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ADAPTIVE DISCRETE PID CONTROLLER

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The Ziegler-Nichols method was used for the setting of controller parameters. The ultimate (critical) gain and period have been calculated from the discrete model of the system obtained from on-line identification. The application of the adaptive controller has been made with good results on distillation column.

Introduction

The *PID* controller setting, developed by Ziegler and Nichols[1] *(Z-N)* half a century ago, is still widely used in industry. In this well-known approach the parameters of controller are calculated from the ultimate (critical) gain K_u and the critical period T_u of the closed-loop system. These critical values are obtained from experimental setting of the *P*-controller in feedback. The gain of controller is increased until oscillation with the constant amplitude is obtained. The oscillating poles of closed loop are then located on imaginary axis. Åström and Hägglund[2] proposed to use a relay in the feedback loop instead of the *P*-controller. The manipulated variable has a form of rectangle impulse train and the shape of controlled variable is nearly sinusoidal. The ultimate gain K_u is then given as a ratio of the first harmonic of the relay amplitude D and the process output amplitude A

$$K_u = \frac{4D}{\pi A} \tag{1}$$

This method is much simpler then the classical *Z-N* method but gives the ultimate values with an error[2,3].

The values of the ultimate gain and the period differ for discrete step function input signal with sampling period T from continuous input signal and thus will be further designated as $K_u(T)$ and $T_u(T)$. Sampling and holding operations can be aproximated by the dead time $T/2$, which brings phase lagging into closed-loop and reduces the critical frequency. The same methods as those described above can be used for discrete system with a selected sampling period. If a discrete process model is known from identification, the ultimate data $K_u(T)$ and $T_u(T)$ may be calculated from the condition that the poles of closed-loop transfer function lie on unit circle of z-plane[4,5]. This method can be used for an adaptive controller, when on-line identification in each sampling period is made, as it will be shown later.

Determining Ultimate Values

The analytic determination of the ultimate values K_u and T_u for the continuous transfer function differs from the discrete one. In the first case the roots of the characteristic equation of closed-loop must lie on the imaginary axis and in left-hand side of the s-plane. The computation of K_u and T_u is made by substituting $s = i\,\omega_u$ in the characteristic equation. This solution is the same as the experimental setting of P-controller by *Z-N* method.

The relationships for a discrete system will be now derived. A single input single output (SISO) system model in the form of difference equation is given by

$$A(z^{-1})y(k) = z^{-d}B(z^{-1})u(k) \tag{2}$$

where u is the input, y is the output, d is the time delay expressed as an integer multiple of the sampling period T and A and B polynomials are defined by

$$A(z^{-1}) = 1 + \sum_{i=1}^{n} a_i z^{-i}, \quad B(z^{-1}) = \sum_{i=1}^{n} b_i z^{-i}$$

The transfer function F_s of the system is given as

$$F_s(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \tag{3}$$

and the transfer function F_c of the controller as

$$F_c(z^{-1}) = K \tag{4}$$

The closed-loop transfer function for set point changes can be written as

$$F(z^{-1}) = \frac{z^{-d}KB(z^{-1})}{A(z^{-1}) + z^{-d}KB(z^{-1})} \tag{5}$$

The roots of characteristic equation lie on the unit circle and inside the unit circle of the z-plane when the condition $K = k_u(T)$ is fulfilled. A pair of complex roots making quadratic three-term $z^2 + Cz + 1$ or a root -1 making two-term $z + 1$ lies on the unit circle. The characteristic equation must be divisible by one of this terms and it leads to the solving of polynomial equations (6) or (7)

$$z^{n+d}\left[A(z^{-1}) + z^{-d}K_uB(z^{-1})\right] = (z^2 + Cz + 1)z^{n+d-2}D(z^{-1}) \tag{6}$$

where

$$D(z^{-1}) = 1 + \sum_{i=1}^{n+d-2} d_i z^{-i}$$

$$z^{n+d}\left[A(z^{-1}) + z^{-d}K_uB(z^{-1})\right] = (z + 1)z^{n+d-1}E(z^{-1}) \tag{7}$$

where

$$E(z^{-1}) = 1 + \sum_{i=1}^{n+d-1} e_i z^{-i}$$

The solution of Eq. (7) is simple. The root -1 forms the oscillating component $(-1)^k$, which correspondes to a continuous function $\cos(\pi t/T)$ with ultimate period $T_u = 2T$. The variable z is defined in Z-transformation as

$$z = e^{i\omega T} = \cos\omega T + i\sin\omega T \tag{8}$$

Equation (8) has the solution $z = -1$ for $\omega_u = 2\pi/T_u = \pi/T$. The ultimate gain for open-loop system is then given by

$$K_u(T) = -\frac{1}{F_s(-1)} \tag{9}$$

Equation (6) can be solved e.g. by comparison of terms with the same powers of variable z. The correct solution must be selected from Eqs (6) or (9) according to the condition that all the other roots of characteristic equation are stable.

