Research Article

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Mathematical approaches for improving the efficiency of railway transport

Abstract: Current business trends push transport companies to improve the efficiency of their entrepreneurial activities, what is closely related to costs reduction and revenues increase. Many transport companies achieve it empirically, but some mathematical algorithms, mainly from operational research, can be used to improve the efficiency of railway transport by finding the optimal solution of specific problems. The paper is focused on using of selected methods from applied mathematics in railway transport. These methods are used mainly for improving the operational efficiency of the transport company, what tends to decrease overall operating costs. Major impact is done by efficient use of railway vehicles and train staff. It can be achieved by optimization as a flow network problem or assignment problem, what is further described in the paper. Mathematical approaches can also positively influence the whole logistic process.

Keywords: railway transport, optimization, applied mathematics, operational efficiency

1 Introduction

Rapid evolution of production and expansion of economic activities mean that managerial decision-making is more complex and responsible. Main reasons are:

- more possibilities to deal with the situation,
- increasing division of work
- deeper division of work also on strategic level
- wider division of work causes that consequences from incorrect decisions are more noticeable [1, 2]

Systematic management of all activities in the company is currently associated with the use of modern information technologies related to well-established information system. Creating a suitable information system requires quantification of economic problems, what relates to the use of mathematical and statistical models and methods. The result of all activities in the field of systematic approaches and especially quantitative approaches for development and application of methods designed to support managerial decisions is a new scientific discipline known as operational research or operational analysis [3].

Mathematical models used to describe economic processes can be divided according to different criteria:

- coincidence
- deterministic
- stochastic
- time factor

  – static
  – dynamic

- usage of mathematical medium

  – linear
  – nonlinear

Operational research can be characterized as a scientific discipline that focus on the analysis of several types of decision-making problems [4].

Mathematical approaches from operational research have these advantages:

- it is possible to find out necessary information when there is not any conclusion directly from surveyed object or in case that it does not already exist;
- accelerate the decision-making process by shortening all processes that currently take a long time;
- facilitate and rationalize management by making the model clear, concise and it provides logically correct descriptions of reality;

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- allow variant solutions with possibility to select the most suitable one according to various circumstances;
- to prevent possible losses due to incorrect decisions [5].

Application of mathematical approaches in the field of transport is described below.

2 Current research on this field of transport

There are not many similar researches focusing on issues that deal with mathematical approaches which are applied to railway transport. Some studies are done with problems of the bus transport within the town. Topics of these researches are vehicle schedule and also bus-driver schedule. The solution based on applied mathematical approaches of bus-driver schedule is a primary research of Stanislav Paluch's (University of Zilina). The author describes some mathematical algorithms that are used to optimize of bus-driver schedule and gives some model examples. Some basic applications of flow-network problems are the content of his scientific and educational publications [6–8].

Other author Jaroslav Kleprlik (University of Pardubice) deals with schedules in general. In his publications, there he suggests using of the Hungarian method as a solution of assignment problem and its practical use in transport optimization. These mathematical approaches that were mentioned above are the content of the paper and its application in railway transport is also there [9].

Some authors, for example Matus Dlugos, Peter Zuber, Rudolf Dzurus, Jozef Staffen, Jozef Bujna (all from University of Zilina) and Robert Geci (University of Presov) deal with similar issues in their bachelor or diploma thesis [10–15].

3 Background of railway transport operation from mathematical point of view

Operational part of the management of railway transport consists of train timetable, vehicle circulation, run of train staff and operating-economic evaluation. Technical things such as maintenance or infrastructure are not direct part of it, therefore the main emphasis is put on operational is-

sues which are mentioned above. Train timetable is the basic schedule of train operation, which is also available for the traveling public [16, 17].

Vehicle circulation and the run of train staff are directly dependent on train timetable so there is an enormous potential to optimise the operation. Selected mathematical approaches, which are suitable for this kind of optimisation, are the major content of this paper. The run of train requisites can be described and explained separately. The run means regularly recurring schedule of usage time. Train requisites are subjects or objects, which are necessary to join into one set, which can move independently in the transport process. Simply, it is the train, which may consist of locomotives, different types of wagons, vehicle-drivers, conductors, stewards etc. [18, 19].

Basic elements in the transport process are transport hubs, where train requisites can switch from one train into another. In transport hubs, there are also flows of passengers in passenger transport or material flows in freight transport. Directional connection of two transport hubs is called the section. Sequence of transport hubs and sections between these transport hubs is called the route [20].

