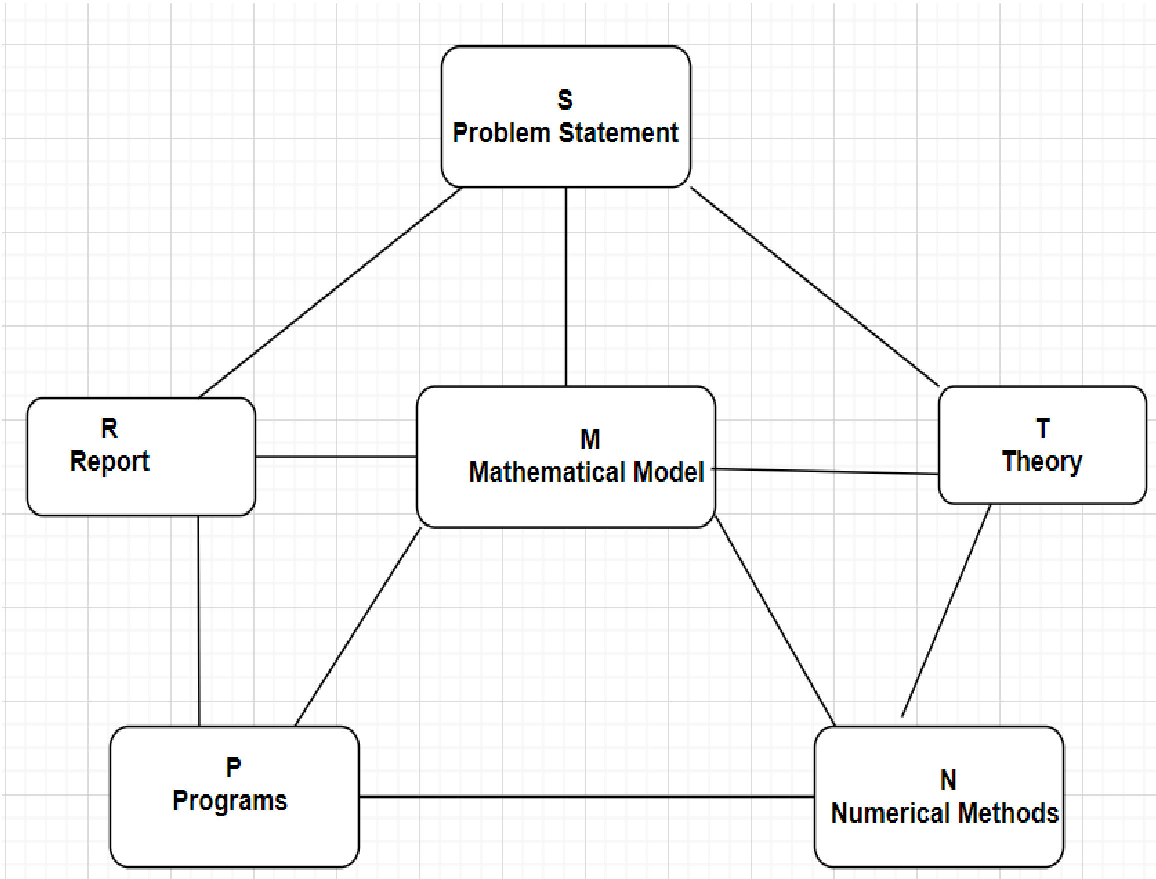

Figure 5 Mathematical Model

(J. Kallrath, 2004)

2.3 Time series analyzing

The time series modeling and analysis represent the important process and are very often demanded in various areas of our life. The answer for the common question of why we need the time series analysis, is that the analysis of time series often leads to better decision making in the future in most cases, also it is needed to predict future values of time series to know about future progress, the available opportunities in the future can be useful to see how given variables change over time and also to examine the changes between the chosen data points and changes shifts in other variables and compare them.

Time Series Forecasting

Time series forecasting uses information regarding associated patterns and historical values to predict future activities. This relates often to issues of seasonality, cyclical fluctuation analysis, and trend analysis. success in the times series forecasting is not guaranteed as with all forecasting methods (Cheryl L. Jennings, 2008).

2.3.1 History of time series analysis

This chapter will provide a brief history about time series development:

- The 1920s and 1930s was the early started of theoretical development on time series analysis by using the stochastics process.
- During the same period was introduced (ARMA) autoregressive moving average to remove periodic fluctuations in the time series. the stationary series models (ARMA) models were introduced by Herman Wold, but those models fail to derive a likelihood function to enable maximum likelihood (ML) estimation of the parameters in that time.
- In 1970 was the time the classic book "Time Series Analysis" was published by G. E. P. Box and G. M. Jenkins containing the full modeling procedure for individual series: specification, estimation, diagnostics, and forecasting.
- The (ARIMA) Autoregressive integrated moving average model was developed by Gwilym Jenkins George Box in the 1970s as an attempt of using the mathematical approach to describe the changes in time series. The names ARIMA models and Box-Jenkins model are mentions as synonyms.
- The common models and techniques used for forecasting and seasonal adjustment nowadays are using Box-Jenkins models, also the other models for times series forecasting are trace back to Box-Jenkins models techniques (Klein, 2005).

2.3.2 Time series applications

The time series models usage has twofold:

- Provide an understanding for the produced observed data the underlying their structure and forces.
- Fit a model and proceed to monitor, feedback, or even forecasting, and feedforward control.

Many applications are used in time series analysis, and examples of these applications are:

- Economic and finance forecasting.
- Price estimation.
- Sales forecasting.

- Budgetary analysis.
- Stock market analysis.
- Yield projections.
- Cluster monitoring.
- Health monitoring.
- Traces.
- Process and quality control.
- Inventory analysis.
- Weather analysis.
- Network data analysis. (Robert H. Shumway, 2017)

2.4 Models

This chapter will be a brief introduction about (AR) autoregressive model, (MA) moving average model, (ARMA) autoregressive–moving-average model, the Box-Jenkins Model, (p, d, q) orders, and (ARIMA) autoregressive integrated moving average model with their equations, processes, and some examples.

2.4.1 Definition of ARIMA model

ARIMA (autoregressive integrated moving average) modeling is an economics statistical flexible and widely used models. the models can be used for forecasting, a better understanding of the data, and univariate times series analysis. The analysis can provide a short-run forecast of large data (Douglas C. Montgomery, 2008).

The ARIMS process contains three processes (AR) autoregressive process, differencing process, and (MA) moving average process.

2.4.2 (AR) Autoregressive process

- **Definition:**

Autoregressive (AR) model uses the forecasts in the past to predicts future values and behavior. The AR model also called Markov models, transition models, or conditional models.

The notation AR(p) refers to the process of order (p) in the following equation, it can be shown by Yue-Walker system equations:

$\rho_1 = \varphi_1 \quad + ...+ \ \varphi_p\rho_{p-1}$

$\rho_2 = \varphi_1\rho_1 + \varphi_2 + ... + \varphi_p\rho_{p-2}$

$= (\rho_p = \varphi_1\rho_{p-1} + \varphi_2\rho_{p-2} + ... + \varphi_p)$ **Equation 1** Yue-Walker system

$= (\rho_p = \varphi_1\rho_{p-1} + \varphi_2\rho_{p-2} + ... + \varphi_p)$ **Equation 2** Yue-Walker system

The solution of the system by using autocorrelation functions.

- **Basic concepts AR processes:**

The variable of predicted dependent designed as a random error term plus a linear function of independent variable:

$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ **Equation 3** AR model linear function

Where:

- Y is a variable. X is variable.
- i = time.
- $\boldsymbol{\varepsilon_i}$ random error term.

$y_i = \emptyset_0 + \emptyset_1 \, y_i - 1 + \varepsilon_i$ **Equation 4** AR model first order

AR (1) Autoregression - first order:

Where:

- Y is a variable and I = Time.
- I-1 is a linear function of (y) value.
- ε_i random error terms.

ε_i~ N(0, σ)

COV $(\varepsilon_i, \varepsilon_j) = 0$ for i ≠ j COV (ε_i,y_j)=0 for all i,j

- The (y_i) mean:

$$\mu = \frac{\emptyset_0}{1 - \sum_{j=1}^{\rho} \emptyset_j}$$

- The (y_i) variance:

$$var(y_i) = \frac{\sigma^2}{1 - \emptyset_1^2}$$

- The autocorrelation (lag h):

$$\rho_h = \emptyset_1^h$$

(Napier, 2020)

- The error terms ϵt:

Constant variable and mean is 0:

$E(\epsilon_t)=0$

$Var(\epsilon_t)= \sigma\ 2/1- \rho\ 2$

The adjacent error terms covariance of:

$Cov(\epsilon_t, \epsilon_{t-1})= \rho\ (\sigma\ 2/1- \rho\ 2)$

The adjacent error terms correlation:

$$Corr(\epsilon_t, \epsilon_{t-1})= Cov(\epsilon_t, \epsilon_{t-1})/ \sqrt{Var(\epsilon_t), Var(\epsilon_t, \epsilon_{t-1})}$$

Equation 5 εi random error terms

AR (2) Autoregression – second order

$$\gamma_i = \emptyset_0 + \emptyset_1\gamma_{i-1} + \emptyset_2\gamma_{i-2} + \boldsymbol{\varepsilon_i}$$

Equation 6 AR model-second order

AR(p) Autorepression –(P) order

$$\gamma_i = \emptyset_0 + \emptyset_1\gamma_{i-1} + \emptyset_2\gamma_{i-2} + \cdots + \emptyset_\rho\gamma_{i-\rho} + \varepsilon_i$$

Equation 7 AR model P-order

(Idris, 2014)

Using AR process terms in the ARIMA models when:

- When the autocorrelation decaying towards zero in the autocorrelation function (ACF).
- The quick cut off in the partial autocorrelation (PACF) plots towards zero.
- Positive lag-1 in the autocorrelation function (ACF) of stationary series (Ratnadip Adhikari, 2013).

Example:

Let find autoregression for wages from 2011 to 2020 in Sweden as is showing the table (1).

Table 1 Wages in Sweden

Year	Wages	Year	Wages
2011	89.582	2016	90.011
2012	89.551	2017	90.142
2013	89.408	2018	90.404
2014	89.668	2019	90.432
2015	89.746	2020	90.465

Source: (Bank, 2021)

Solution:

We can find the AR autoregression by using Microsoft Excel in the following steps:

- Firstly, preparing the table in the Microsoft Excel.
- Then manipulating the table to get (Y_t, Y_{t-1}, Y_{t-2}, Y_{t-3}) as shown in the following table:

Table 2 AR model Y_t

Years	Y_t	Y_{t-1}	Y_{t-2}	Y_{t-3}
2014	89.668	89.408	89.551	89.582
2015	89.746	89.668	89.408	89.551
2016	90.011	89.746	89.668	89.408
2017	90.142	90.011	89.746	89.668
2018	90.404	90.142	90.011	89.746
2019	90.432	90.404	90.142	90.011
2020	90.465	90.432	90.404	90.142

Source: Author

- Last step by using data analysis in Microsoft Excel to find the final solution as shown in the following tables (3, 4, 5):

Table 3 AR model results

Regression Statistics	
Multiple R	0.99
R Square	0.98
Adjusted R Square	0.96
Standard Error	0.06
Observations	**7**

Table 4 ANOVA

Anova	df	SS	MS	F	Significance F
Regression	3	0.64	0.21	55.16	0.004012
Residual	3	0.01	0.003		
Total	6	0.65			

Table 5 Statistics table

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	30.63	9.44	3.24	0.05	0.59	60.66	0.59	60.66
Yt-1	0.72	0.17	4.23	0.02	0.18	I.26	0.18	I.26
Yt-2	0.56	0.22	2.55	0.08	-0.14	I.26	-0.14	I.26
Yt-3	-0.62	0.23	-2.65	0.08	-1.36	0.12	-1.36	0.12

Source: Author

- If the (P-Value ≤ Alpha (0.05)) then the results significant and we reject the null hypothesis
- If the (P-value > Alpha (0.05)) then the result is statistically non-significant, and we fail to reject the null hypothesis.

As it is shown in figure (6), the wages in Sweden are show on the vertical axis and time series years is shown on the horizontal. From the graph we noticed that the wage in Sweden is increasing during time (Source: Author).

Figure 6 Wages in Sweden

(Source: Author (MS excel))

2.4.3 Differencing Process:

It is mentioned as a letter (d) degree of differencing and it is the most important process to prepare data to be used in ARIMA models. By using this process, it can transform a non-stationary times series to stationary time series (series without seasonality or trend).

The first differencing value:

It can be calculated by the difference between the previous time and the current time. We find the second differencing if the first differencing values fail to go around a variance and constant mean by using the first differencing values. We repeat the process till we have a stationary series (Meyler, 1998).

Rules of Identifying the (d) differencing order:

- First rule: it needs a higher differencing order if the time series has a positive autocorrelation and a high number of lags.
- Second rule: the time series does not need a higher differencing order. If the autocorrelation of lag-1 is negative, zero or the autocorrelations are all small.
- Third rule: the differencing order where the standard deviation is lowest is often the optimal differencing order.
- Fourth rule: (mean-reverting) the original series is stationary when there are no differencing orders. The original series has one differencing order when the model has a constant average trend. The original series has a time-varying when the model has two orders of total differencing.
- Fifth rule: the model with a constant term (non-zero mean value) when there is no differencing order in the model. The model without a constant term when there is two total differencing order in the model. We include in a non-zero average trend when the model with a constant term and one total differencing order (Nau, 2020).

2.4.4 (MA) Moving-average process

Definition:

MA model: moving average model or in other words moving average process. It is common and specifies a model of analysis, observing and calculate the output variable between similarities current values and various past errors values. We use this model to account for the unpredictable events in time series analysis meaning.

(MA) moving average process in statistics means calculation by creating a series of averages of different subsets of the full data set used to analyze data points.

Moving average (MA) process in finance means technical analysis of stock indicators and it used to create updated average prices that helps smooth out prices.

MA (q) moving averages is the order of (q) in the ARIMA models and its sums the (q) most rent errors plus the current random error (Chan, 2010).

(MA) Moving average process formulas:

The first order of the moving average model is shown as:

$$x_t = \mu + \omega_t + \emptyset_1 \omega_{t-1}$$

Equation 8 MA model 1st order

The second order of the moving average model is shown as:

$$x_t = \mu + \omega_t + \emptyset_1 \omega_{t-1} + \emptyset_2 \omega_{t-2}$$

Equation 9 MA model 2nd order

The (q) order of the moving average model is shown as:

$$x_t = \mu + \omega_t + \emptyset_1 \omega_{t-1} + \emptyset_2 \omega_{t-2} + \cdots + \emptyset_q \omega_{t-q}$$

Equation 10 MA model q order

Where:

- X_t = the values of "X" in the current period – t.
- μ=constant factor.
- θ_1 = constant factor.
- w_t and w_{t-1}= represent the residuals for the current and the previous period.
- MA(q) process mean is μ.

