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A CONTRIBUTION TO STUDY OF DEEP BED FILTRATION USING WOUND CARTRIDGE FILTERS PART I: MATHEMATICAL MODEL

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A quantitative relationship between the pressure drop of a clogged cylindrical fibrous filter bed and the extent of particle deposition is presented. The starting point is the modified model of the flow through filtration medium which combines the tube flow and flow around solid bodies. The effect of particle deposition is assumed to increase the fibre diameter and to decrease the local porosity. To complete the description, the macroscopic conservation equation and the deposition rate equation are also specified. The method of solution of these equations, which yields the values of effluent concentration, specific deposit, and pressure as functions of time and position, is also proposed and discussed.

Introduction

Wound cartridge filters rank among the most widely used types of deep bed filters which have the advantage of being simple, with the minimum installation cost. Areas of common use are in fields as diverse as water purification, waste water treatment, activated sludge processes, and cleaning of electroplating solutions. They are also used for protection of hydraulic and lubricating systems, as well as in beverage manufacturing, and many other industrial solid-liquid separation processes.

Wound cartridges consist of spun stable fibres of wool, cotton, glass and/or various synthetic materials. The fibres are wound on a hollow perforated core until the desired cartridge thickness is achieved. The porosity and pore size of the filtration medium is determined by the fibre nature and diameter and by the control of winding pitch and tension.

The separation process is realised as the deep bed filtration of particles which takes place within the pores created by a fibrous material. The particles attached on individual fibres decrease the available pore volume and change the geometry and the structure of the medium. In the constant velocity filtration mode, the pressure drop across the filter bed increases due to the loss of permeability, and, eventually, filtration efficiency is changed.

Deep bed filtration has been studied by many researchers. Detailed reviews of aspects of these studies that include theoretical considerations were given for example by authors[1-4]. The theories can be grouped into two categories: macroscopic and microscopic[1]. The macroscopic studies are aimed at the phenomenological description of the filtration process, which is usually based on the prediction of the time dependence of the effluent concentration and on the pressure drop required to maintain a given throughput. Microscopic theories are intended to provide information on and insight into the mechanism of particle deposition.

Although the problem of great practical importance, the mathematical modelling of deep bed filtration using wound tubular fibrous media has not been satisfactorily solved yet. Thus the main objective of this study is to apply the model of the flow of Newtonian liquids through fixed beds of particles suggested by authors[5,6] to the flow through wound filtration cartridges, and, consequently, to develop a phenomenological model enabling to calculate the pressure drop during the deep bed filtration run.

Theoretical

The following considerations are based on the modified capillary fixed bed model which was proposed in Refs[5,6] for the calculation of a steady laminar flow of Newtonian and purely viscous non-Newtonian fluids through fixed beds of spherical and nonspherical particles. The model is based on a modification of the linear characteristic dimension of the system combining the ideas of the flow through capillaries and the flow around solid bodies.

Let us consider a steady laminar flow of a Newtonian fluid of constant density ρ and constant viscosity μ through an elementary cylindrical volume of bed of fibrous medium having a radius r and thickness dr. The flow results from a difference in pressure *dp* at the both cylindrical surfaces of the elementary volume. The system is illustrated in Fig. 1. Denoting the superficial fluid velocity u (we consider $u > 0$ in the case of the flow inward the cylinder) and the porosity of the clean filter medium ε_0, the modified capillary model[5] can be expressed as

$$\frac{dp}{dr} l_{ch} = \mu \frac{2u}{\varepsilon_0 l_{ch}} \tag{1}$$

Here

$$l_{ch} = \frac{\varepsilon_0}{a_p \varphi (1 - \varepsilon_0)} \tag{2}$$

is the characteristic linear dimension of the filter medium void space where

$$a_p = \frac{4}{d_0} \tag{3}$$

is the specific surface of the fibres, and d_0 is the diameter of fibres composing the filter medium. The quantity φ (bed factor) is a dynamical characteristic of the filter medium representing a ratio of the total and friction drags of the filter medium. For Newtonian fluids it has a constant value in the creeping flow region[5,6]. For a random bed of spherical particles it is $\varphi = 1.5$, for a bed of nonspherical particles (including beds of fibres) it can be simply determined experimentally[6].

