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MULTIVARIABLE CONTROL OF A DISTILLATION COLUMN

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Two methods of automatic control of multivariable systems design are compared in this paper: Inverse Nyquist Array and decoupling method. Both methods reduce the non-diagonal elements of the system transfer functions matrix by means of suitable compensator. The multivariable control problem is in such way, simplified to single variable control loops. The results of the simulation and experiments on the real pilot plant distillation column are given.

Introduction

Consider an m-input m-output linear system described by the discrete model

$$\underline{F}(z^{-1}) = \underline{A}(z^{-1})^{-1}\underline{B}(z^{-1}) \tag{1}$$

where $\underline{A}(z^{-1})$ and $\underline{B}(z^{-1})$ are the $(m \times m)$ polynomial matrices of n-order polynomials (the backward shift operator z^{-1} is omitted for simplification)

$$\underline{A} = \underline{I} + \underline{A}_1 z^{-1} + ... + \underline{A}_n z^{-n}$$

$$\underline{B} = \underline{B}_1 z^{-1} + \dots + \underline{B}_n z^{-n}$$

The parameters of the model can be identified by using the least- squares algorithm. When the identification is repeated for every output using model in a form

$$Y_i = \sum_{j=1}^{m} \frac{B_{ij}}{A_{ii}} U_j \qquad i = 1, 2, \dots, m \tag{2}$$

the matrix $\underline{A}$ is diagonal. The compensator described by the transfer function $\underline{G}$ is placed ahead of system (Fig.1) and the resulting transfer function $\underline{H}$ is given by

$$\underline{H} = \underline{F}\underline{G} \tag{3}$$

The problem of interactions is avoided when the matrix $\underline{H}$ is diagonal or diagonally dominant. Then the closed-loop system equation can be written as (see Fig. 2)

$$\underline{A}^{-1}\underline{B}\underline{G}\underline{K}^{-1}\underline{P}^{-1}\underline{R}\underline{W} = \left(\underline{I} + \underline{A}^{-1}\underline{B}\underline{G}\underline{K}^{-1}\underline{P}^{-1}\underline{Q}\right)\underline{Y} \tag{4}$$

where $\underline{U}$ is the $(m \times 1)$ input vector, Y is the $(m \times 1)$ output vector, $\underline{W}$ is the $(m \times 1)$ reference signal vector and $\underline{K}$, $\underline{P}$, $\underline{Q}$ and $\underline{R}$ are $(m \times m)$ diagonal polynomial matrices of controller. The integral action $\underline{K}$ is taken in the form $K_{ii} = 1 - z^{-1}$, $i = 1, 2, \dots, m$.

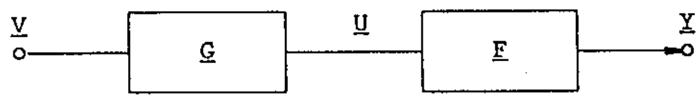


Fig. 1 System with compensator

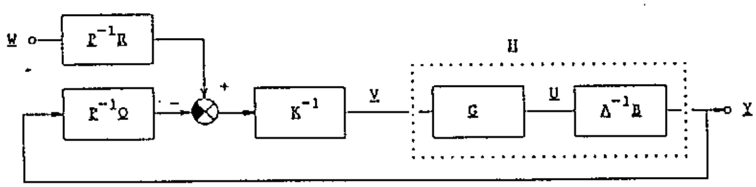


Fig. 2 Block diagram of the control system

Compensator for Diagonal Dominance

The compensator design was based on the idea of Rosenbrock's Inverse Nyquist Array method (INA)[1]. The method was modified by authors to the z-plane for the purpose of discrete control. The original software for PC, enabling the compensator design by means of the classic INA method in p-plane as well as by means of the modification in z-plane, have been elaborated.

The modification in z-plane was shown to have some advantages in comparison with p-plane:

- Frequency responses are closed periodical curves with the unambiguously defined frequency range for design.
- Elementary operations with closed curves lead more quickly and more unambiguously to the final goal.

Design procedure:

a) The compensator matrix $\underline{G}$ is the unit matrix at the beginning.
b) The inverse matrix $\underline{H}^{-1}$ is computed.
c) Array of frequency responses of the matrix $\underline{H}^{-1}$ in z-plane is presented graphically by the computer.
d) Off-diagonal frequency responses are reduced in comparison to the diagonal ones by means of the elementary rows operations and so the compensator matrix elements are changed.
e) The point d) is repeated until reasonable diagonal dominance is reached.
f) The resulting compensator matrix elements are printed out.