MA(q) variance is:

$$var\ \gamma_{i=}\sigma^2\ (1 + \theta_1^2 + \cdots + \theta_q^2)$$

Equation 11 MA(q) variance

MA (1) autocorrelation process is:

$$\rho_{1=} \frac{\theta_1}{1 + \theta_1^2} \quad \rho_{h=0\ for\ h>1}$$

Equation 12 MA (1) autocorrelation

The of an MA (2) autocorrelation process is:

$$\rho_{2=} \frac{\theta_1 + \theta_1\theta_2}{1 + \theta_1^2 + \theta_2^2} \qquad \rho_{2=} \frac{\theta_2}{1 + \theta_1^2 + \theta_2^2} \quad \rho_{h=0\ for\ h>2}$$

Equation 13 MA (2) autocorrelation

MA(q) autocorrelation function process is:

$$\rho_{h=} \frac{\theta_h + \sum_{j=1}^{q-h} \theta_j \theta_{j+h}}{1 + \sum_{j=1}^{q} \theta_j^2}$$

Equation 14 MA (q) autocorrelation

(Hyndman, 2011)

Using Ma process terms in the ARIMA models when:

- Lag-1 we have Negative autocorrelation.
- After a few- lags the autocorrelation functions drop sharply.
- More gradually decreasing of a partial autocorrelation (Ratnadip Adhikari, 2013).

Example:

The following table (6) is showing the monthly sales of a company.

Calculate the moving average of January 2020:

Table 6 Monthly Sales in a company

Year	Month	Sales CZK	Year	Month	Sales CZK
2019	January	36562	2019	August	35455
2019	February	29645	2019	September	45454
2019	March	54555	2019	October	30556
2019	April	30256	2019	November	54585
2019	May	54515	2019	December	65565
2019	June	38454	2020	January	?
2019	July	54545			

Solution:

First, we will use the formula:

$F_t = (A_{t-1} + A_{t-2} + A_{t-3})/3$

F_t = (sales in December + sales in November + sales in October)/3

By using Microsoft Excel, we will get the table (7):

Table 7 Moving average

Year	Month	Sales CZK	MA	Year	Month	Sales CZK	MA
2019	January	36562		2019	August	35455	49171
2019	February	29645		2019	September	45454	42818
2019	March	54555		2019	October	30556	45151
2019	April	30256	40254	2019	November	54585	37155
2019	May	54515	38152	2019	December	65565	43532
2019	June	38454	46442	2020	January	?	50235
2019	July	54545	41075				

The moving average (MA) of January of 2020 =50235 CZK

(Source: Author (MS excel))

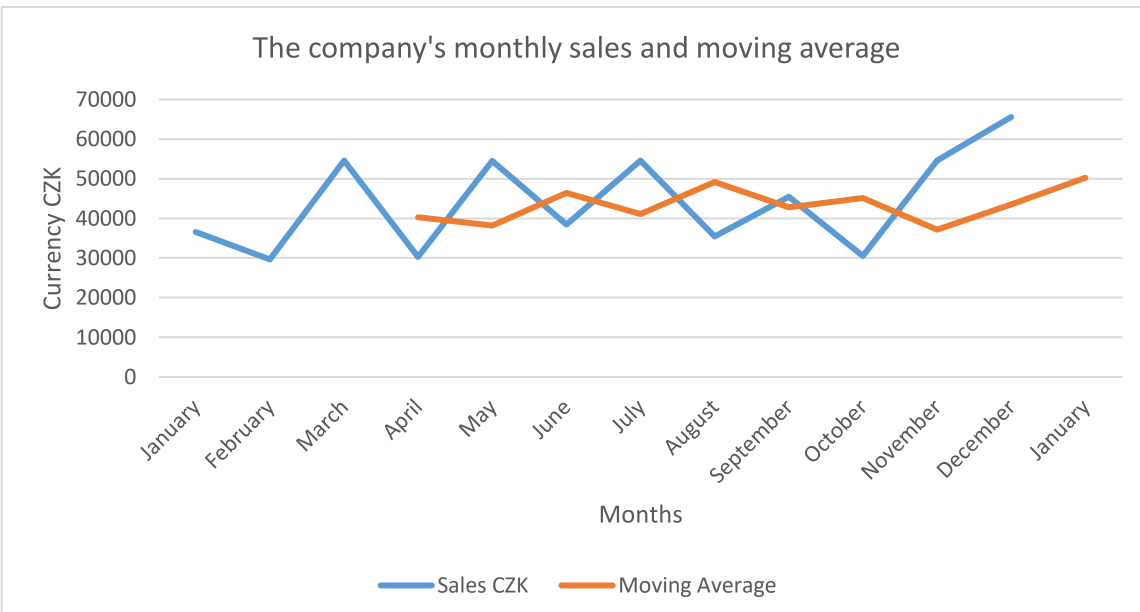

Figure 7 Monthly sales for a company and moving average

(Source: Author (MS excel)

As it is shown in figure (7), the monthly sales are show on the vertical axis and the months is shown on the horizontal. From the figure (7) we noticed that the sales from November to the end of the year is increasing during time.

.

From the figure (7) the moving average process of the monthly sales in the company is shown on the vertical axis and the months are shown on the horizontal. From the figure (7) we noticed that the monthly sales look more smoothly after the moving average process (Source: Author).

2.4.5 ARMA model

(ARMA) autoregressive moving average model or (ARMA) process of order (p, q):

It is combining AR (p) autoregressive process terms and MA (q) moving average processes terms together. We used ARMA (p, q) because it is a stationary random sequence, a more flexible representation of correlation in the short-term, and described by fewer parameters than the pure AR (p) and MA (q) process. Where p and q mean:

- p = the order of autoregressive polynomial
- q = the order of the moving average polynomial

and it can be expressed by the given equation:

$$\gamma_i = \emptyset_0 + \emptyset_1 \gamma_{i-1} + \emptyset_2 \gamma_{i-2} + .. + \emptyset_\rho \gamma_{i-\rho} + \varepsilon_i + \emptyset_1 \varepsilon_{i-1} + .. + \emptyset_q \varepsilon_{i-q}$$

Equation 15 ARMA model equation

where:

- ϕ = constant.
- φ = the autoregressive model's parameters.
- θ = the moving average model's parameters.
- ε = error terms (white noise).
- Y = variable.
- i = time (Dickey, 2018).

2.4.6 ARIMA model

(ARIMA) autoregressive integrated moving average models: in theory, ARIMA models mean the most general models for forecasting a time series which time series should be stationary which means over time it should show similar behavior in form of the constant, autocovariance, expected value, and in conjunction with nonlinear transformations such as deflating or logging. In the

ARIMA models, we called a random variable is stationary in the series time if statistical properties are constant over time and stationary time series has no trend. By allowing the differencing of the time series the ARMA model extends to nonstationary times series and the result we can have ARIMA models.

The ARIMA (p, d, q): known as non-seasonal models with three parameters (p, d, q):

- p = autoregressive model order (number of terms)
- d = differencing degree
- q = moving average order (number of lagged forecast errors) (HOBOKEN, 2008).

Example:

Suppose X_t is non-stationary series, we will follow the following step:

First X_t to ΔX_t:

To be stationary series, we will take the first difference, the ARIMA (p, 1, q) model will be:

$$\Delta \mathrm{X}_{\mathrm{t}} = c + \alpha_1 \Delta \mathrm{X}_{\mathrm{t-1}} + \cdots + \alpha_p \Delta \mathrm{X}_{\mathrm{t-p}} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \cdots - \theta_{\mathrm{q}} \varepsilon_{t-q},$$

where $\Delta \mathrm{X}_{\mathrm{t}} = \mathrm{X}_{\mathrm{t}} - \mathrm{X}_{\mathrm{t-1}}$.

In other case if p = q = 0 in the equation then the classified of ARIMA (0, 1, 0), the model becomes a random walk model.

Equation:

We obtain a non-seasonal ARIMA model by combining a moving average model and differencing with the autoregression model. The equation of time series forecasting ARIMA is a linear equation as it is shown:

$$\acute{\gamma}_t = c + \emptyset_1 \acute{\gamma}_{t_{-1}} + \cdots + \emptyset_\rho \acute{\gamma}_{t_{-\rho}} + \emptyset_1 \varepsilon_{t-1} + \cdots + \emptyset_q \varepsilon_{t-q} + \varepsilon_t$$

Equation 16 ARIMA equation

Where:

- y'_t = differenced time series in time t.
- ε_t = is white noise t time.
- C = constant.

ARIMA models: Special cases of ARIMA (p, d, q):

Table 8 Cases of ARIMA model

NO	Description	ARIMA (p,d,q)
1	Autoregression	ARIMA (p,0,0)
2	Random walk with drift	ARIMA (0,1,0) with a constant
3	Random walk	ARIMA (0,1,0) with no constant
4	Moving average	ARIMA (0,0,q)
5	White noise	ARIMA (0,0,0)

(Athanasopoulos, 2018)

ARIMA modeling stages:

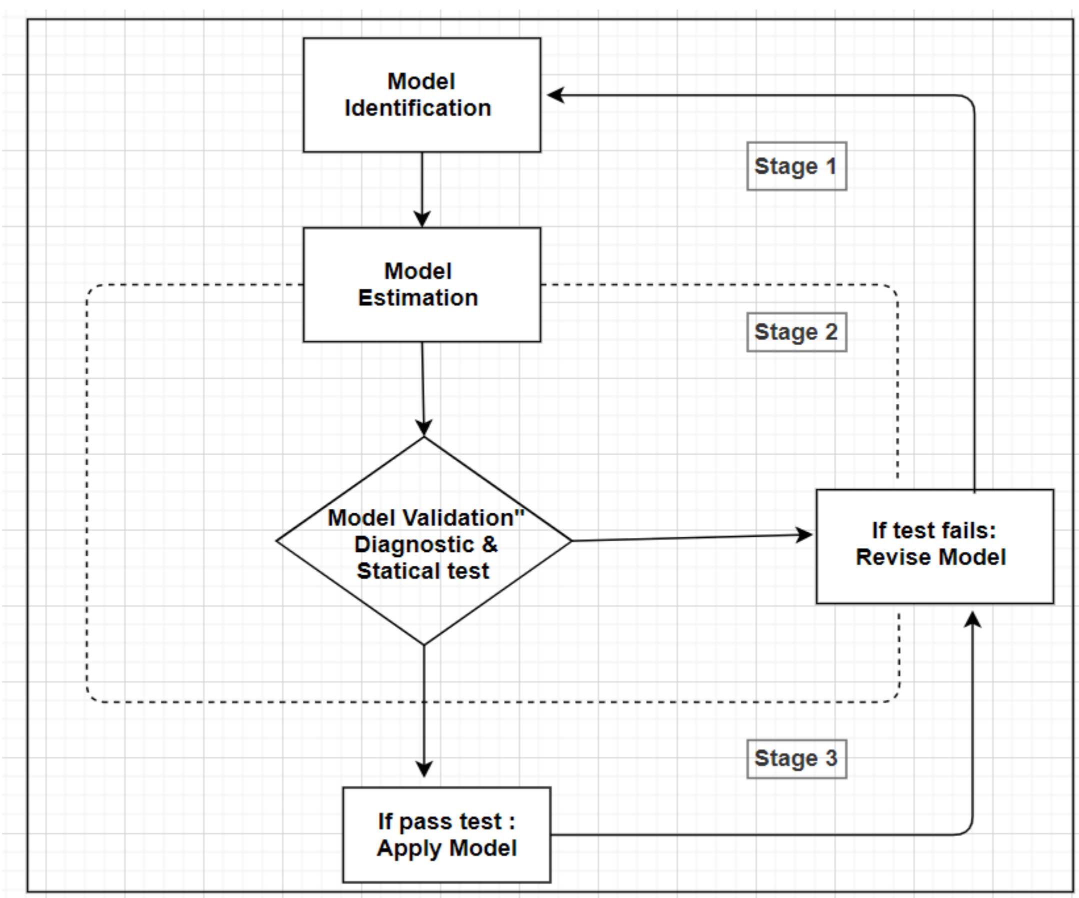

Figure 8 ARIMA model stages

(Srivastava, 2016)

As is shown in the figure (8), the ARIMA model contains three stages, each stage has its own processes.

The first stage of the model is responsible in the identification of the model, when the stage second is responsible in the estimation, validation, diagnostic and test of the model and the third stage is responsible to make the decision of applying the model or refuse it, if the model pass the testing process from the second stage then we will apply the model if the model didn't pass the test then the model refused and needs a revising.

The ARIMA model processes are:

- Identification model.
- Estimation model.
- Validation, diagnostic, statistical model.
- If the test fails, then revise the model.
- If the test passes, then apply the mode as showing in the following figure (8) (Eberly, 2021).

2.4.7 SARMA and SARIMA

SARMA

A seasonal autoregressive moving average (SARMA): its special case and extension of the ARMA model. Where the polynomials of the (AR) autoregressive model and (MA) moving average are factored into (B) monthly polynomial and (B12) seasonal polynomial or annual polynomial. It is useful to forecast univariate data with seasonality and trends in time series in other words we use SARMA with ARMA models that have a seasonal component.

Equation:

SARMA (p, q) $\times$ (P, Q) 12

$$\emptyset(B)\Phi(B^{12})(\gamma_n - \mu) = \psi(B)\psi(B^{12})\epsilon_n$$ **Equation 17** SARMA

The seasonal and non-seasonal components:

Seasonal components:

$$Seasonal\ AR = \Phi(B^S) = 1 - \Phi_1 B^S - \cdots - \Phi_p B^{PS}$$
$$Seasonal\ MA = \theta(B^S) = 1 + \theta_1 B^S - \cdots - \theta_Q B^{QS}$$

Equation 18 Seasonal components

Non seasonal AR:

$$AR: \emptyset(B) = 1 - \emptyset_1 B - \cdots - \emptyset_P B^P$$
$$MA: \theta(B) = 1 + \theta_1 B - \cdots - \theta_q B^q$$

Equation 19 Non-seasonal components

SARIMA

A seasonal autoregressive integrated moving average (SARIMA): It is a method useful to forecast univariate data with seasonality and trends in time series and it is an extension of the ARIMA model. We use SARIMA models with seasonal component because we notice the limitation of the ARIMA model when it comes to time series with repeating cycle and at the same time it cannot handle forecasting time series that have seasonal components for that we use the SARIMA model for supporting modeling seasonal in the time series forecasting.

Equation:

The equation of SARIMA contains both seasonal multiplies with non-seasonal factors $ARIMA\ (p, d, q)\ x\ (P, D, Q)S$

Where

d = differencing (non-seasonal), p = AR order (non-seasonal), q = MA order (non-seasonal), S = time of repeating (seasonal pattern), P = AR order (seasonal), Q = MA order (seasonal) and D = seasonal differencing (Hylleberg, 1992).