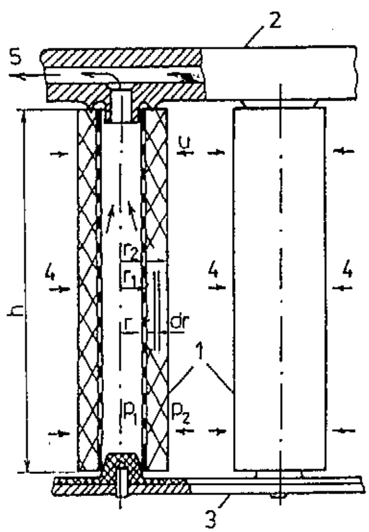

Fig. 1 Schematic diagram of a typical wound cartridge filter: 1 wound cartridge, 2 liquid distributor, 3 sealing end, 4 feed inlet, 5 filtrate outlet

The superficial velocity u of the flow at the distance r from the axis of cartridge core is given as

$$u = \frac{\dot{V}}{2\pi r h} \tag{4}$$

where $\dot{V}$ is the fluid volumetric flow rate. Substituting for u in Eq. (1) from Eq. (4), and integrating over the limits of p_1 to p_2 and from r_1 to r_2, the pressure drop expression for the flow through clean cartridge is obtained in the form

$$\Delta p = p_2 - p_1 = \frac{\mu \dot{V}}{\pi h \varepsilon_0 l_{ch}^2} \ln\left(\frac{r_2}{r_1}\right) \tag{5}$$

To modify Eq. (1) to the dynamic behaviour of a deep bed filter, we should include the influence of the porous medium permeability reduction due to pore clogging. If we take $\sigma(r, t)$ as the volume of deposited particles per unit volume of filter medium (which is usually referred to as specific deposit), and if we assume that the linear dimension l_{ch} depends only on the bed porosity ε and the modified fibre diameter d (the bed factor φ does not vary with retention), we obtain after some rearranging of Eq. (1)

$$\frac{dp}{dr} = \frac{\mu \dot{V}}{\pi r h \varepsilon[\sigma(r,t)]\, l_{ch}^2\{\varepsilon[\sigma(r,t)]\, d[\sigma(r,t)]\}} \tag{6}$$

The porosity (or void fraction) of the filter medium ε changes with the time as the particle accumulation within the bed increases. If the deposited matter of the average porosity ε_d forms relatively smooth coating outside the filter fibres, ε and σ can be related by the simple expression[1]

$$\varepsilon = \varepsilon_0 - \frac{\sigma}{1 - \varepsilon_d} \tag{7}$$

The effect of deposition is assumed simultaneously to increase the fibre diameter. We suppose that the porous medium is composed of n cylindrical fibres of diameter d_0 and length L per unit of bulk volume. By regular coating of the fibres, the diameter increases to d and porosity of porous medium changes from

$$\varepsilon_0 = 1 - nL\frac{\pi d_0^2}{4} \tag{8}$$

to

$$\varepsilon = 1 - nL\frac{\pi d^2}{4} \tag{9}$$

Combining Eqs (8), (9), and (7), we get

$$d = d_0 \sqrt{1 + \frac{\sigma}{(1 - \varepsilon_d)(1 - \varepsilon_0)}} \tag{10}$$

To complete the description, the macroscopic conservation equation and the deposit rate (kinetic) equation need to be specified. The conservation equation is similar to that used in any fixed bed process involving the transport of matter from the mobile to the stationary phase. For a radial flow with a changing cross-sectional area, this can be written as

$$-\frac{1}{r}\frac{\partial(ruc)}{\partial r} + \frac{\partial(\sigma + \varepsilon c)}{\partial t} = 0 \tag{11}$$

The independent variables are the radial distance r and the time t. The quantity c refers to the volume concentration of particulate matters in the liquid. It is often assumed that all moving particles are neglected with respect to the retained ones[1]. This simplification is only reasonable if εc is small compared with σ. The second term in balance equation (11) is then simplified considerably.

The equation of rate of filtration, in general, can be expressed as[1,2]

$$\frac{\partial \sigma}{\partial t} = uG(c, \sigma, \vec{\omega}) \tag{12}$$

Here $\vec{\omega}$ is the parameter vector including quantities such as fibre size, density difference, fluid viscosity, deposit morphology, etc. The empirical determination of the functional form (12) from experimental data has been attempted by a number of investigators[1-4].

Solution of the Model

To obtain a theoretical relation between the pressure drop Δp and the time t during the deep bed filtration run, it is necessary to establish first the dependence of the specific deposit σ on the radius r for various retention times. This can be determined from the kinetic and the conservation equations (12) and (11), considering the appropriate initial and boundary conditions.