Almost everything what is related to optimization of the run of train requisites could be described by a graph. The graph is a mathematical object, which consists of ordered pair of sets $G = (V, E)$. First set consists of vertices and other set consists of edges. Basic precondition is that each edge connects two vertices. In case of transport process, vertices might represent trains and edges might represent possible switchings from one train to another. The run of train requisites is specific, because train requisites can switch from one train to another only in one direction therefore the graph must be oriented. Vertices and edges represent values of some attributes very often. In transport process, switching of train requisites might be represented by edges. For example: vehicle-drivers, conductors, stewards etc. can switch from one train to another therefore each edge must have information about duration or cost of that switching. It means that the graph is ordered triplet of sets $G = (V, E, d)$, where d is a function $d: H \rightarrow R^+$ which assign the nonnegative real number to each edge and it represents the duration or costs of the switching. These switchings and their values may represent not only duration or costs, because there can be considered many different quantities. The most important things by choosing values is a way of solution, what means that the result should be minimum (duration, costs…) or maximum (efficiency, usage…). In this paper, there is the method for seeking the minimum [21].

Switching of train requisites within each operating day can be solved by flow network. Basic unit in regu-

lar railway passenger transport is a train t_i. Every train is marked by unique number and its complete characteristics are stated in the train timetable, where are all stations and stops of this train and its temporal position. Majority of trains are short from temporal point of view therefore one train cannot fulfil day-long operation of the railway vehicle or train staff members. This is the reason, why each railway vehicle or train staff member have got the sequence of more trains per one operating day. Train t_i foregoes train t_j in case, when there is on identical place (station or stop) enough time to t_j departure after t_i arrival. The difference between t_j departure and t_i arrival is a temporal dissipation and it is marked as d_{ij} with its value in minutes. There is a set of trains $T = \{t_1, t_2, \ldots, t_n\}$ and the main task is to schedule these trains into operating days. Every train would be just in one operating day. Number of operating days should be as small as possible because number of railway vehicles and train staff members directly depends on number of operating days. In case of train staff members, higher legal standards, mostly labour Code, must be also considered. It specifies maximum working time, duration of breaks and many other associated criteria. This task can be solved by graph G with set of vertices $V_1 \cup V_2 \cup \{s, m\}$, where V_1 is the set of all train arrivals from the set T, V_2 is the set of all train departures from the set T, s is the fictitious source and m is the fictitious mouth. Symbols v_i and w_j are signs for vertices of V_1 and V_2 sets, which represent train t_i and t_j. Set of oriented edges E in the graph G consists of three types of edges. First type includes oriented edges (s, v_i) where $v_i \in V_1$. Second type includes oriented edges (w_j, m) where $w_j \in V_2$. Third type includes oriented edges (v_i, w_j) in case when t_i foregoes t_j with $d(v_i, w_j)$ [6].

4 Case study of optimization

Finding the optimal solution of this problem by flow network is applied on chosen long-haul railway route represented by shown timetable below. This route was chosen for its relevance as closed system, where vehicles from these trains are used only on this route.

The easiest solution is to use the same number of railway vehicles and train staff as number of trains in the timetable. It is possible to reduce the number of railway vehicles and train staff, when some edge (v_i, w_j) of the graph G is used. It means that the railway vehicle and train staff members switch from the train t_i to the train t_j. Every other used edge means saving of another railway vehicle and train staff members. Among used edges, there must not be any edges with the same vertex. It is an assignment of train

departures to train arrivals and as many as possible train departures should have assigned some train arrival. This can be done by searching for the maximum flow in the network. In case of train staff, maximum work time and minimum break time must be respected [22].

Maximum flow in Figure 1 means that there are not any other alternative ways where the flow could be put. Circulation of train staff members may be linked to vehicle circulation, but final number of train staff members should equal or higher than number of vehicles. When the optimal solution of switchings is known, the sequence of these switchings must be done. In the example of the route Kosice – Zvolen, basic temporal period for these trains in the timetable is 24 hours. It means, that each train runs every 24 hours (everyday) repeatedly. Solution for operating days (one operating day = one 24 hours period) can be showed as the permutation $P (i \rightarrow j)$ what shows switchings of railway vehicles from one train to another.

Example of permutation

$$P = \begin{pmatrix} 930° & 931° & 932° & 933° & 934° \\ 933° & 934° & 935° & 936° & 937° \end{pmatrix}$$

Basic precondition is that every train in the timetable is implemented only once during the temporal period. Stated example of the permutation must be completed with switchings between temporal periods (operating days). This can be solved similarly as solution for one operating day. Then it is necessary to change the permutation P into permutation $P^\star$. This adapted permutation is showed in oriented graph (Figure 2), where vertices represent trains and oriented edges represent switchings from one train to another.