2.4.8 ARMAX, ARIMAX and SARIMAX

ARMAX

The ARMAX is an extension of the ARMA model and ARIMAX is an extension of the ARIMA model with adding an independent variable we called it an exogenous variable to improve the forecasting time series performance model and explain the dependent variable.

Equation:

$$P_t = c + \beta X + \emptyset_1 P_{t-1} + \theta_1 \epsilon_{t-1} + \varepsilon_i$$

ARIMAX

Equation:

$$\Delta P_t = c + \beta X + \emptyset_1 \Delta P_{t-1} + \theta_1 \epsilon_{t-1} + \varepsilon_i$$

Where

P_t and P_{t-1} = time (current and past period), $\epsilon_t$and ϵ_{t-1} = the error terms, c = constant factor, ϕ_1 and θ_1= estimating parameters, β =coefficient X= exogenous variable.

SARIMAX

Seasonal autoregressive integrated moving averages with exogenous regressors SARIMAX: it is an extension of the ARIMA model and ARIMAX model and it is used for the data that have (seasonality) seasonal cycle. The SARIMAX does not require only the set of (p, d, q) as ARIMA model but required more X exogenous regressors and (P, D, Q) s the seasonality aspects (Stylianos Vagropoulos, 2016).

2.4.9 The Box-Jenkins Model

It is a mathematical model created by the two statisticians George Box and Gwilym Jenkins in 1970. This model created to forecasting the data from specified in the time series.

The Box-Jenkins model good for forecasting time series data that have short-term period and we can apply (ARIMA) autoregressive integrated moving average or (ARMA) autoregressive moving average to search for the best fit of time series with past value to the times series model.

The Box-Jenkins model methodology:

The methodology of this model refers to the following steps:

- Identification.
- Estimation.
- Diagnostic checking.
- Modeling.
- Use model.

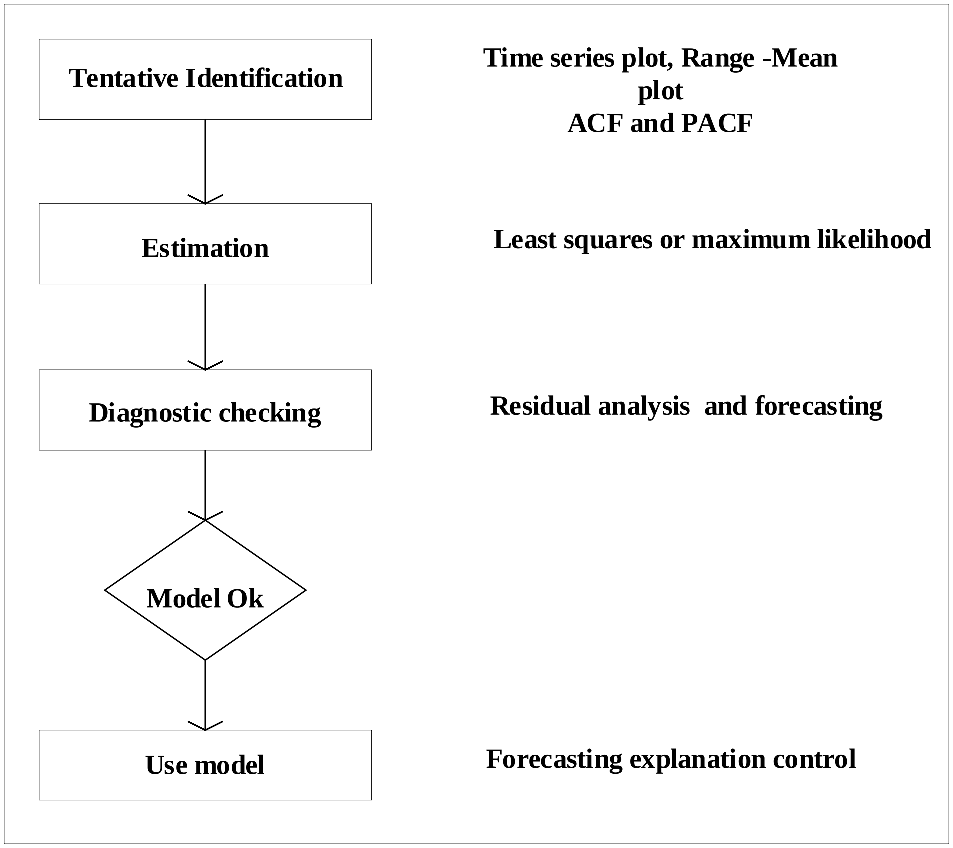

Figure 9 The Box-Jenkins Model

(Pelgrin, 2011)

As is shown in the figure (9), the Box-Jenkins model processes it is similar to the ARIMA model process.

The process of building the Box-Jenkins model is shown in the following step:

- Identification of model it is the step of making the plots such as time series plot, range-mean plot, (PACF) a partial auto-correlation function plot and (ACF) Autocorrelation plot.
- Estimation it is the step of estimate the model by the maximum likelihood or least square processes.
- Diagnostic checking by using residual and forecasting analysis.
- Testing of the model if the model passes the testing, then it can be used in forecasting if the model did not pass the testing it is refused to be used and need revising.
- After the model pass the test the Box-Jenkins model can be used for explaining, controlling, and forecasting processes (Anderson, 2003).

2.5 Applying ARIMA model example

In the approach of time series forecasting. Based on this data, we will propose the ARIMA model by using IBM SPSS statistic method and then use them to forecast the Qatari (GNI) gross national income for the next ten years (from 2019 to 2029).

In the table (9), there is shown the Qatari GNI (from 2000 to 2019).

Table 9 Qatari GNI

Year	GNI	Year	GNI
2000	5,28E+10	2010	2,20E+11
2001	5,85E+10	2011	2,61E+11
2002	6,32E+10	2012	2,91E+11
2003	6,88E+10	2013	3,06E+11
2004	7,97E+10	2014	3,03E+11
2005	8,29E+10	2015	2,33E+11
2006	1,17E+11	2016	2,19E+11
2007	1,42E+11	2017	2,49E+11
2008	1,69E+11	2018	2,54E+11
2009	1,83E+11	2019	2,60E+11

Source: World bank

Solution:

In the table (10), there is shown the Qatari GNI model description:

Table 10 Data description

Model Name	Qatari GNI
Series or Sequence	GNI
Transformation	None
Non-Seasonal Differencing	0
Seasonal Differencing	0
Length of Seasonal Period	No periodicity
Horizontal Axis Labels	Year
Intervention Onsets	None
Reference Lines	None
Area Below the Curve	Not filled

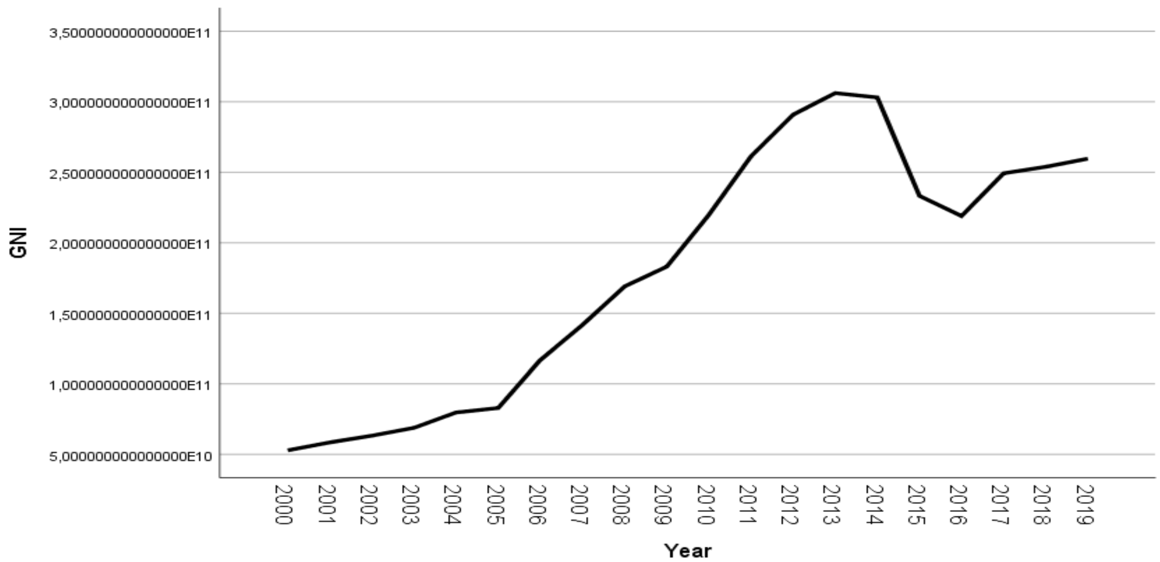

Figure 10 GNI

Source: SPSS

As it is shown in figure (10), GNI are show on the vertical axis and the years is shown on the horizontal.

ARIMA (0,1,0)

In the following table (11), there is shown the GNI statistics:

Table 11 GNI statistic

Model	Number of Predictors			Model Fit statistics	Ljung-Box Q(18)			Number of Outliers
				R^2	Statistics	DF	Sig.	
GNI-Model_1	0	0,92	48,01	10,53	10,53	18	0,913	0

In the table (12), there is shown the GNI difference table:

Table 12 GNI difference

			Estimate	SE	t	Sig.
GNI	No Transformation	Constant	1,09E+10	5,72E+09	1,9	0,1
		Difference	1			

In the table (13), there is shown the GNI statistics fit:

Table 13 GNI fit statistic

Fit Statistic	Mean	Minimum	Maximum	Percentile	
				5	10
Stationary R^2	-6,66E-16	.	-7,00E-16	-7,00E-16	-7,00E-16
R-squared	9,20E-01	.	9,00E-01	9,00E-01	9,00E-01
RMSE	2,46E+10	.	2,00E+10	2,00E+10	2,00E+10
MAPE	9,01E+00	.	9,00E+00	9,00E+00	9,00E+00
MaxAPE	3,46E+01	.	3,00E+01	3,00E+01	3,00E+01
MAE	1,64E+10	.	2,00E+10	2,00E+10	2,00E+10
MaxAE	8,07E+10	.	8,00E+10	8,00E+10	8,00E+10
Normalized BIC	4,80E+01	.	5,00E+01	5,00E+01	5,00E+01
	25	50	75	90	95
Stationary R^2	-7,00E-16	-7,00E-16	-7,00E-16	-7,00E-16	-7,00E-16
R-squared	9,00E-01	9,00E-01	9,00E-01	9,00E-01	9,00E-01
RMSE	2,00E+10	2,00E+10	2,00E+10	2,00E+10	2,00E+10
MAPE	9,00E+00	9,00E+00	9,00E+00	9,00E+00	9,00E+00
MaxAPE	3,00E+01	3,00E+01	3,00E+01	3,00E+01	3,00E+01
MAE	2,00E+10	2,00E+10	2,00E+10	2,00E+10	2,00E+10
MaxAE	8,00E+10	8,00E+10	8,00E+10	8,00E+10	8,00E+10
Normalized BIC	5,00E+01	5,00E+01	5,00E+01	5,00E+01	5,00E+01

In the table (14), there are shown the ACF and PACF residual values:

Table 14 GNI ACF and PACF

Residual ACF			Residual PACF			Residual ACF			Residual PACF		
Model	ACF	SE	Model	PACF	SE	Model	ACF	SE	Model	PACF	SE
1	0,41	-0,02	1	0,42	-0,24	10	0,23	0,27	10	0,23	0,23
2	0,23	0,27	2	0,23	0,23	11	0,41	-0,02	11	0,42	-0,24
3	0,41	-0,02	3	0,42	-0,24	12	0,23	0,27	12	0,23	0,23
4	0,23	0,27	4	0,23	0,23	13	0,41	-0,02	13	0,42	-0,24
5	0,41	-0,02	5	0,42	-0,24	14	0,23	0,27	14	0,23	0,23
6	0,23	0,27	6	0,23	0,23	15	0,41	-0,02	15	0,42	-0,24
7	0,41	-0,02	7	0,42	-0,24	16	0,23	0,27	16	0,23	0,23
8	0,23	0,27	8	0,23	0,23	17	0,41	-0,02	17	0,42	-0,24
9	0,41	-0,02	9	0,42	-0,24	18	0,23	0,27	18	0,23	0,23

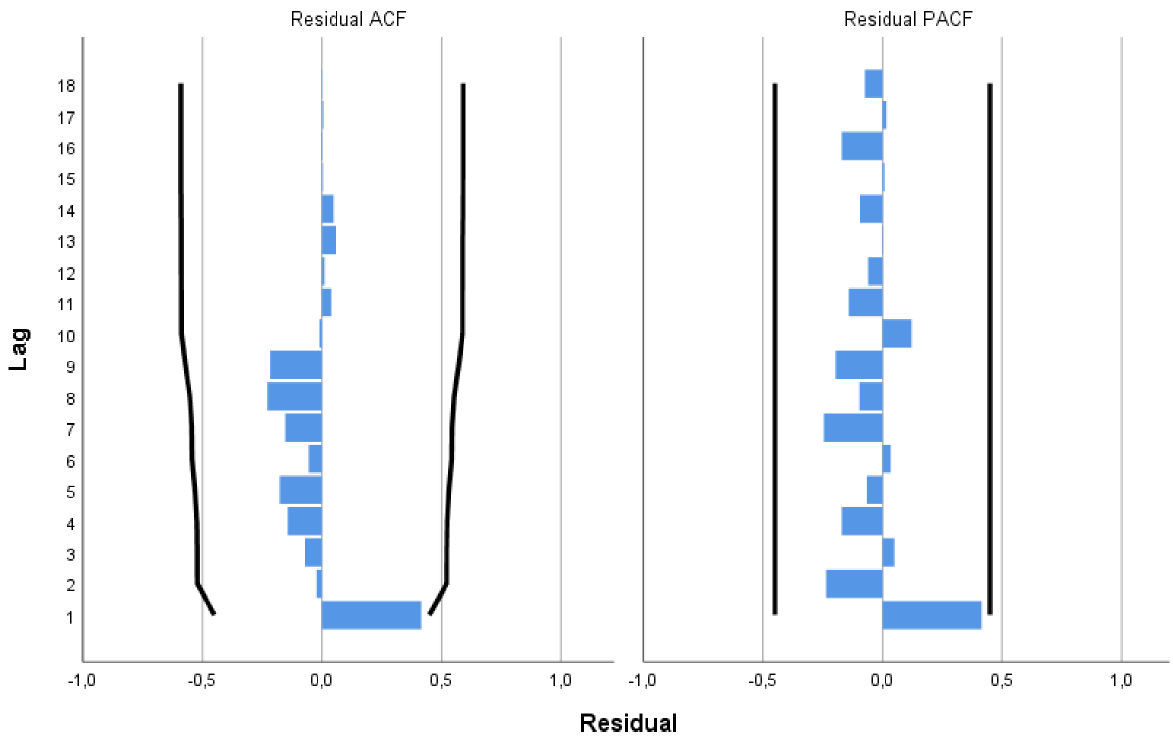

Figure 11 Residual ACF and PACF

(Source: SPSS)

As it is shown in figure (11), the lags are show on the vertical axis residuals are shown on the horizontal.