Compensator for Decoupling

Decoupling has been the topic of several papers such as e.g.[2-6]. The compensator matrix design is not unique. The decoupling conditions are fulfilled, after the product of matrices $\underline{B}$ and $\underline{G}$ is a diagonal matrix, as resulted from Eq. (4), when the other matrices are diagonal as expected. The two-dimensional model of the investigated distillation column is further supposed. The decoupling conditions for this case are expressed by the Eq. (5)

$$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \times \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} H_{11} & 0 \\ 0 & H_{22} \end{bmatrix} \tag{5}$$

All elements in (5) are polynomials with the backward shift operator z^{-1} and

$$F_{ij} = \frac{B_{ij}}{A_{ii}} \qquad i = 1, 2 \qquad j = 1, 2$$

On the assumption that all elements of matrix $\underline{G}$ are polynomials the compensator has the transfer function matrix[3,4,6] as follows

$$\underline{G} = z\,adj\,\underline{B} = z\begin{bmatrix} B_{22} & -B_{12} \\ -B_{21} & B_{11} \end{bmatrix} \tag{6}$$

and diagonal elements of matrix $\underline{H}$ are given by

$$H_{11} = z \det \underline{B}/A_{11}$$
$$H_{22} = z \det \underline{B}/A_{22} \tag{7}$$

where $\det \underline{B} = B_{11}B_{22} - B_{12}B_{21}$.

Experimental Results

The compensators have been verified by the automatic control of the distillation column separating the binary mixture methanol - water (Fig.3). The electric power of the reboiler (u_1) and the reflux flow (u_2) were used as the input variables, compositions on the second (y_1) and the sixth (y_2) tray as output variables. The compositions of the mixture were measured indirectly by means of the tray temperatures. The distillation column was connected with the computer PC/AT by the interface unit CTRL 51 (Ref.[7]).

At first the simulation study was made for the column model in the form of Eq. (1) with the 2nd order polynomials. The model was obtained by the identification of column by the two pseudorandom binary signals brought at the same time to both inputs. The sampling interval was T = 120 sec. This model had the folowing matrices

$$\underline{A}_1 = \begin{bmatrix} -0.80338 & 0 \\ 0 & -1.06726 \end{bmatrix}$$

$$\underline{B}_1 = \begin{bmatrix} 0.03851 & -0.01062 \\ 0.01262 & -0.01990 \end{bmatrix}$$

$$\underline{A}_2 = \begin{bmatrix} 0.02616 & 0 \\ 0 & 0.14709 \end{bmatrix}$$

$$\underline{B}_2 = \begin{bmatrix} -0.00386 & -0.00192 \\ 0.00316 & 0.00157 \end{bmatrix}$$

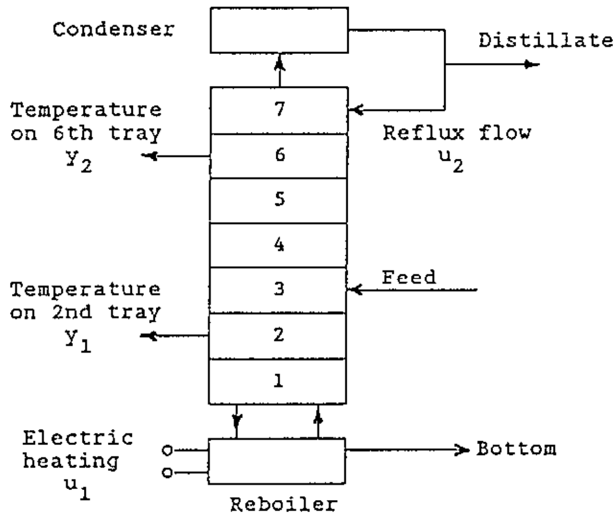


Fig. 3 Distillation column

The measuring instruments and control elements were included into the model so that all input and output values were represented by the electric voltage. All signals were taken as differences from theirs steady state values. The time responses for step changes in set points w_1 and w_2 for a selected time sequence were computed. The controllers were designed by means of the pole placement method for the 10% overshoot of controlled variable. The array of frequency responses of the inverse transfer function matrix $\underline{H}^{-1}$ in z-plane as starting point of the compensator for the diagonal dominance design is shown in Fig. 4. The matrix becomes diagonally dominant after two elementary row operations are performed - see Fig. 5. The corresponding compensator matrix $\underline{G}$ is

$$\underline{G} = \begin{bmatrix} 1 & 0.33 \\ 0.64 & 1.2112 \end{bmatrix} \tag{8}$$

The system matrix $\underline{F}$ with zero off-diagonal elements has been used for the controller design.