Herzig et.al.[2] demonstrated, for planar sand filter, that if the kinetic equation (12) is of the first order, which means that the rate of the transfer of particles to the porous medium is a linear function of the concentration c, then Eqs (11) and (12) are equivalent to a pair of ordinary differential equations. In this study we have modified this approach for the computations in the case of deep bed filtration using tubular fibrous media.

We consider a cylindrical porous bed limited by two surfaces with a radius r and the outer radius r_2. The total mass balance for the time interval

[0, t] is then given by

$$\int_0^t \dot{V} c_0(\hat{t})\, dt = \int_0^t \dot{V} c(r, \hat{t})\, d\hat{t} + \int_r^{r_2} 2\pi h \hat{r} \sigma\, d\hat{r} \tag{13}$$

At the depth r, the first order kinetic equation can be written as

$$c(r, t) = \frac{1}{u} \frac{1}{F(\sigma, \vec{\omega})} \frac{\partial \sigma}{\partial t} \tag{14}$$

By substituting the expression of $c(r,t)$ and $\dot{V}$ into the first right-hand side term of Eq. (13) and by changing the variable (i.e., if σ is substituted for t), Eq. (13) becomes

$$\int_0^t \dot{V} c_0(\hat{t})\, d\hat{t} = \int_0^{\sigma} 2\pi r h \frac{1}{F(\sigma, \vec{\omega})}\, d\sigma + \int_r^{r_2} 2\pi h \hat{r} \sigma\, d\hat{r} \tag{15}$$

If Eq. (15) is differentiated with respect to r, we obtain after some rearranging

$$\frac{d\sigma}{dr} = F(\sigma, \vec{\omega}) \left(\sigma - \frac{1}{r} \int_0^{\sigma} \frac{d\sigma}{F(\sigma, \vec{\omega})} \right) \tag{16}$$

This is an ordinary differential equation from which, using boundary condition $\sigma = \sigma_2(t)$ for $(r, t) = (r_2, t)$, the specific deposit distribution through filter medium at the given time t can be obtained.

To complete the description of the filtration process the specific deposit in the inlet layer σ_2 is calculated from the kinetic equation as

$$\frac{d\sigma_2}{dt} = F(\sigma_2, \vec{\omega})\, u(r_2)\, c_0 \qquad \sigma_2(t{=}0) = 0 \tag{17}$$

and the concentration profile through filter medium as

$$\frac{dc}{dr} = F(\sigma, \vec{\omega})\, c \qquad c(r{=}r_2, t) = c_0(t) \tag{18}$$

If the specific deposit $\sigma(r,t)$ is known from the solution of the equations (16) and (17) the pressure drop across the filter medium can be obtained by the integration of Eq. (6) over the limits from r_1 *to* r_2.

This can only be made for some simple functions $F(\sigma, \vec{\omega})$. Most of the proposed expressions can be derived from general equations suggested by Ives[7] and Mohanka[8]. Here we consider a simple form of this function as

$$F(\sigma, \bar{\omega}) = \lambda\left(1 - \frac{\sigma}{\sigma_{max}}\right) \quad (19)$$

in which λ is the filter coefficient and σ_{max} is the maximum value of σ to be achieved during the filtration run. Using Eq. (19), the integration of Eq. (17) leads to

$$\sigma_2 = \sigma_{max}\left(1 - \exp\left[-\frac{u_2 \lambda c_0}{\sigma_{max}} t\right]\right) \quad (20)$$

It can also be shown that

$$\int_0^{\sigma} \frac{d\sigma}{F(\sigma, \bar{\omega})} = -\frac{\sigma_{max}}{\lambda} \ln\left(1 - \frac{\sigma}{\sigma_{max}}\right) \quad (21)$$

To solve the system of equations (16), (18), and (6) using relations (20) and (21), the Runge-Kutta-Merson method with an automatic adjustment of integration step was used. The calculations were carried out using a Pascal computer code. The estimates of the model parameters are given in Table I. The plots of pressure drop-time dependencies for chosen model parameters variations are given in Figs 2 - 4. In all cases, the filtration medium is assumed to be initially free of deposited particles.