In the table (15), there are shown the GNI forecasting values:

Table 15 GNI forecasting

Model		2020	2021	2022	2023	2024
GNI	Forecast	2,70E+11	2,81E+11	2,92E+11	3,03E+11	3,14E+11
	UCL	3,22E+11	3,55E+11	3,82E+11	4,07E+11	4,30E+11
	LCL	2,19E+11	2,08E+11	2,03E+11	2,00E+11	1,98E+11
Model		2025	2026	2027	2028	2029
GNI	Forecast	3,25E+11	3,36E+11	3,47E+11	3,58E+11	3,68E+11
	UCL	4,52E+11	4,73E+11	4,93E+11	5,13E+11	5,32E+11
	LCL	1,98E+11	1,99E+11	2,00E+11	2,02E+11	2,05E+11

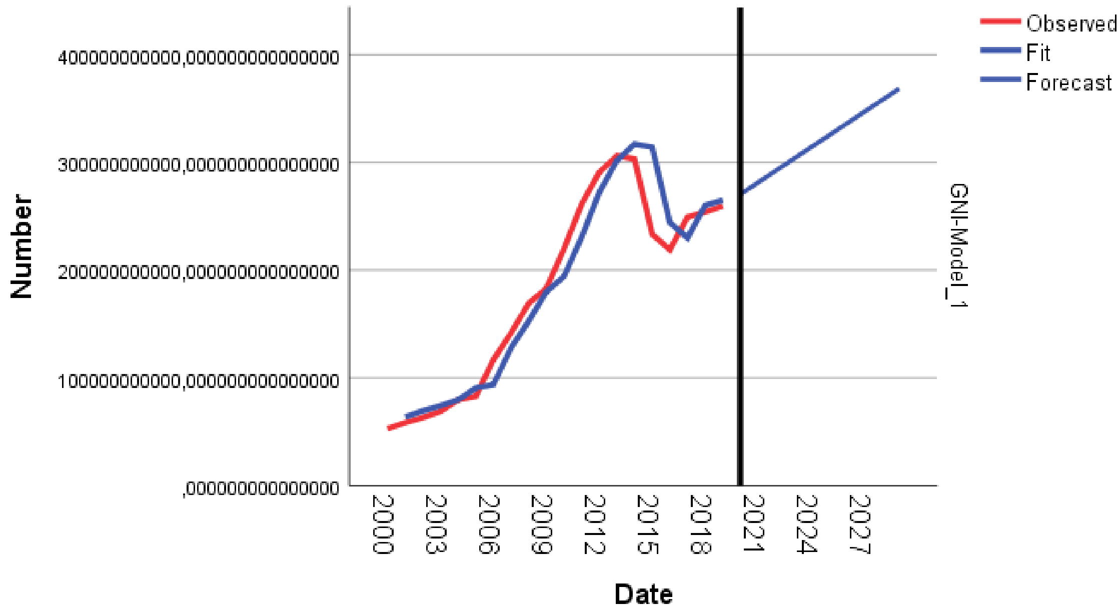

Figure 12 GNI forecasting

(Source: SPSS)

As it is shown in figure (12), the GNI is show on the vertical axis and years are shown on the horizontal. The results from applying the GNI ARIMA (0,1,0) model in the IBM SPSS program showed that the future Qatari GNI values will continue increasing in the future.

ARIMA (1,1,0)

In the tables (16, 17), there are shown the GNI ARIMA (1,1,0) statistics and parameters:

Table 16 GNI statistic (1,1,0)

Model	Number of Predictors		Model Fit statistics	Ljung-Box Q(18)			Number of Outliers
			R-squared	Statistics	DF	Sig.	
GNI-Model_1	0	0,93	48,03	5,22	17	1	0

Table 17 ARIMA Parameters

		Estimate	SE	t	Sig.
GNI	Constant	1,09E+10	8,65E+09	1,257	0,23
No Transformation	AR Lag 1	0,4	0,22	1,8	0,1
	Difference	1			

In the tables (18,19), there are shown the GNI ARIMA (1,1,0) fit statistics and ACF and PACF residuals:

Table 18 GNI fit statistic (1,1,0)

Fit Statistic	Mean	Minimum	Maximum	Percentile	
				5	10
Stationary R^2	0,17	.	0,17	0,17	0,17
R-squared	0,93	.	0,93	0,93	0,93
RMSE	2,31E+10	.	2,31E+10	2,31E+10	2,31E+10
MAPE	7,72	.	7,72	7,72	7,72
MaxAPE	32,23	.	32,23	32,23	32,23
MAE	1,40E+10	.	1,40E+10	1,40E+10	1,40E+10
MaxAE	7,52E+10	.	7,52E+10	7,52E+10	7,52E+10
Normalized BIC	48,03	.	48,03	48,03	48,03
	25	50	75	90	95
Stationary R^2	0,17	0,17	0,17	0,17	0,17
R-squared	0,93	0,93	0,93	0,93	0,93
RMSE	2,31E+10	2,31E+10	2,31E+10	2,31E+10	2,31E+10
MAPE	7,72	7,72	7,72	7,72	7,72
MaxAPE	32,23	32,23	32,23	32,23	32,23
MAE	1,40E+10	1,40E+10	1,40E+10	1,40E+10	1,40E+10
MaxAE	7,52E+10	7,52E+10	7,52E+10	7,52E+10	7,52E+10
Normalized BIC	48,03	48,03	48,03	48,03	48,03

Table 19 ACF and PACF Table

Residual ACF			Residual PACF			Residual ACF			Residual PACF		
Model	ACF	SE	Model	PACF	SE	Model	ACF	SE	Model	PACF	SE
1	0,11	-0,19	1	0,11	-0,21	10	0,23	0,23	10	0,23	0,23
2	0,23	0,23	2	0,23	0,23	11	0,11	-0,19	11	0,11	-0,21
3	0,11	-0,19	3	0,11	-0,21	12	0,23	0,23	12	0,23	0,23
4	0,23	0,23	4	0,23	0,23	13	0,11	-0,19	13	0,11	-0,21
5	0,11	-0,19	5	0,11	-0,21	14	0,23	0,23	14	0,23	0,23
6	0,23	0,23	6	0,23	0,23	15	0,11	-0,19	15	0,11	-0,21
7	0,11	-0,19	7	0,11	-0,21	16	0,23	0,23	16	0,23	0,23
8	0,23	0,23	8	0,23	0,23	17	0,11	-0,19	17	0,11	-0,21
9	0,11	-0,19	9	0,11	-0,21	18	0,23	0,23	18	0,23	0,23

In the table (20), there is shown the GNI ARIMA (1,1,0) forecasting:

Table 20 Forecasting table

Model		2020	2021	2022	2023	2024
GNI	Forecast	2,79E+11	2,89E+11	3,00E+11	3,11E+11	3,22E+11
	UCL	3,62E+11	4,02E+11	4,37E+11	4,70E+11	5,00E+11
	LCL	1,95E+11	1,76E+11	1,62E+11	1,52E+11	1,44E+11
Model		2025	2026	2027	2028	2029
GNI	Forecast	3,32E+11	3,43E+11	3,54E+11	3,65E+11	2,79E+11
	UCL	5,28E+11	5,55E+11	5,80E+11	6,05E+11	3,62E+11
	LCL	1,37E+11	1,32E+11	1,28E+11	1,25E+11	1,95E+11

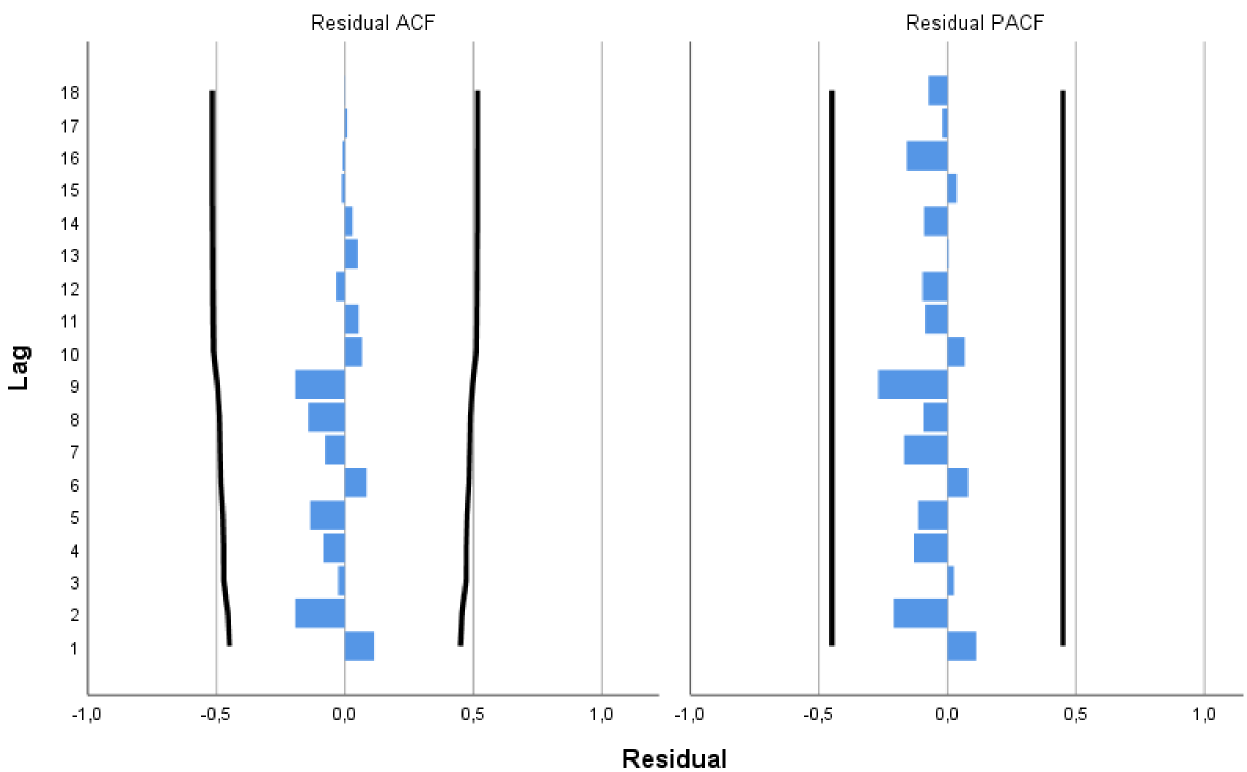

Figure 13 ACF and PACF residuals ARIMA (1,1,0)

Source: SPSS

As it is shown in figure (13), the lags are shown on the vertical axis and residuals are shown on the horizontal.

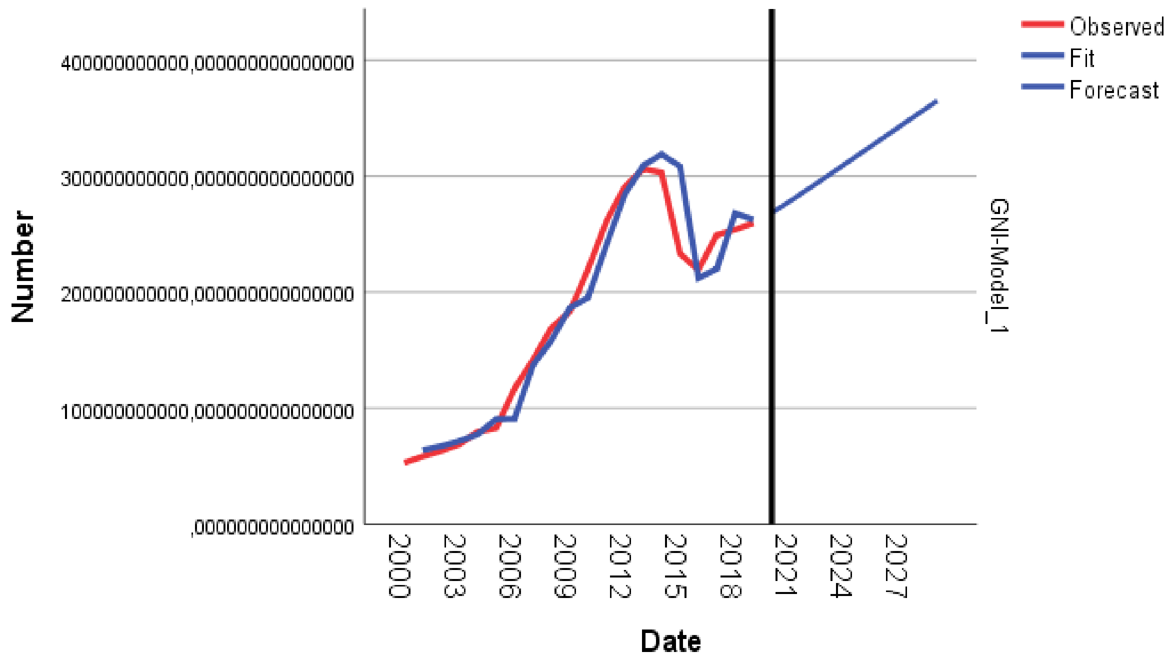

Figure 14 Forecasting ARIMA (1,1,0)

Source: SPSS

As it is shown in figure (14), the GNI is show on the vertical axis and years are shown on the horizontal. The results from applying the GNI ARIMA (1,1,0) model in the IBM SPSS program showed that the future Qatari GNI values will continue increasing in the future.

The results from ARIMA (0,1,0) and ARIMA (1,1,0) models show that Qatari GNI will continue increasing in the future.

Comparison between the ARIMA models:

Related to the normalized Bayesian information criterion (BIC) values in the fit statistic tables, the model that has smaller normlized BIC values fits more than the one that has bigger normalized BIC values (Clement, 2014).

From the tables (13,18) we noticed that the ARIMA (1,1,0) model fitted more than the ARIMA (0,1,0) with dataset (Source: Author).

3 RESEARCH METHODOLOGY

Gross domestic product (GDP) is a standard measurement tool to value all final services and goods from a nation in the market in a given period including foreign trade balance, paid-in construction costs, government purchase, private inventories, and personal consumption. The GDP has three ways that can be measured, first way is the income approach, the second way the expenditure approach, and the last one the production approach. The GDP expenditure can be calculated by measuring the total spent on services and goods in way of adding up all the cash spent by the government, businesses, and consumers in the given period. Also, we can measure the income earned approach by adding up all the cash received by all the economy participants in the given period.