The term $z^2 + Cz + 1$ has its equivalent in continuous form as the denominator $z^2 - 2\cos\omega T + 1$ of Z-transformation of function $\cos\omega t$ or $\sin\omega t$ and the constant C can be then expressed as

$$C = -2\cos\omega_u T \tag{10}$$

The equation for computing ultimate period can be easely derived from Eq. (10)

$$T_u(T) = \frac{2\pi T}{\cos^{-1}\left(-\frac{C}{2}\right)} \tag{11}$$

Example 1

For $n = 2$ and $d = 0$ the critical gain is (from Eq. (6))

$$K_u(T) = \frac{1 - a_2}{b_2} \qquad \text{if} \quad (a_1 + K_u b_1)^2 - 4(a_2 + K_u b_2) \le 0$$

or else from Eq. (9)

$$K_u(T) = \frac{a_1 - a_2 - 1}{b_2 - b_1}$$

The determination of discrete ultimate values was simulated on underdamped, non-minimum phase and dead-time processes. Discrete ultimate values approach the continuous ones only if the sampling period T is very small with regard to the time constants of the process. The difference of ultimate gain and period is smaller than 5% when sampling interval is about 1/50 of the dominant time constant of the process. The discrete ultimate values from relay experiment differ from the calculated ones especially for the larger sampling period.

Example 2

The process with transfer function

$$F_s(s) = \frac{1}{(4s+1)(2s+1)(s+1)}$$

has $K_u = 11.25$ and $T_u = 6.717$. The discrete ultimate values for various sampling periods are shown in Table I[6]

Table 1 Ultimate gains and periods

Sampling period T	$K_u(T)$ from Eqs (6) and (11)	$T_u(T)$ from Eqs (6) and (11)	$K_u(T)$ from relay	$T_u(T)$ from relay
0.1	10.36	6.98	9.57	7.2
0.2	9.62	7.23	8.65	7.6
0.5	8.00	7.93	7.82	8.0
1.0	6.39	8.97	7.82	8.0

Controller Setting

The transfer function of the discrete *PID* controller can have various forms according to how the *I* and *D* action factors are discretised and whether control deviation $e(k)$ or output signal $y(k)$ is employed (e.g.[7,8]). Here the equation of *PID* controller was used with replacement of the integral by the trapezium method

$$u(k) = p_0 e(k) + p_1 e(k-1) + p_2 e(k-2) + u(k-1) \tag{12}$$

where

$$e(k) = y_r(k) - y(k)$$

$$p_0 = K\left[1 + \frac{1}{2T_i} + \frac{T_d}{T}\right] \tag{13}$$

$$p_1 = K\left[1 + \frac{2T_d}{T} - \frac{T}{2T_i}\right] \tag{14}$$

$$p_2 = K\frac{T_d}{T} \tag{15}$$

The discrete version of *Z-N* method was suggested by Takahashi et al.[9], where the sampling period T is used as a parameter in equations for the calculation of controller constants

$$K = 0.6K_u\left[1 - \frac{T}{T_u}\right] \tag{16}$$

$$T_1 = 0.6\,T_u\left[1 - \frac{T}{T_u}\right] \tag{17}$$

$$T_d = \frac{0.125\,T_u}{1 - \frac{T}{T_u}} \tag{18}$$

These constants are equal to the classical *Z-N* setting when the sampling period $T = 0$. The discrete controller has the similar behaviour as the continuous one when the discrete values $K_u(T)$ and $T_u(T)$ are used. It is also possible to use many other method for contoller design from ultimate values, based e.g. on the pole placement or on the amplitude and phase margin.

Adaptive Control

The algorithm for the adaptive control (self-tuning controller) contains the following procedure for each sampling period T:

1) The parameters of the model (2) are estimated by on-line identification.
2) The ultimate values $K_u(T)$ and $T_u(T)$ are calculated from Eqs (6) and (11) or from Eq. (9) and consequently $T_u(T) = 2T$.
3) The setting of the controller parameters is made by Eqs (12) to (18).

Problems of an adaptive control and a on-line identification are described in greater detail e.g. in[8,10].