The compensator for decoupling was computed from the Eq. (6) and the controller design was performed for the resulting transfer function matrix $\underline{H}$ according to Eq. (7).

The simulated time responses of the control system for the case of no compensator in place is shown in Fig. 6a. The interaction of both controllers is evident. The compensator computed according to Eq. (8) strongly reduces the interaction - see Fig. 7a. The interaction is not observable by using the compensator as the result of Eq. (6) - see Fig. 8a. The manipulated variables were limited to the ∓ 5 V range.

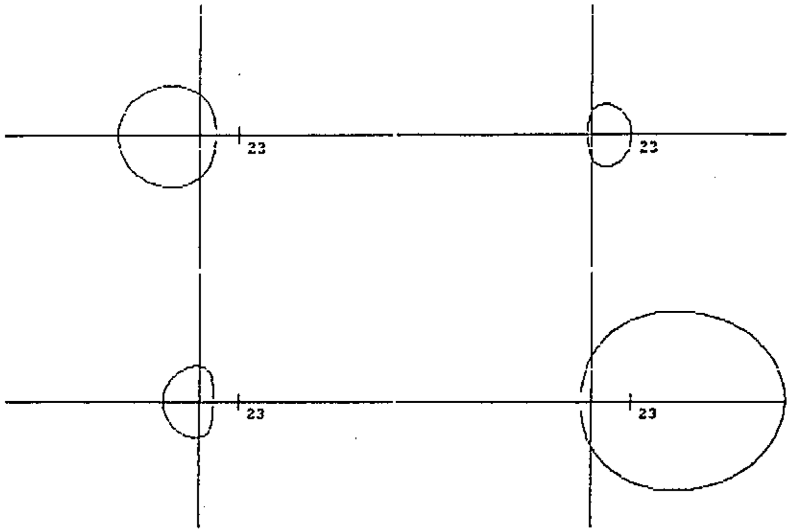


Fig. 4 Starting point of the compensator design

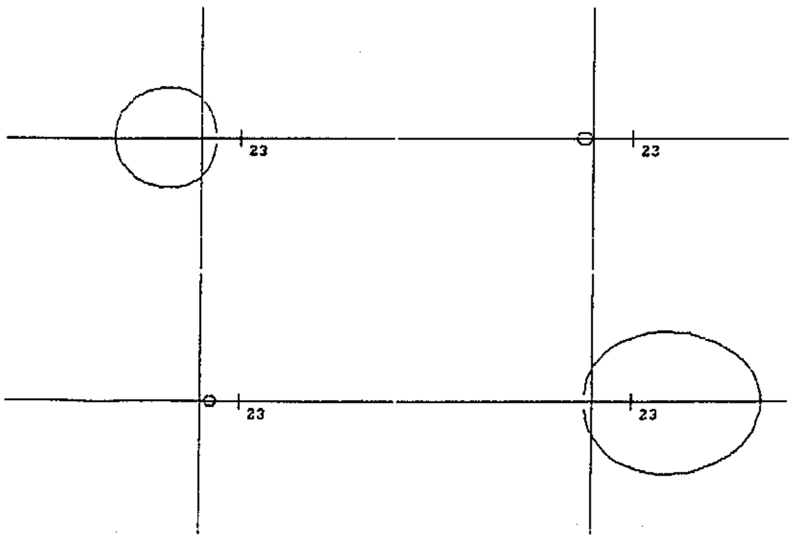
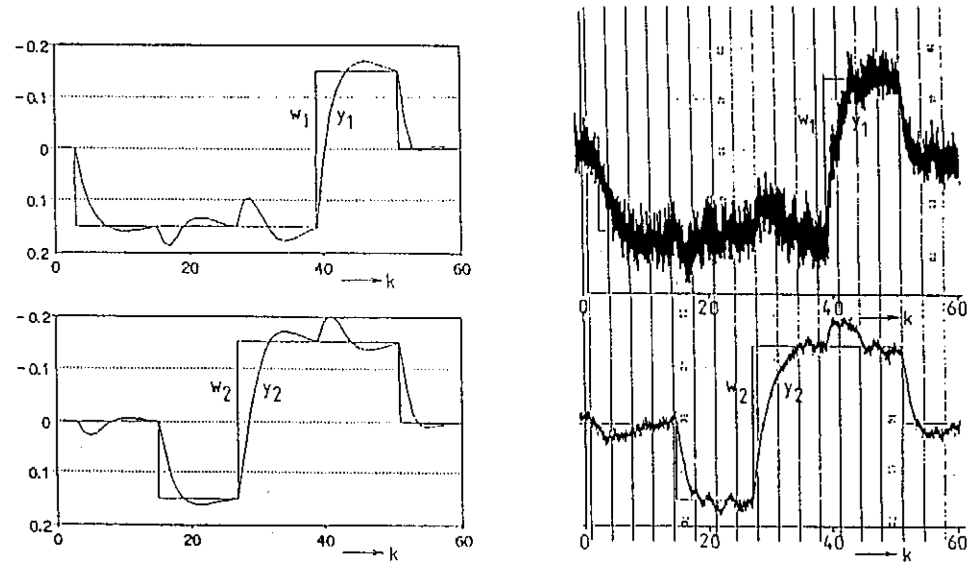