The GDP measurements can assist the countries to set up policies, plans, and development strategies for the economy, judging the status of the operations of the macroeconomy as a whole and according to the economists GDP measurements is the best tool to measure the performance of the economy.

The method that has been used in this study for analyzing and forecasting the Qatari GDP is the statistical techniques in time series forecasting the ARIMA model. Qatari GDP is going through the ARIMA model process as identify data, estimate, checking diagnostic, testing data, and then apply the model if the data passed the testing process.

The results of this method will be shown on the following pages. Those results will assist to measure the performance of the Qatari economy and make future predictions of it.

3.1 Qatar

Qatar is an independent emirate, Islamic monarchy and it is officially called the State of Qatar. It is located in the west of the Asia continent and the northeast of the Arabian Peninsula coast. Qatar has land borders in the south only with the kingdom of Saudi Arabia, Qatar also shares maritime borders with the United Arab Emirates, Bahrain, and Iran. Qatar has 2.726.922 inhabitants, 11,581 km^2, two official languages Arabic and English, and 77.5% of the population are Muslim and 22.5 % are other religions.

Since 1850 Qatar has been ruled by the Al-Thani family and the current prince of Qatar is Tamim bin Hamad bin Khalifa Al Thani. The largest and capital city of Qatar is called Doha and it is in the east of the county; Doha has been the capital city since 1971. Doha is called the Capital of Sports and has hosted many sports events, the most important of which are the 2006 Asian Games, the 2011 Pan Arab Games the 2011 AFC Asian Cup, clubs World Cup, hosted the 2012 UNFCCC climate negotiations, and set to host of the 2022 FIFA World Cup (Barrientos, 2020).

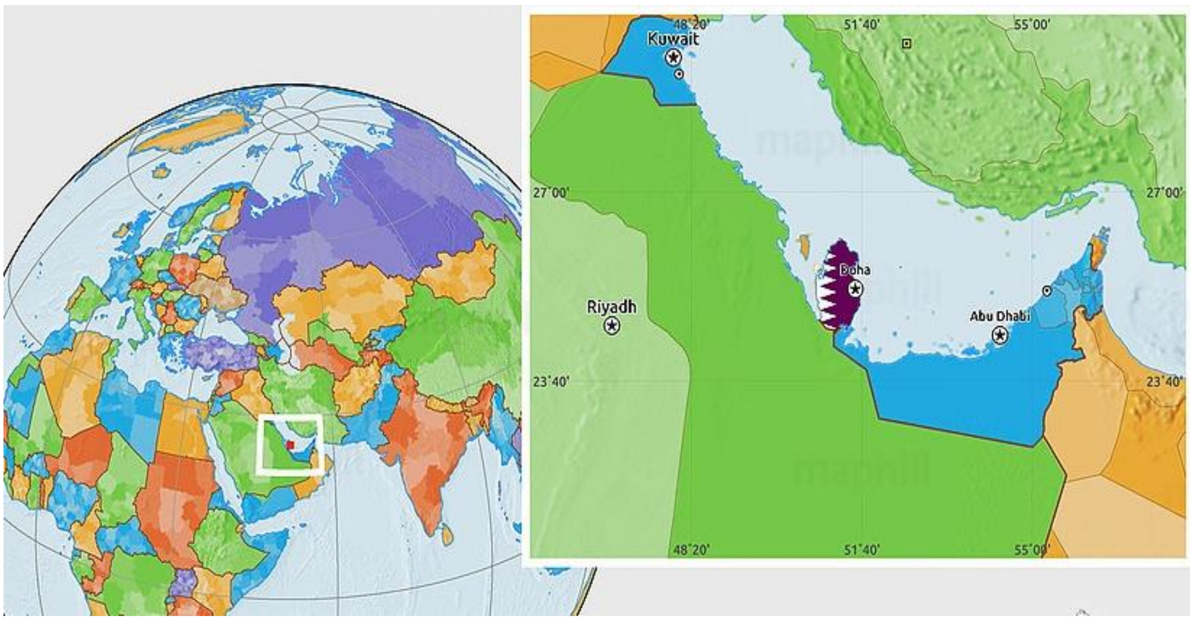

Figure 15 Flag Location Map of Qatar

(Maphill, 2020)

The figure (15) shows the location and the flag of Qatar.

3.1.1 History of Qatar

- In 1825 - Qatar tries to establish independence from Bahraini suzerainty, and the Al-Thani family establishes control over Qatar.
- In 1871 - Qatar submit to Ottoman overlordship.
- In 1916 - a signed deal between Qatar and Britain which Britain seeks to guarantee Qatar's protection in return for controls Qatar's external affairs.
- 1939 - Qatar's Oil reserves discovered. The oil Exploitation delayed was because of the Second World War. The oil became Qatar's main source of revenue instead of pearling and fishing.
- In 1971 - Qatar get full independence from British colonialism.

- In 1996 - Qatar changed the face of Arab broadcasting by sets up international Al-Jazeera TV.
- In 2005, June - Qatar provides some democratic reforms by Qatar's first written constitution comes into effect.
- In 2017, June - Diplomatic crisis between Qatar Saudi Arabia, Bahrain, UAE, and Egypt. Saudi Arabia launches an Arab transport blockade with the goal to get Qatar to cut its connections with Islamist groups, Turkey, and Iran (Affiliates, 2021).

3.1.2 Qatari economy overview

The Qatari economy is the 11^{th} largest economy in the world and 1^{st} in Arabic countries. Qatar is a member of the GCC (Gulf cooperation council), a member of WTO (World trade organization since 1996, a member of the OPEC (organization petroleum exporting countries) and has ranked 45 in the HDI (Human development index) by score 0.848.

Qatar was a poor country in the past but since exploration and production of natural gas and oil, the revenue increased dramatically, moving Qatar's rank out from one of the poorest countries to the highest per capita incomes in the world. The petroleum sector participates in the economy exports around 85 % and 70 % of the total national revenue.

With the small size of population (around 2.639 million) and with high vast revenue from the petroleum sector, Qatar enjoys the economic benefits and surplus from the petroleum sector and make Qatari government one of the rare governments that do not levy any taxes from its citizen and let them enjoy their full earnings and provide them many services for free (Anthony, 2020).

In the following some information about the Qatari economy:

- Qatari economy size is $ 160.00 billion.
- Qatar is the first in the world GDP per capita ($ 129,000).
- Qatar has the third-largest proven natural gas reserve in the world around 12 %.
- The second-largest exporter of natural gas.
- Petroleum sector participates in more than 60 % of GDP.
- Qatar's economic freedom is the 31^{st} freest with a score of 72.0 in 2021.
- Qatar is the 11^{th}-largest wealth fund in the world with $300 billion of assets.

- The trade surplus balance in 2019 is around $ 43.76 billion.
- Average per capita income of $ 166,000 per year in Qatar.
- Inflation rate -0.7 in 2019.
- Unemployment rate 0.12 in 2019 (Bank, 2021).

3.1.2.1 Sources of the income in Qatari economy

Before the discovery of oil, the Qatari economy focused on fishing and pearls. After the discovery of the oil, the Qatari economy focused on oil, gas, petrochemicals, agriculture, industry, services, exports, investment, and others (non - oil sectors) (Dake, 2020).

3.1.2.2 Taxes

Personal taxes

There is no tax on the income of individuals' salaries, which means that employees receive their wages and salaries without deducting any taxes. You can get complete information about personal income tax.

Companies' taxes

The tax rate is fixed at 10 % of the company's total income from the state and is paid annually.

Tax exemptions

- Small craft (three employees or less).
- The income of companies belongs to agriculture, fishing, and transport.
- Qataris residing in the country.

Other taxes

There are no other taxes. Qatar is one of the few countries in the world that put a small tax rate, which makes it an attractive working environment for people, companies, and investors (Dulebenets, 2021).

3.1.2.3 Investment

Qatari Investments:

- Investments in finance (banks and finance fund).
- Towers, skyscrapers, real estate, and hotels.
- Industry
- Transport (Qatar Airways – the best airline in the world2019)
- Sports (Paris Saint-Germain – Be-in sport channels – World cup 2022)
- Invest in energy and others (Affairs-Qatar, 2019).

3.1.2.4 Policy to increase the income

- Qatar's vision is to achieve economic diversification and reduce dependence on oil and gas.
- The Government plans invest inside and outside the state.
- Attract foreign investment by:
  - Providing a good environment for investment.
  - Flexible tax and custom system
  - Developing of the investment law to protect the investors (GCO, 2021).

3.2 GDP

Gross domestic product (GDP): is the standard measure of the total monetary value of all finished goods and services in a specific period in a country. The GDP makes a snapshot of the country's economy which helps in the estimation process of the growth rate and size of the economy in the country. The calculation of the GDP can be calculated on a fiscal quarter basis or on an annual basis by counting incomes, production, or expenditure in the economy.

GDP has several measurements which it can be useful for different purposes:

- Nominal GDP: it is used to measure the value of goods and services in the market by using current prices either in U.S dollars currency or local currency.
- Real GDP: It is an inflation-adjusted used to measure the reflection between the changes in prices with economic output in a given period.
- GDP Growth Rate: it is used to measure economic growth by comparing the GDP for the current period to the GDP for the previous period.

- GDP per capita: is a measurement of the amount of income or output per person in a country economy (Kushnir, 2018).

As shown in figure (16), the GDP per capita and GDP with current currency (U.S dollar) will show in the vertical axis and the years will show on the horizontal axis. This figure shows the Qatari GDP during many years and shows the affection of some political events on it.

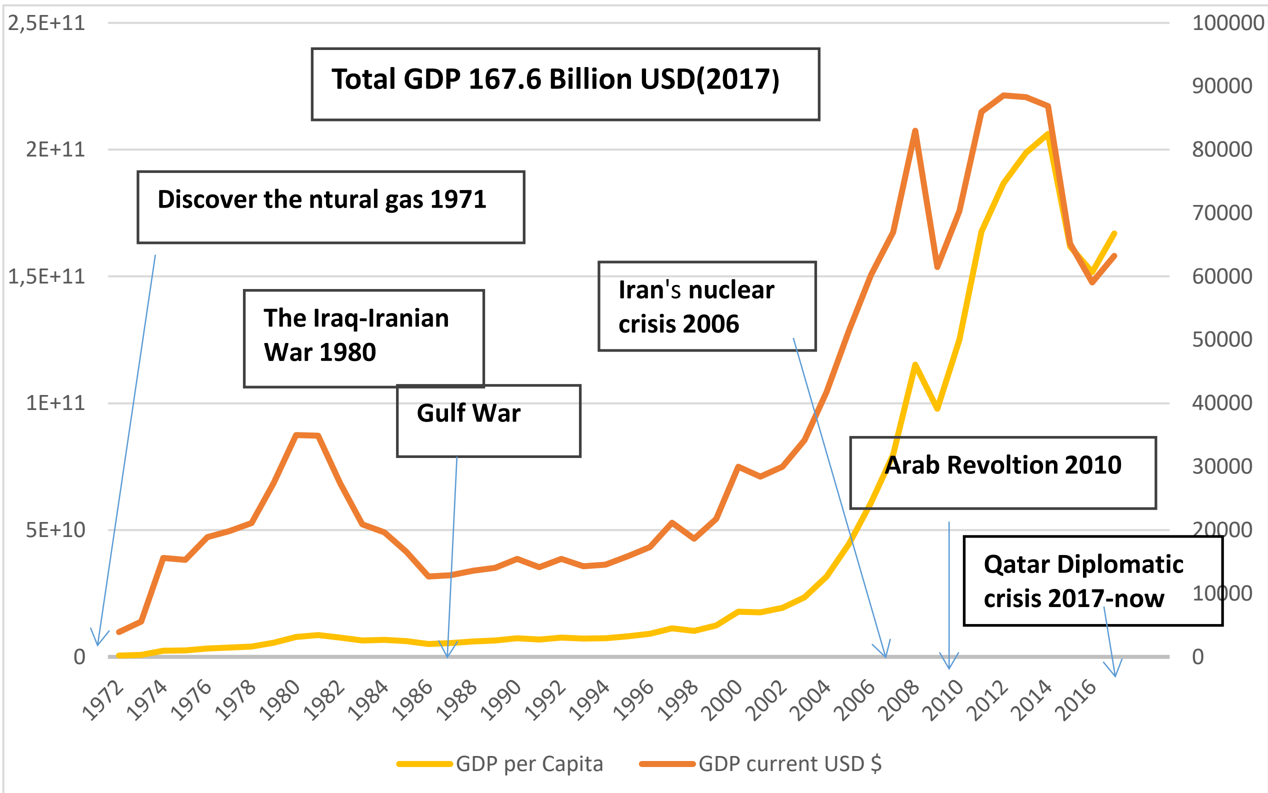

Figure 16 Qatari GDP

Source: World bank

3.3 Qatari GDP forecasting

In this study, the annual GDP of Qatar was obtained from World Bank from 1980 to 2019. This means that we have 39 observations of GDP that satisfy the rule of having over 39 observations in the approach of time series forecasting. Based on this data, we will propose the appropriate models and then use them to forecast the Qatari GDP for the next six years (from 2019 to 2025).

Qatari GDP data set:

In the table (21), there is shown Qatari GDP data set.

Table 21 Qatari GDP dataset

Year	GDP	Year	GDP	Year	GDP	Year	GDP
1980	7,83E+09	1990	7,36E+09	2000	1,78E+10	2010	1,25E+11
1981	8,66E+09	1991	6,88E+09	2001	1,75E+10	2011	1,68E+11
1982	7,60E+09	1992	7,65E+09	2002	1,94E+10	2012	1,87E+11
1983	6,47E+09	1993	7,16E+09	2003	2,35E+10	2013	1,99E+11
1984	6,70E+09	1994	7,37E+09	2004	3,17E+10	2014	2,06E+11
1985	6,15E+09	1995	8,14E+09	2005	4,45E+10	2015	1,62E+11
1986	5,05E+09	1996	9,06E+09	2006	6,09E+10	2016	1,52E+11
1987	5,45E+09	1997	1,13E+10	2007	7,97E+10	2017	1,61E+11
1988	6,04E+09	1998	1,03E+10	2008	1,15E+11	2018	1,83E+11
1989	6,49E+09	1999	1,24E+10	2009	9,78E+10	2019	1,76E+11

AR model

In the tables (22, 23, 24), there are shown the AR model results.