Experiments on the Binary Distillation Column

The experiments were carried out on the distillation column, placed in the laboratory of The Department of Process Control and Computer Techniques. The distillation column consists of 7 cap trays diameter of which is 150 mm. It has an electrically heated reboiler and a total condenser. The binary mixture methanol-water at its boiling point entred the third tray. The power of the reboiler in steady state was 7.5 kW and feed flow was 20 mol min^{-1}. The composition measurement on each tray was replaced by the temperature measurement which is, for known atmospheric pressure, directly proportional to the composition.

The control loop with power of the reboiler as input signal and the temperature on the second tray as output signal was used for the purpose of these experiments. The input variable was represented by the numbers on the input of D/A convertor and the output variable by the electric voltage. The control was realized by the microcomputer PP-06 (compatible with the computer PC/XT IBM) connected with the distillation column by an interface unit. The sampling period T had the duration of 120 s.

The experiments can be divided into two parts. Firstly the three methods for determining of discrete ultimate values were compared and secondly adaptive control was realized.

First of all $K_u(T)$ and $T_u(T)$ were determined by the classical experimental way according to *Z-N* method. The courses of the controlled and manipulated variable for the found out ultimate gain $K_u(T) = 26\,000\ V^{-1}$ are shown in Fig. 1. The manipulated variable $u(k)$ was limited in D/A converter on values $\mp 2\ 047$ what corresponds to the changes of power in scope ∓ 700 W. The ultimate period was $T_u(T) = 240$ s.

The results of the method with relay in a feedback loop are shown in Fig. 2. The amplitude of $y(t)$ was about 0.1 V, the amplitude of the relay was $\mp 2\ 047$ and from Eq. 1 $K_u(T)$ was about 26 000 V^{-1}. The ultimate period $T_u(T)$ was longer than 240 s owing to the noise for some time intervals.

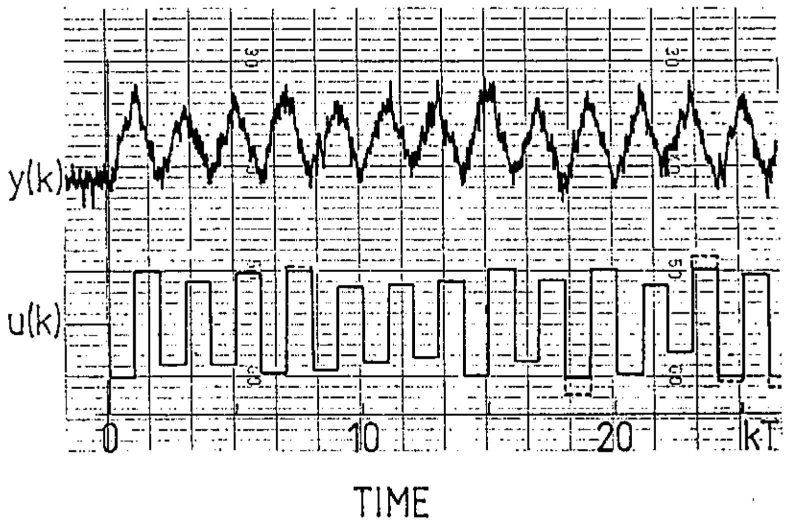

Fig. 1 Closed-loop response for *P*-controller gain K_u = 26 000 V^{-1} (scale for $y(k)$: 1 division = 0.01 V)

Calculation of ultimate values from difference equation was used as the third method. The identification was made by a recursive (on-line) least-squares-method (RLS). The pseudorandom binary (PRB) signal with amplitude of ∓ 400 W was employed as the input signal. The model of second order without time delay was chosen as a good aproximation of the dynamic behaviour. The parameters of model changed in every step of identification owing to a nonlinearity of column and noise signal. The values of ultimate gain $K_u(T)$ was computed in the range of 20 000 - 35 000 V^{-1}, ultimate period $T_u(T)$ was permanently $2T$ = 240 s.

It can be seen that the critical values computed from all three experiments agree very well. The third method can be then used for the adaptive control algorithm.