Example of permutation

$$P^\star = \begin{pmatrix} 930° & 931° & 932° & 933° & 934° & 935° & 936° & 937° \\ 933° & 934° & 935° & 936° & 937° & 930° & 931° & 932° \end{pmatrix}$$

Required number of railway vehicles is equal to the number of temporal periods of train switchings. The result can be described in the Table 2.

Table 1: Train timetable on the route Kosice – Zvolen (source: authors).

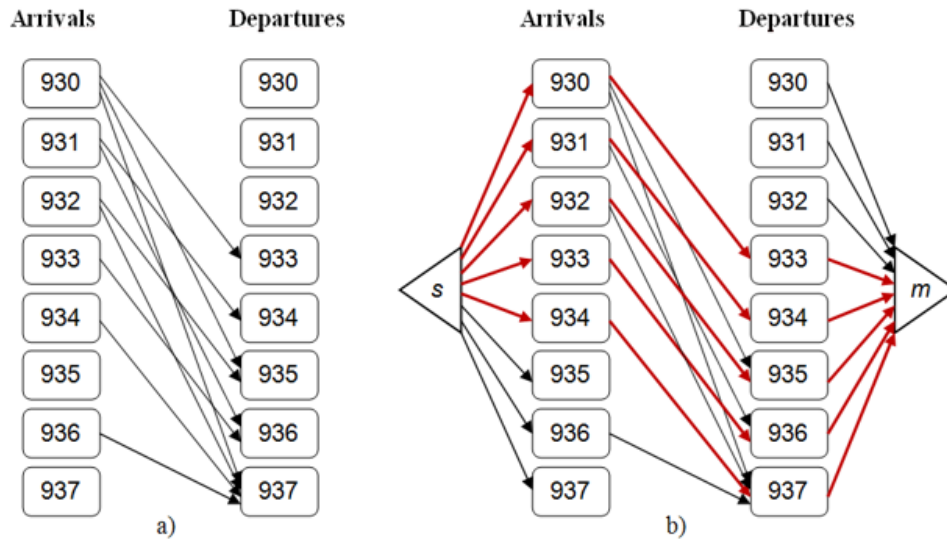| TRAIN NUMBER	DEPARTURE		ARRIVAL	
930	Kosice	5:23	8:49	Zvolen
931	Zvolen	7:13	10:37	Kosice
932	Kosice	9:23	12:49	Zvolen
933	Zvolen	11:13	14:37	Kosice
934	Kosice	13:23	16:49	Zvolen
935	Zvolen	15:13	18:37	Kosice
936	Kosice	15:23	18:49	Zvolen
937	Zvolen	19:13	22:37	Kosice

Figure 1: a) digraph for the route b) graph *G* with maximum flow. All edges in *G* have got capacity = 1
Resultant operating days: D1: 930 ⟶ 933 ⟶ 936; D2: 931 ⟶ 934 ⟶ 937; D3: 932 ⟶ 935

Table 2: Number of railway vehicles and their schedule (source: authors)

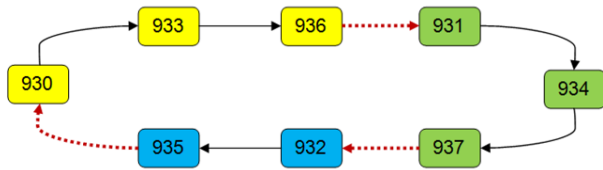| RAILWAY VEHICLE	OPERATING DAY		
	1	2	3
1	930 ⟶ 933 ⟶ 936	931 ⟶ 934 ⟶ 937	932 ⟶ 935
2	931 ⟶ 934 ⟶ 937	932 ⟶ 935	930 ⟶ 933 ⟶ 936
3	932 ⟶ 935	930 ⟶ 933 ⟶ 936	931 ⟶ 934 ⟶ 937

Figure 2: Graphical representation of permutation *P** (source: authors).

Final solution of this type of assignment problem, for example by "Hungarian method" based on the theorem of Egervary and Konig, is going to be found in the matrix $D = (d_{ij})_{i,j=1,2,...,n}$ of *n* elements. Each two of these elements are not located in the same column not even in the same row. Sum of the values that represent elements must be as minimal as possible [9].