Table 22 AR model results

Regression Statistics	
Multiple R	0.98
R Square	0.96
Adjusted R Square	0.96
Standard Error	14672339536.99
Observations	37

Table 23 GDP ANOVA

Anova	df	SS	MS	F	Significance F
Regression	3.00E+00	1.82E+23	6.06E+22	2.81E+02	1.44E-23
Residual	3.30E+01	7.10E+21	2.15E+20		
Total	3.60E+01	1.89E+23			

Table 24 GDP statistics

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95%	Upper 95%
Intercept	4,00E+09	3,00E+09	1,00E+00	2,00E-01	-3,00E+09	1,00E+10	-3,00E+09	1,00E+10
Y_{t-1}	1,00E+00	2,00E-01	7,00E+00	4,00E-08	9,00E-01	2,00E+00	9,00E-01	2,00E+00
Y_{t-2}	-3,00E-01	3,00E-01	-1,00E+00	3,00E-01	-9,00E-01	3,00E-01	-9,00E-01	3,00E-01
Y_{t-3}	6,00E-02	2,00E-01	3,00E-01	8,00E-01	-3,00E-01	4,00E-01	-3,00E-01	4,00E-01

In the table (24), there is shown the autoregressive results, from the results, we will compare P-value with alpha (0.05):

- If the (P-Value ≤ Alpha (0.05)) then the results significant and we reject the null hypothesis
- If the (P-value > Alpha (0.05)) then the result is statistically non-significant, and we fail to reject the null hypothesis.

From the results table we noticed the P-value of X_1, X_2 and X_3 is > than alpha.

The results are statistically non-significant, and it fails to reject the null hypothesis.

3.3.1 ARIMA model

In the approach of time series forecasting. Based on this data, we will propose the ARIMA model by using IBM SPSS statistic method and then use them to forecast the Qatari GDP for the next six years (from 2019 to 2025).

3.3.1.1 ARIMA (1,1,1)

In the tables (25, 26, 27, 28), there are shown the ARIMA (1,1,1) results.

Table 25 ARIMA (1,1,1) Fit statistics

Fit Statistic	Mean	Minimum	Maximum	Percentile	
				5	10
Stationary R^2	0,09	0,09	0,09	0,09	0,09
R^2	0,96	0,96	0,96	0,96	0,96
RMSE	1,44E+10	1,44E+10	1,44E+10	1,44E+10	1,44E+10
MAPE	15,33248	15,33248	15,33248	15,33248	15,33248
MaxAPE	41,39395	41,39395	41,39395	41,39395	41,39395
MAE	7,56E+09	7,56E+09	7,56E+09	7,56E+09	7,56E+09
MaxAE	5,50E+10	5,50E+10	5,50E+10	5,50E+10	5,5E+10
Normalized BIC	47,06	47,06	47,06	47,06	47,06
	25	50	75	90	95
Stationary R^2	-0,1	-0,1	-0,1	-0,1	-0,1
R^2	0,96	0,96	0,96	0,96	0,96
RMSE	1,46E+10	1,46E+10	1,46E+10	1,46E+10	1,46E+10
MAPE	13,51	13,51	13,51	13,51	13,51
MaxAPE	30,85	30,85	30,85	30,85	30,85
MAE	8,48E+09	8,48E+09	8,48E+09	8,48E+09	8,48E+09
MaxAE	4,45E+10	4,45E+10	4,45E+10	4,45E+10	4,45E+10
Normalized BIC	46,90	46,90	46,90	46,90	46,90

Table 26 ARIMA (1,1,1) ACF and PACF residuals

Residual ACF			Residual PACF			Residual ACF			Residual PACF		
Model	ACF	SE	Model	PACF	SE	Model	ACF	SE	Model	PACF	SE
1	0,035	0,160	1	0,035	0,160	13	-0,026	0,187	13	-0,015	0,160
2	-0,089	0,160	2	-0,091	0,160	14	0,010	0,187	14	-0,044	0,160
3	0,067	0,162	3	0,075	0,160	15	-0,090	0,187	15	0,138	0,160
4	-0,018	0,162	4	-0,032	0,160	16	-0,032	0,188	16	-0,084	0,160
5	-0,069	0,162	5	-0,055	0,160	17	-0,024	0,188	17	-0,051	0,160
6	0,290	0,163	6	0,292	0,160	18	0,012	0,188	18	-0,047	0,160
7	0,001	0,176	7	-0,042	0,160	19	-0,067	0,188	19	-0,061	0,160
8	-0,058	0,176	8	0,004	0,160	20	-0,063	0,189	20	-0,043	0,160
9	-0,270	0,176	9	-0,341	0,160	21	-0,008	0,190	21	-0,100	0,160
10	-0,004	0,187	10	0,047	0,160	22	-0,046	0,190	22	-0,032	0,160
11	0,012	0,187	11	-0,001	0,160	23	-0,015	0,190	23	0,016	0,160
12	-0,049	0,187	12	-0,100	0,160	24	-0,011	0,190	24	-0,034	0,160

Table 27 ARIMA (1,1,1) Statistics model

Model	Number of Predictors	Model Fit statistics	Ljung-Box Q(18)			Number of Outliers
		R^2	Statistics	DF	Sig.	
GDP-Model_1	0	0,09	9,84	16	0,88	0

Table 28 ARIMA (1,1,1) Forecasting

Model		2020	2021	2022	2023	2024	2025
GDP	Forecast	1,81E+11	1,87E+11	1,93E+11	2,00E+11	2,08E+11	2,16E+11
	UCL	2,17E+11	2,45E+11	2,73E+11	3,01E+11	3,29E+11	3,58E+11
	LCL	1,47E+11	1,34E+11	1,24E+11	1,16E+11	1,09E+11	1,03E+11

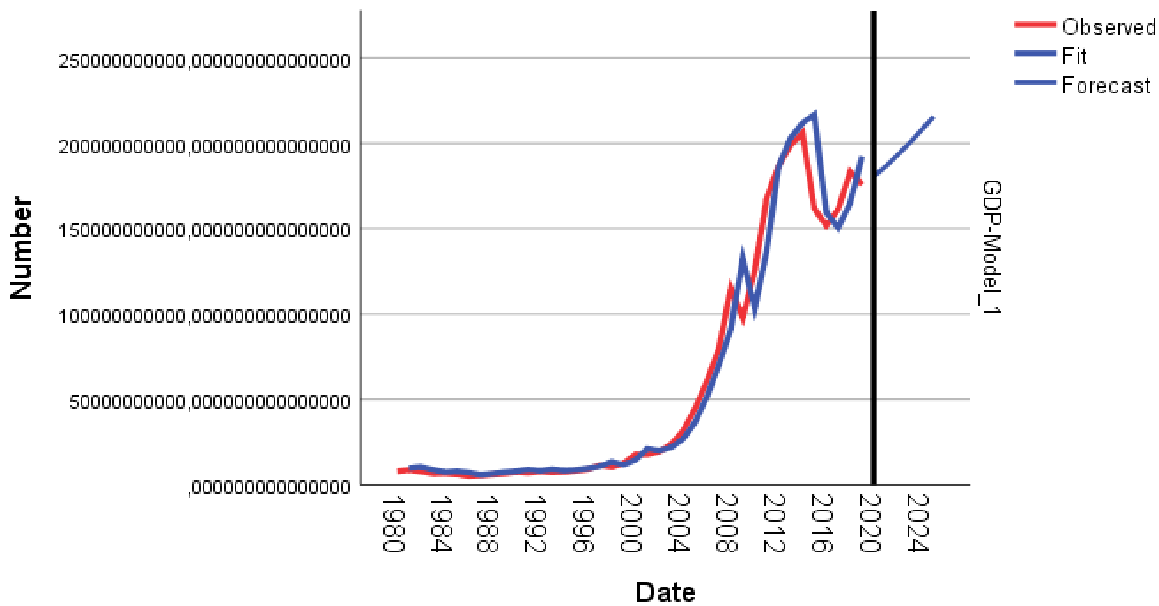

Figure 17 Forecasting ARIMA (1,1,1)

Source: SPSS

As it is shown in figure (17), the GDP is show on the vertical axis and years are shown on the horizontal. The results from applying the GDP ARIMA (1,1,1) model in the IBM SPSS program showed that the future Qatari GDP values will be stable in the future.

3.3.1.2 ARIMA (0,1,0)

In the tables (29, 30, 31, 32), there are shown the ARIMA (0,1,0) results:

Table 29 ARIMA (0,1,0) Fit statistics

Fit Statistic	Mean	Minimum	Maximum	Percentile	
				5	10
Stationary R^2	1,11E-16	1,11E-16	1,11E-16	1,11E-16	1,11E-16
R^2	9,53E-01	9,53E-01	9,53E-01	9,53E-01	9,53E-01
RMSE	1,54E+10	1,54E+10	1,54E+10	1,54E+10	1,54E+10
MAPE	1,29E+01	1,29E+01	1,29E+01	1,29E+01	1,29E+01
MaxAPE	3,99E+01	3,99E+01	3,99E+01	3,99E+01	3,99E+01
MAE	8,15E+09	8,15E+09	8,15E+09	8,15E+09	8,15E+09
MaxAE	6,45E+10	6,45E+10	6,45E+10	6,45E+10	6,45E+10
Normalized BIC	47	47	47	47	47
	25	50	75	90	95
Stationary R^2	1,11E-16	1,11E-16	1,11E-16	1,11E-16	1,11E-16
R^2	9,53E-01	9,53E-01	9,53E-01	9,53E-01	9,53E-01
RMSE	1,54E+10	1,54E+10	1,54E+10	1,54E+10	1,54E+10
MAPE	1,29E+01	1,29E+01	1,29E+01	1,29E+01	1,29E+01
MaxAPE	3,99E+01	3,99E+01	3,99E+01	3,99E+01	3,99E+01
MAE	8,15E+09	8,15E+09	8,15E+09	8,15E+09	8,15E+09
MaxAE	6,45E+10	6,45E+10	6,45E+10	6,45E+10	6,45E+10
Normalized BIC	47	47	47	47	47

Table 30 ARIMA (0,1,0) ACF and PACF residuals

Residual ACF			Residual PACF			Residual ACF			Residual PACF		
Model	ACF	SE	Model	PACF	SE	Model	ACF	SE	Model	PACF	SE
1	0,299	0,16	1	0,299	0,16	13	-0,144	0,209	13	-0,082	0,16
2	0,185	0,174	2	0,105	0,16	14	-0,178	0,212	14	-0,14	0,16
3	0,32	0,179	3	0,266	0,16	15	-0,147	0,216	15	0,023	0,16
4	0,15	0,193	4	-0,014	0,16	16	-0,046	0,218	16	0,108	0,16
5	0,108	0,196	5	0,018	0,16	17	-0,176	0,219	17	-0,059	0,16
6	0,126	0,197	6	0,009	0,16	18	-0,091	0,222	18	-0,058	0,16
7	0,076	0,199	7	0,004	0,16	19	-0,182	0,223	19	-0,215	0,16
8	0,089	0,2	8	0,041	0,16	20	-0,198	0,227	20	-0,009	0,16
9	-0,199	0,201	9	-0,321	0,16	21	-0,136	0,231	21	-0,022	0,16
10	-0,09	0,206	10	0	0,16	22	-0,185	0,233	22	-0,105	0,16
11	-0,024	0,207	11	-0,009	0,16	23	-0,075	0,237	23	0,006	0,16
12	-0,133	0,207	12	0,007	0,16	24	-0,069	0,238	24	-0,061	0,16

Table 31ARIMA (0,1,0) Statistics model

Model	Number of predictors	Model Fit statistics	Ljung-Box Q(18)			Number of Outliers
		R-squared	Statistics	DF	Sig.	
GDP-Model	0	0,96	24,24	18	0,15	0

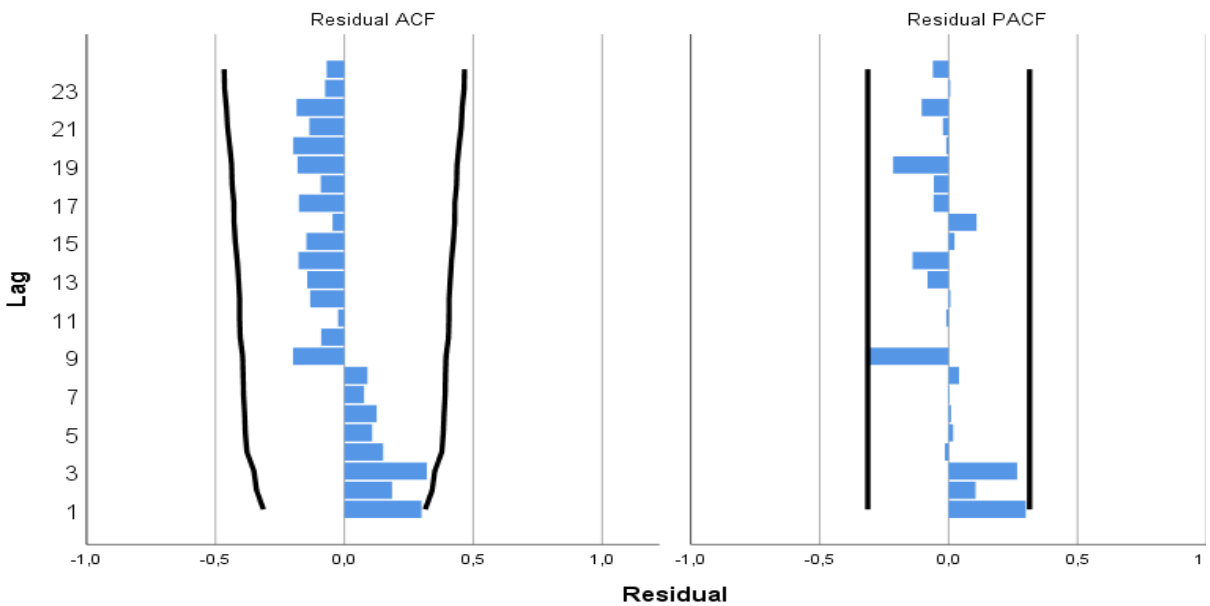

Figure 18 ACF and PACF residuals ARIMA (0,1,0)

Source: SPSS

As it is shown in figure (18), the lags are show on the vertical axis and residuals are shown on the horizontal.