It can be noted from Figs 2 - 4 that the presence of accumulating deposits modifies the pressure drop in a complex manner. Initially it increases and then it is levelling at an upper limiting value Δp_{lim}. At this stage of the filtration process, the interstitial velocity reaches a critical value at σ_{max} when no further deposition takes place (the rate of detachment equals that of adherence). The plots of the pressure drop vs. time indicate that for lower values of σ_{max} and ε_d, the increase in pressure drop is nearly linear becoming curved at the end to reach the limiting value. If the parameters σ_{max} and ε_d have larger values, the presence of deposited particles has nearly negligible effect on the pressure drop in the first stage of the process. Also a larger time is required to reach the limiting steady state value of pressure drop in this case.

A graphical representation of the dependence of the upper limiting pressure drop Δp_{lim} on σ_{max} and ε_d is shown in Fig 5. As it can be seen the value of Δp_{lim} increases rapidly for ε_d higher then 0.9 as well as for increasing σ_{max}. On the other hand, the limiting pressure drop is not affected by the value of the filter coefficient λ. For the model parameters used it reaches the constant value of 1267 Pa.

Table I Model parameters

Parameter	Fig. 2	Fig. 3	Fig. 4	Fig. 5
Characteristics of the filtration medium				
Inner diameter, r_1, m	0.016	0.016	0.016	0.016
Outer diameter, r_2, m	0.032	0.032	0.032	0.032
Height, h, m	0.3	0.3	0.3	0.3
Product of $a_p\varphi$, m^{-1}	156410	156410	156410	156410
Clean medium porosity, ε_0	0.61	0.61	0.61	0.61
Feed and process characteristics				
Superficial velocity, u_2, ms^{-1}	3.3×10^{-4}	3.3×10^{-4}	3.3×10^{-4}	3.3×10^{-4}
Water viscosity, μ, mPas	1.006	1.006	1.006	1.006
Feed concentration, c_0	1.43×10^{-4}	1.43×10^{-4}	1.43×10^{-4}	1.43×10^{-4}
Variables tested				
Filter coefficient, λ, m^{-1}	50	50	5 - 500	50
Maximum specific deposit, σ_{max}	0.003 - 0.04	0.02	0.02	0.003 - 0.05
Deposit porosity, ε_d	0.9	0.7 - 0.97	0.9	0.7 - 0.97

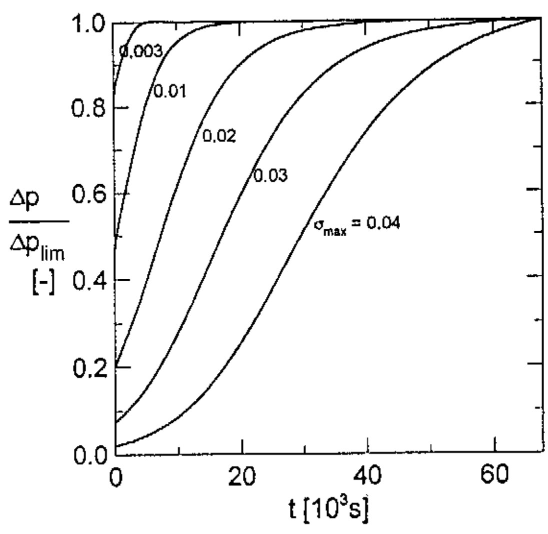

Fig. 2 Dimensionless pressure drop as a function of time for various values of maximum specific deposit

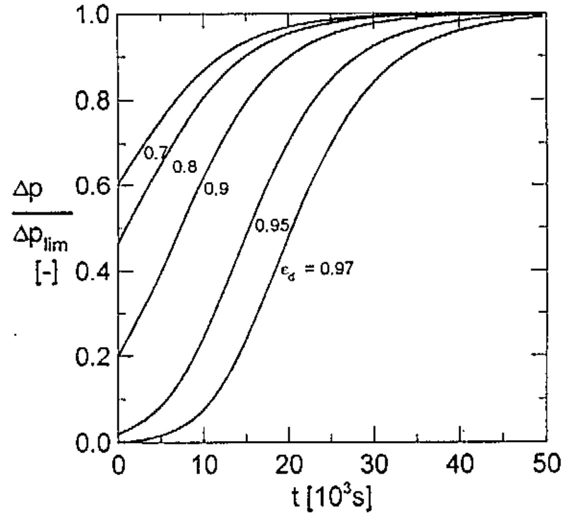

Fig. 3 Dimensionless pressure drop as a function of time for various deposit porosities