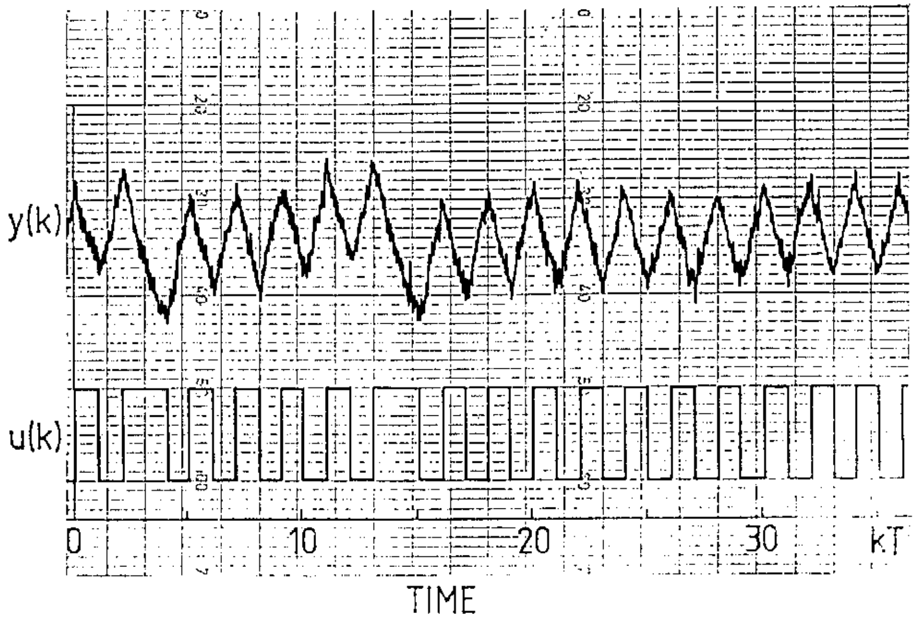

Fig. 2 Closed loop response for relay in feedback loop

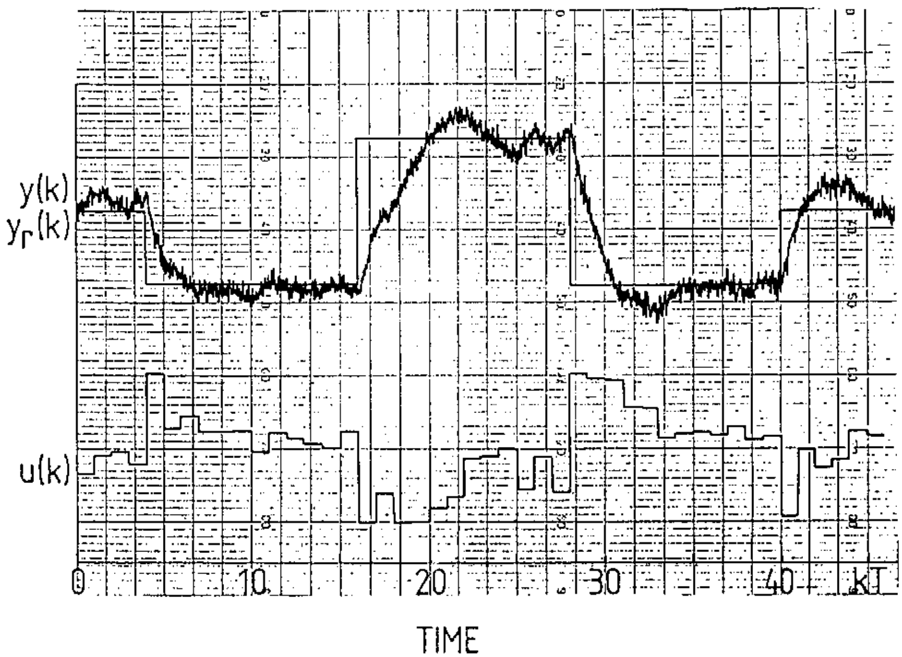

Fig. 3 Adaptive control of distillation column

The control of the distillation column was realized in the following way: In the first 15 sampling instants, only the identification in an open loop was realized. The input PRB signal was generated in computer and model parameters were computed by RLS method with a forgetting factor changed exponentially from 0.6 to 0.98. Afterwards the control loop was closed and the control algorithm worked with variable set point y_r adjustmented in the computer programme. The identification RLS algorithm used then a variable forgetting factor. The performance of this experiment is shown in Fig. 3. The calculated ultimate gain ranged from 19 800 to 38 500 V^{-1}, the ultimate period was mostly 240 s, but it was for some models a little longer - max. 331 s.

Indefined disturbances were found in the operation of column (e.g. in steps k = 18 - 20 in Fig. 3), but the controller reduced them well. The quality of the tested algorithm is comparable with the other methods, as the dead-beat, the quadratic criterion and the minimum variance controllers tested on the same column, which results are refered in[11].

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