Table 32 ARIMA (0,1,0) Forecasting

Model		2020	2021	2022	2023	2024	2025
GDP	Forecast	1,93E+11	2,12E+11	2,32E+11	2,55E+11	2,80E+11	3,07E+11
	UCL	2,64E+11	3,27E+11	3,93E+11	4,65E+11	5,44E+11	6,32E+11
	LCL	1,37E+11	1,30E+11	1,27E+11	1,26E+11	1,26E+11	1,27E+11

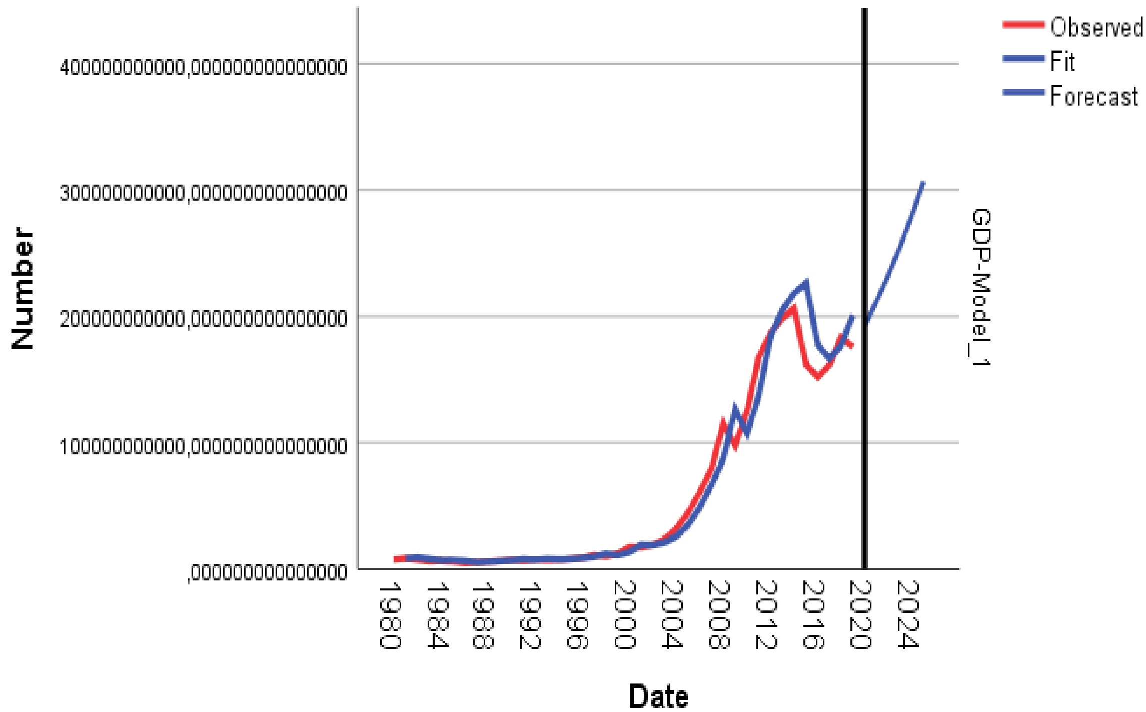

Figure 19 Forecasting ARIMA (0,1,0)

Source: PSS

As it is shown in figure (19), the GDP is show on the vertical axis and years are shown on the horizontal. The results from applying the GDP ARIMA (0,1,0) model in the IBM SPSS program showed that the future Qatari GDP values will continue increasing in the future.

3.3.1.3 ARIMA (1,1,0)

In the tables (33, 34, 35, 36), there are shown the ARIMA (1,1,0) results:

- (p=1) autoregressive model order (number of terms).
- (d=1) differencing degree.
- (q=0) moving average order (number of lagged forecast errors).

Table 33 ARIMA (1,1,0) Statistics model

Model	Number of predictors	Model Fit statistics	Ljung-Box Q(18)			Number of Outliers
		R^2	Statistics	DF	Sig.	
GDP-Model	0	0,96	8,76	17	0,95	0

Table 34 ARIMA (1,1,0) Fit statistics

Fit Statistic	Mean	Minimum	Maximum	Percentile	
				5	10
Stationary R^2	0,059	0,059	0,059	0,059	0,059
R^2	0,963	0,963	0,963	0,963	0,963
RMSE	1,39E+10	1,39E+10	1,39E+10	1,39E+10	1,4E+10
MAPE	28,212	28,212	28,212	28,212	28,212
MaxAPE	84,102	84,102	84,102	84,102	84,102
MAE	8,29E+09	8,29E+09	8,29E+09	8,29E+09	8,3E+09
MaxAE	4,96E+10	4,96E+10	4,96E+10	4,96E+10	5E+10
Normalized BIC	46,90	46,90	46,90	46,90	46,90
	25	50	75	90	95
Stationary R^2	0,059	0,059	0,059	0,059	0,059
R^2	0,963	0,963	0,963	0,963	0,963
RMSE	1,39E+10	1,39E+10	1,39E+10	1,39E+10	1,4E+10
MAPE	28,212	28,212	28,212	28,212	28,212
MaxAPE	84,102	84,102	84,102	84,102	84,102
MAE	8,29E+09	8,29E+09	8,29E+09	8,29E+09	8,3E+09
MaxAE	4,96E+10	4,96E+10	4,96E+10	4,96E+10	5E+10
Normalized BIC	46,90	46,90	46,90	46,90	46,90

Table 35 ARIMA (1,1,0) ACF and PACF residuals

Residual ACF			Residual PACF			Residual ACF			Residual PACF		
Model	ACF	SE	Model	PACF	SE	Model	ACF	SE	Model	PACF	SE
1	0,019	0,16	1	0,019	0,16	13	-0,023	0,185	13	0,005	0,16
2	-0,064	0,16	2	-0,065	0,16	14	-0,003	0,185	14	-0,018	0,16
3	0,041	0,161	3	0,044	0,16	15	-0,054	0,185	15	0,127	0,16
4	-0,006	0,161	4	-0,012	0,16	16	-0,032	0,186	16	-0,165	0,16
5	-0,1	0,161	5	-0,095	0,16	17	-0,019	0,186	17	-0,02	0,16
6	0,307	0,163	6	0,313	0,16	18	-0,013	0,186	18	-0,029	0,16
7	-0,049	0,177	7	-0,095	0,16	19	-0,038	0,186	19	0,006	0,16
8	-0,095	0,177	8	-0,046	0,16	20	-0,036	0,186	20	-0,071	0,16
9	-0,202	0,179	9	-0,252	0,16	21	-0,018	0,186	21	-0,108	0,16
10	0,062	0,184	10	0,093	0,16	22	-0,03	0,186	22	0,054	0,16
11	-0,039	0,185	11	-0,011	0,16	23	-0,028	0,186	23	-0,046	0,16
12	-0,03	0,185	12	-0,124	0,16	24	-0,022	0,187	24	-0,014	0,16

Table 36 ARIMA (1,1,0) Forecasting

Model		2020	2021	2022	2023	2024	2025
GDP	Forecast	1,77E+11	1,81E+11	1,85E+11	1,89E+11	1,94E+11	1,98E+11
	UCL	2,06E+11	2,26E+11	2,43E+11	2,58E+11	2,72E+11	2,84E+11
	LCL	1,49E+11	1,36E+11	1,27E+11	1,21E+11	1,16E+11	1,12E+11

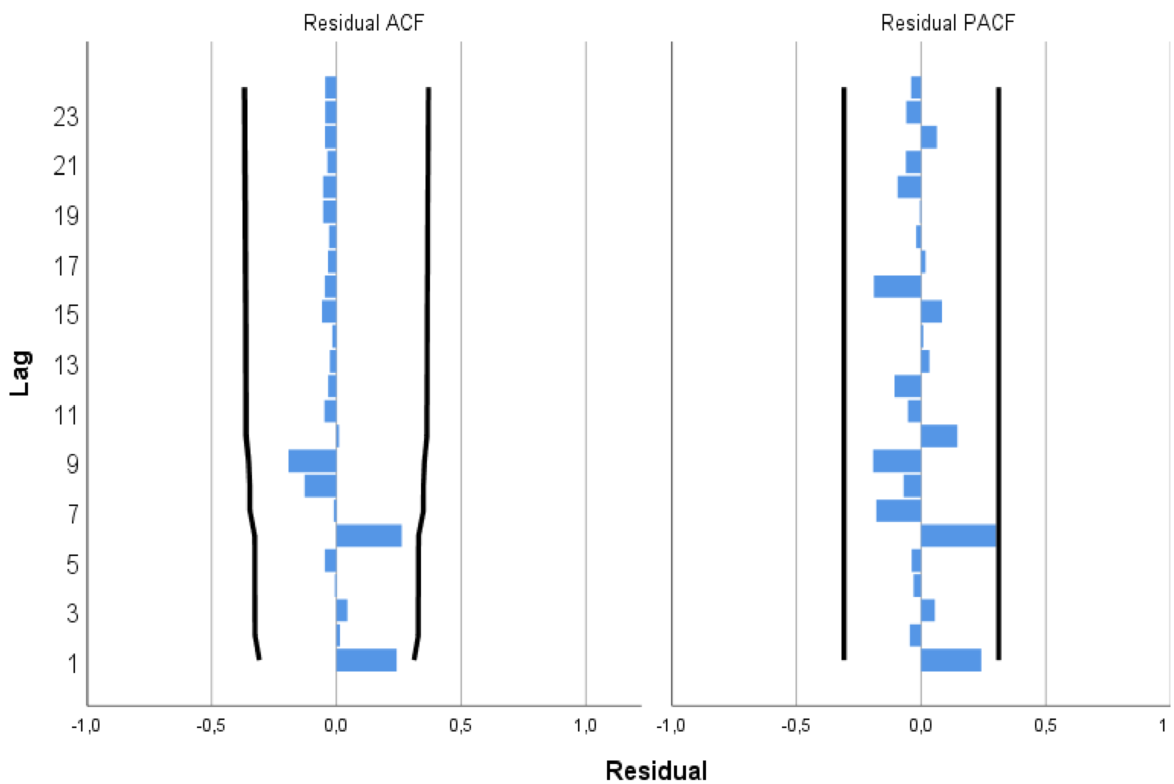

Figure 20 ACF and PACF residuals ARIMA (1,1,0)

Source: SPSS

As it is shown in figure (20), the lags are show on the vertical axis and residuals are shown on the horizontal.

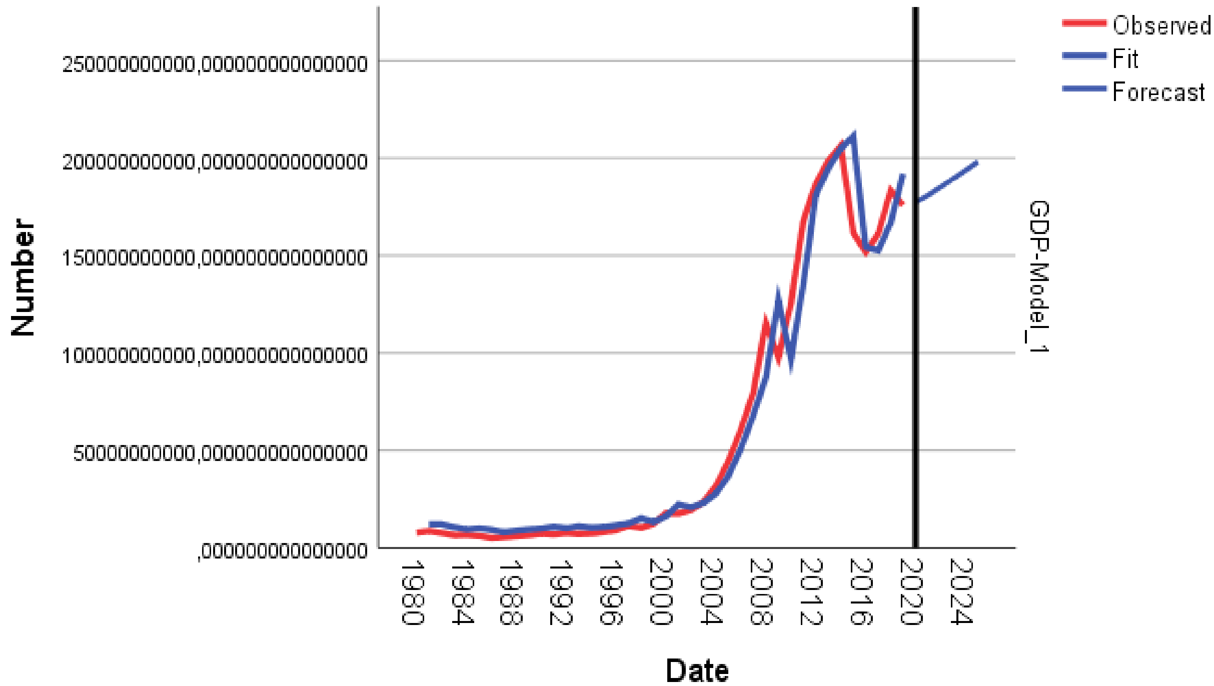

Figure 21 Forecasting ARIMA (1,1,0)

Source: SPSS

As it is shown in figure (21), the GDP is show on the vertical axis and years are shown on the horizontal. The results from applying the GDP ARIMA (1,1,0) model in the IBM SPSS program showed that the future Qatari GDP values will continue increasing in the future.

3.3.1.4 ARIMA (3,1,1)

In the tables (37, 38, 39, 40), there are shown the ARIMA (3,1,1) results:

- (p=3) autoregressive model order (number of terms).
- (d=1) differencing degree.
- (q=1) moving average order (number of lagged forecast errors).

Table 37 ARIMA (3,1,1) Statistics model

Model	Number of predictors	Model Fit statistics	Ljung-Box Q(18)			Outliers
		R^2	Statistics	DF	Sig.	
GDP-Model_	0	0,96	8,69	14	0,85	0

Table 38 ARIMA (3,1,1) Fit statistics

Fit Statistic	Mean	Minimum	Maximum	Percentile	
				5	10
Stationary R^2	0,07	0,07	0,07	0,07	0,07
R^2	0,96	0,96	0,96	0,96	0,96
RMSE	1,45E+10	1,45E+10	1,45E+10	1,45E+10	1,4E+10
MAPE	28,14	28,14	28,14	28,14	28,14
MaxAPE	82,49	82,49	82,49	82,49	82,49
MAE	8,22E+09	8,22E+09	8,22E+09	8,22E+09	8,2E+09
MaxAE	4,97E+10	4,97E+10	4,97E+10	4,97E+10	5E+10
Normalized BIC	47,26	47,26	47,26	47,26	47,26
	25	50	75	90	95
Stationary R^2	0,07	0,07	0,07	0,07	0,07
R^2	0,96	0,96	0,96	0,96	0,96
RMSE	1,45E+10	1,45E+10	1,45E+10	1,45E+10	1,4E+10
MAPE	28,14	28,14	28,14	28,14	28,14
MaxAPE	82,49	82,49	82,49	82,49	82,49
MAE	8,22E+09	8,22E+09	8,22E+09	8,22E+09	8,2E+09
MaxAE	4,97E+10	4,97E+10	4,97E+10	4,97E+10	5E+10
Normalized BIC	47,26	47,26	47,26	47,26	47,26