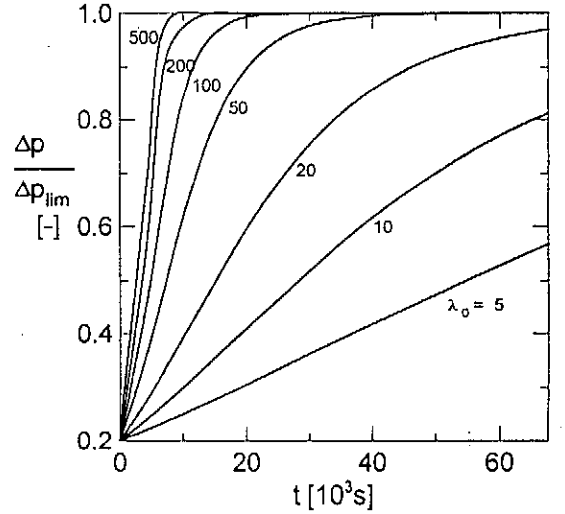

Fig. 4 Dimensionless pressure drop as a function of time for various filter coefficients

In this connection it is necessary to note that a behaviour has also been mentioned in literature in which the pressure drop-time dependence can be a line curving upwards. This is usually due to blocking of pores along with formation of a surface layer of deposit on the inlet face of the filter material. To describe these phenomena, the blocking deposit morphology and the cake formation mechanism should be incorporated into the model.

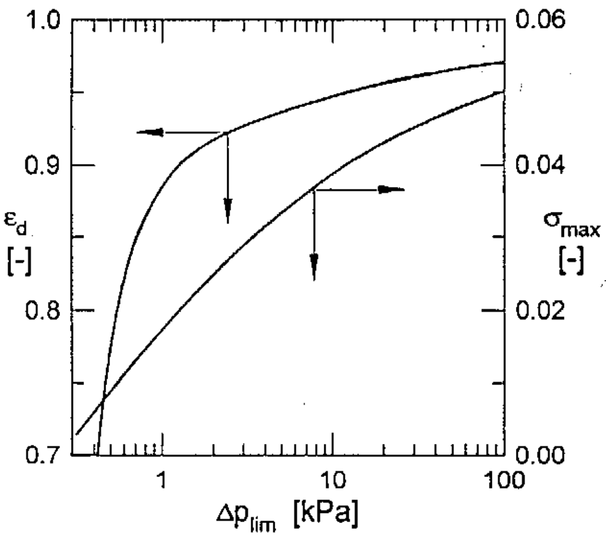

Fig. 5 Upper limiting pressure drop as a function of deposit porosity and maximum specific deposit

Conclusion

In this paper, a model governing the transport and capture of suspended solid particles in a cylindrical fibrous filtration medium is presented. The effect of deposition is assumed to increase the fibre diameter and to decrease the local porosity.

The increase in pressure drop of clogged filter as a function of the maximum specific deposit σ_{max}, deposit porosity ε_d, and filter coefficient λ is also calculated. On the basis of the assumed morphology (smooth coating) it is shown that the model predicts, at least qualitatively, the deep bed filtration using commercial wound cartridge filters.

However, the results which were obtained by the solution of the model presented should be viewed as estimates. The comparison of calculated results with experimental data will be given in a subsequent paper.

Symbols

a_p	fibre specific surface, m $^{-1}$
c	volume concentration of solid particles
d	modified fibre diameter, m
d_0	fibre diameter, m
$F(\sigma, \bar{\omega})$	function in Eq. (12), m $^{-1}$
$G(c, \sigma, \bar{\omega})$	function in Eq. (10), m $^{-1}$

h	height of the filtration cartridge, m
l_{ch}	characteristic linear dimension of filtration medium, m
L	fibre length, m
n	number of fibres
p	pressure, Pa
Δp	pressure drop, Pa
r	cartridge radius, m
$\hat{r}$	radial variable, m
t	time, s
$\hat{t}$	time variable, s
u	superficial velocity, $m s^{-1}$
$\dot{V}$	volumetric flow rate, $m^3 s^{-1}$
ε	variable porosity
ε_0	clean filtration medium porosity
ε_d	deposit porosity
λ	filter coefficient, m^{-1}
μ	fluid viscosity, Pas
σ	specific deposit
σ_{max}	maximum specific deposit
$\vec{\omega}$	vector of parameters in Eq. (10), m^{-1}
φ	bed factor

Indexes

0	initial value
1	outlet (inner radius)
2	inlet (outer radius)

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