Assignment problem is defined as the problem that deals with the optimal assignment of elements from the set, which consists of *n* elements, to the same number of elements from other set. It is assumed that all right-side coefficients values a_i, $i = 1, 2, \ldots, n$ and b_j, $j = 1, 2, \ldots, n$ is equal 1. Matrix *X* consists of variables x_{ij}, which value is equal 1 in case, when there is the *j*-elements assigned to

i-element. When it is not true, x_{ij} is equal 0. Coefficients d_{ij} could be presented as temporal dissipations. Assignment problem can be defined this way:

$$\min z(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \qquad (1)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, n$$

$$x_{ij} \in \{0, 1\} \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, n$$

Practical usage of this formula is shown in the Table 4 below. Appropriate optimization of train requisites schedule in railway passenger transport is an important task of transport technology. Minimizing required number of train requisites and provide maximum efficiency of their usage are main goals [23].

Many attributes have significant influence to solution of this task. In the Table 3, there are time differences between departures and arrivals of every train on the route

Table 3: Time differences between departures and arrivals of trains.

min	930	931	932	933	934	935	936	937
930		1344		144		384		624
931	1126		1366		166		286	
932		1104		1344		144		384
933	886		1126		1366		46	
934		864		1104		1344		144
935	646		886		1126		1246	
936		744		984		1224		24
937	406		646		886		1006	

Table 4: Solution of assignments on the route Kosice – Zvolen.

	930	931	932	933	934	935	936	937
930	0	0	0	1	0	0	0	0
931	0	0	0	0	1	0	0	0
932	0	0	0	0	0	1	0	0
933	0	0	0	0	0	0	1	0
934	0	0	0	0	0	0	0	1
935	1	0	0	0	0	0	0	0
936	0	1	0	0	0	0	0	0
937	0	0	1	0	0	0	0	0

Kosice – Zvolen. Each train is signed with its number. Values of time differences are written only for those trains where the arrival station of one train is the same as the departure station of other train. Values of time differences are converted and written in minutes according to formula 2.

$$T_d = D_{ij} - A_{ij} \qquad (2)$$

where: T_d = Time difference converted to minutes [min]
D_{ij} = Departure of the train marked by ij [hh:mm]
A_{ij} = Arrival of the train marked by ij [hh:mm]

In the Table 4, assigned departure of one train to the arrival of the other train is shown with value 1 in the relevant cell of the table. Every value 0 mean that there is not any assignment between those trains. All values in the Table 4 represent the optimal solution of assignment problem. These values are results of calculation according to formula 1 and there are many approaches and algorithms for doing it. In case of many inputs (trains), using of some software is more relevant than manual calculation from temporal point of view. There are many mathematical programs, which can be useful and applicable for this issue. In this paper, there was used Solver with restrictive conditions according to formula 1. Other restrictive condition is that all values must be binary. This issue is linear problem,

therefore Simplex LP algorithm is chosen method to solve it.

The result of assignment problem solution is the same as result of flow network. Assignment problem is better for cases where there are many trains. Flow network solution is friendlier for these users, who prefer graphical approaches and outputs.

5 Conclusions

Mathematical approaches are very suitable for optimization of all problems that are connected to railway transport operation. In the paper, optimization of the run of train requisites in passenger transport is shown. Two possible approaches are used there. Firstly, this problem is solved on the flow network as a simple task of finding the maximum flow. Secondly, optimization task is transformed to assignment problem and the optimal assigns represent also the optimal solution of the whole task. It is necessary to pay more attention to using these and other mathematical methods in a field of transport, because they can make the railway transport operation more efficient. Every software, which can solve these mathematical tasks fast, can

be used and applied to field of railway transport. Its application could help in all aspect of transport business either in operational or strategic level of decision-making and the whole managerial issues. Synergic effects are directly influenced by efficiency of train operation. The most relevant impact is that costs would decrease by efficient using of train requisites mainly railway vehicles and train staff members such as vehicle-drivers, conductors and stewards. Mathematical solution is better than empirical solution, because there do not have to be any person who make working schedules since everything is done automatically. There is also the possibility to find many alternative solutions is case that some circumstances are changed. In case when the train is delayed, the software, which use mathematical methods, can easily calculate and remake the plan of working schedule. Then the vehicle from delayed train could be used on another train than it was previously planned. Current trends are very supportive for using automatic procedures and techniques therefore these issues are very actual. They can provide new benefits and different point of views in the field of railway transport.

Conclusions should be summarized:

1. Operating processes are more efficient and less time-consuming,
2. Decision-making on operational and strategic level is more flexible,
3. Costs reduction mostly in direct costs influenced by operation efficiency,
4. People could focus on creative work because mechanic work is done by software.

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