Table 39 ARIMA (3,1,1) ACF and PACF residuals

Residual ACF			Residual PACF			Residual ACF			Residual PACF		
Model	ACF	SE	Model	PACF	SE	Model	ACF	SE	Model	PACF	SE
1	0,007	0,16	1	0,007	0,16	13	-0,022	0,185	13	0,028	0,16
2	-0,001	0,16	2	-0,001	0,16	14	-0,004	0,185	14	-0,026	0,16
3	-0,011	0,16	3	-0,011	0,16	15	-0,054	0,185	15	0,121	0,16
4	0,01	0,16	4	0,011	0,16	16	-0,029	0,185	16	-0,16	0,16
5	-0,096	0,16	5	-0,096	0,16	17	-0,022	0,185	17	-0,03	0,16
6	0,301	0,162	6	0,305	0,16	18	-0,014	0,185	18	-0,01	0,16
7	-0,075	0,175	7	-0,099	0,16	19	-0,037	0,186	19	-0,014	0,16
8	-0,058	0,176	8	-0,055	0,16	20	-0,038	0,186	20	-0,056	0,16
9	-0,224	0,177	9	-0,238	0,16	21	-0,019	0,186	21	-0,116	0,16
10	0,063	0,184	10	0,083	0,16	22	-0,029	0,186	22	0,065	0,16
11	-0,042	0,184	11	0,008	0,16	23	-0,03	0,186	23	-0,053	0,16
12	-0,029	0,185	12	-0,147	0,16	24	-0,022	0,186	24	-0,019	0,16

Table 40 ARIMA (3,1,1) Forecasting

Model		2020	2021	2022	2023	2024	2025
GDP	Forecast	1,76E+11	1,81E+11	1,85E+11	1,89E+11	1,94E+11	1,98E+11
	UCL	2,05E+11	2,29E+11	2,45E+11	2,60E+11	2,74E+11	2,87E+11
	LCL	1,47E+11	1,34E+11	1,25E+11	1,18E+11	1,13E+11	1,09E+11

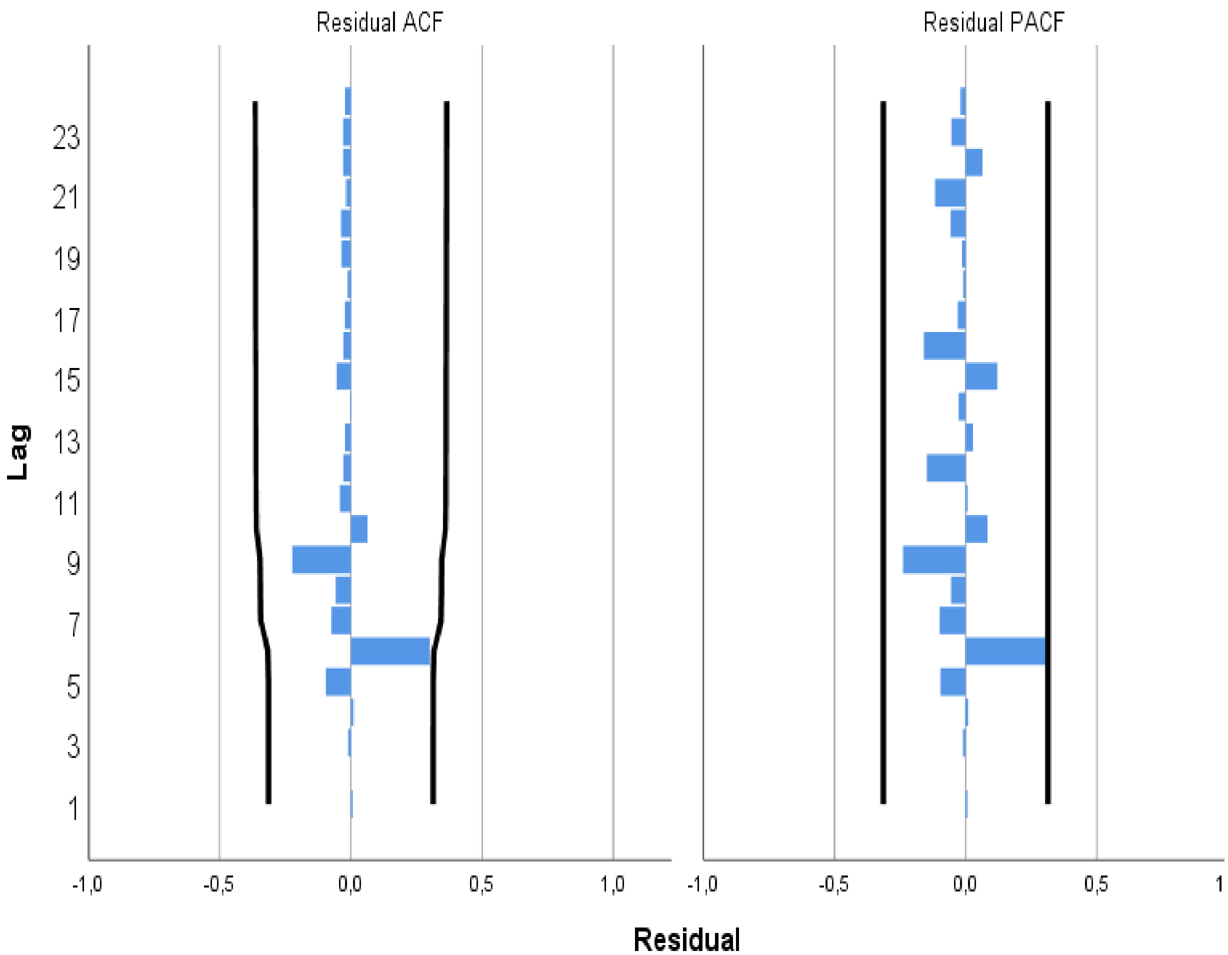

Figure 22 ACF and PACF residuals ARIMA (3,1,1)

Source: SPSS

As it is shown in figure (22), the lags are show on the vertical axis and residuals are shown on the horizontal.

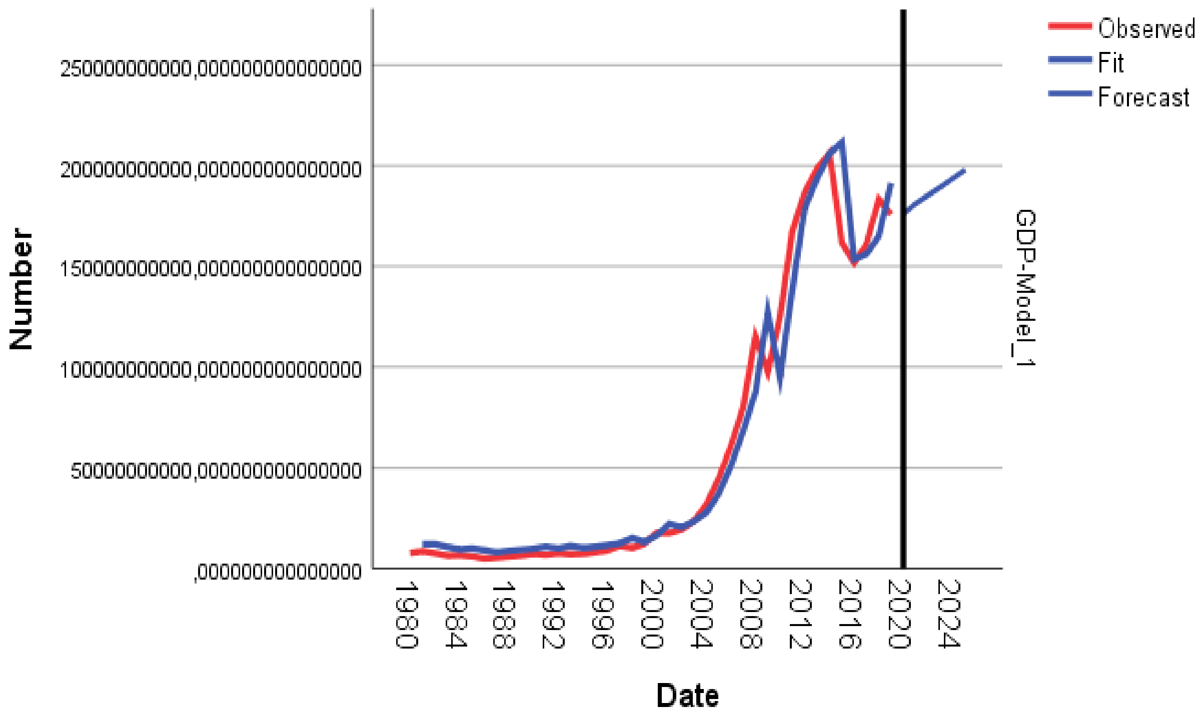

Figure 23 Forecasting ARIMA (3,1,1)

Source: SPSS

As it is shown in figure (23), the GDP is show on the vertical axis and years are shown on the horizontal. The results from applying the GDP ARIMA (3,1,1) model in the IBM SPSS program showed that the future Qatari GDP values will continue increasing in the future.

Summary of the forecasting results:

- The results from ARIMA (1,1,1) model show that Qatari GDP will continue increasing in the future as it is shown in the figure 17.
- The results from ARIMA (0,1,0) model show that Qatari GDP will continue increasing in the future as it is shown in the figure 19.
- The results from ARIMA (1,1,0) model show that Qatari GDP will GDP will continue increasing in the future as it is shown in the figure 21.
- The results from ARIMA (3,1,1) model show that Qatari GDP will continue increasing in the future as it is shown in the figure 23.

3.3.1.5 Comparison of the ARIMA models

Related to the normalized Bayesian information criterion (BIC) values in the fit statistic tables, the model that has smaller normlized BIC values fits more than the one that has bigger normalized BIC values (Clement, 2014).

Table 41 Comparison of the ARIMA models

No	ARIMA (p, d, q)	Normalized BIC value	Fitting order
1	ARIMA (1,1,0)	46,90	1st
2	ARIMA (0,1,0)	47	2nd
3	ARIMA (1,1,1)	47,06	3rd
4	ARIMA (3,1,1)	47,26	4th

Related to the normalized BIC values in the tables (25, 29, 34, 38), the table (41) shows the order of the most fitting models to the datasets and they were ordered descending from the most to the least.

From the table (41) we noticed that the ARIMA (1,1,0) model fitted more with dataset than the other models (Source: Author).

4 RESULTS

GDP is the specific metric that has prevailed since World War II for measuring the success of nations. As it is noticed the GDP has directly proportional to the living standards and wealth.

The goal of this thesis is to answer the research questions through applying, analyzing, and forecasting GDP through available models. The Qatari GDP was chosen in this research to apply the ARIMA model in the data which were selected and predict the future values of it.

Q1: What is the ARIMA model and its parameters (p, d, q)?

ARIMA (autoregressive integrated moving average) modeling is an economics statistical flexible and widely used model. The models can be used for forecasting, a better understanding of the data, and univariate times series analysis. The analysis can provide a short-run forecast of large data (Douglas C. Montgomery, 2008).

The ARIMA (p, d, q): known as non-seasonal models with three parameters (p, d, q):

- p = autoregressive model order (number of terms).
- d = differencing degree.
- q = moving average order (number of lagged forecast errors).

Q2: What are the processes of applying ARIMA model?

The ARIMA model contains three processes: (AR) autoregressive process, differencing process, and (MA) moving average process.

Q3: What are the other types of economic models we can use for forecasting time series data?

From the research and case study, we explained many types of economic models that can be used for forecasting and analyzing future values like:

(ARMA) autoregressive moving average, (SARMA) a seasonal autoregressive average, (SARIMA) a seasonal autoregressive integrated moving average, (ARIMAX) autoregressive integrated moving average with exogenous variables, (ARMAX) autoregressive moving average with exogenous variables and (SARIMAX) a seasonal autoregressive integrated moving average with an exogenous.

Q4: What are the results of the Qatari GDP dataset forecasting?

In the purpose of time series forecasting of Qatari GDP based on this data was selected, the IBM SPSS statistic program was used to apply the ARIMA model and predict the future value of the Qatari GDP.

The results from applying the ARIMA model in the IBM SPSS program showed that the future Qatari GDP values will continue increasing in the future. As shown in the summary of the results:

- The results from ARIMA (1,1,1) model show that Qatari GDP will continue increasing in the future as it is shown in the figure 17.
- The results from ARIMA (0,1,0) model show that Qatari GDP will continue increasing in the future as it is shown in the figure 19.
- The results from ARIMA (1,1,0) model show that Qatari GDP will GDP will continue increasing in the future as it is shown in the figure 21.
- The results from ARIMA (3,1,1) model show that Qatari GDP will continue increasing in the future as it is shown in the figure 23.

The results show that the ARIMA (1,1,0) model fitted more with Qatari GDP dataset than the other models.

4.1 Recommendation

Due to the analyzing research of the Qatari GDP, it seems the Qatari GDP much depends on gas and oil income.

Firstly, it is recommended Qatar government to make a vision to achieve economic diversification and to reduce dependence on oil and gas as the main source for the GDP.

Secondly, it is recommended Qatar government to make strategies to increase the investment inside and outside the country.

Thirdly, it is recommended to facilitate investment laws and procedures to attract foreign investors to invest in the country such as providing a good environment for investment, flexible tax, and custom system, developing the investment law to protect the investors.

It has been noticed also that Qatari GDP is affected by the political and economic problems that happen around the region. Therefore, it is recommended to make strategies and diplomatic solutions to limit this effect in the future.

5 CONCLUSION

In conclusion, the aim of this paper was to model and forecast Qatari GDP based on the ARIMA approach based on the annual data (from 1980 to 2019). The thesis undertook the effort to identify the most important models that can be used for predicting and forecasting economic activity and helps countries to make better decisions and future strategies according to model results.

The theoretical section was divided into four parts, the first part was a background of economics including microcosmic and macroeconomic, and its components, as well as demand and supply curves indicators, were reviewed.

The second part was an introduction to modeling in the economy and some models used by economists to help them to show, predict, and provide solutions for the economic problems, additionally their types, classification, and own charts.

The third part was an introduction about time series analysis, its history, uses, and examples of applications that are using time series analysis to predict the future values related to its past values in aim to provide a better solution for the problems that could be faced in the future. For any large organization, the analysis by using time series analysis is one of the most important data analytics aspect that helps companies understanding the factors which they are highly crucial for business such as randomness, trends, seasonality, cyclicality, distribution and other.

The fourth part was a brief introduction about (AR) autoregressive model, differencing process: (MA) moving average model, (ARMA) autoregressive–moving-average model, the Box-Jenkins model, (p, d, q) orders, and (ARIMA) autoregressive integrated moving average model with their equations, processes, stages, types, and some examples.

In the practical section, the first aim of this part was to model and to forecast Qatari GDP based on the ARIMA approach and based on the given annual data, and the second aim to be familiar with time series analysis and forecasting by using the ARIMA models. The stages of the ARIMA approach are conducted to obtain an appropriate ARIMA model for the Qatari GDP, and we used this model to forecast the Qatari GDP for the next six years (from 2019 to 2025).

Time series plots and correlogram plots were used for testing the stationarity of the data. Various ARIMA models with different order of autoregressive and moving-average terms are used too. Also, all the results of forecasting processes in the practical part of this research were provided.

From the case study and its results we noticed that Qatar has one of the richest economies in the world, it has one of the highest GDP, and the highest income per person in the world. We expect that the Qatari GDP will continue to rise according to the forecasted values from our model.

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