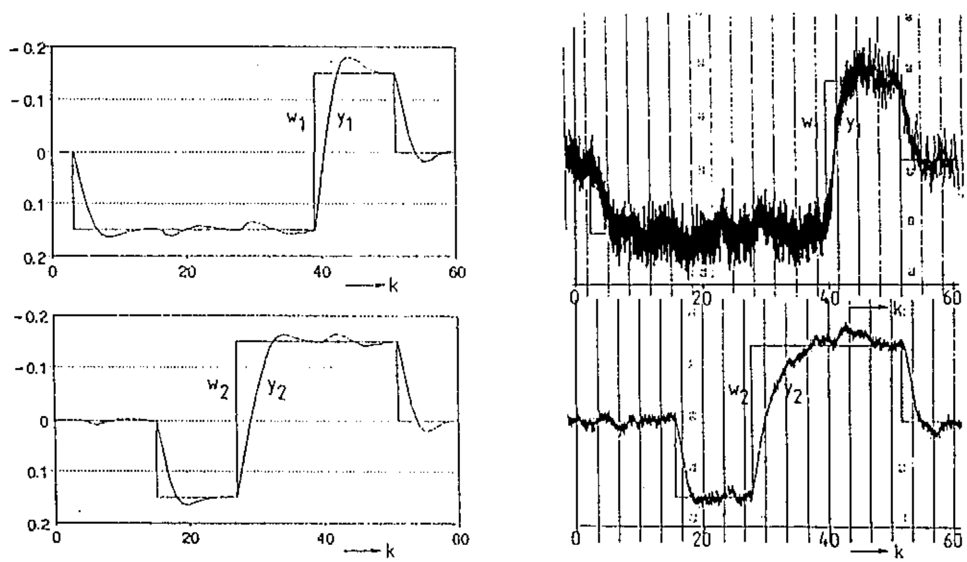
Fig. 5 Achieving the diagonal dominance

After the reasonable results of the computer simulation the both methods have been tested also on a real distillation column. The measured time responses of the controlled variables contain a comparatively high level of the noise signal, as it is shown in Figs 6b, 7b and 8b. Therefore the signals were filtered by means of the analog filter for reducing of quick noise component and by means of discrete filtering of the values measured in the case of the shorter sampling



a) b)

Fig. 6 Time responses of the distillation column without compensator: a) simulation, b) real system



a) b)

Fig. 7 Time responses of the distillation column with compensator for diagonal dominance: a) simulation, b) real system interval. The record of controlled variables measured without compensator is shown in Fig. 6b, that measured by the diagonal dominance compensator in Fig. 7b, and that measured by decoupling compensator in Fig. 8b.

The correspondence of the real and simulated time responses shows a relatively good agreement in spite of describing the non-linear column dynamics by the linear model and in spite of the occurrence of non-defined disturbances apart from the noise in the course of the experiment.

The results presented prove that both the types of compensators are satisfactory for the two-dimensional control of a distillation column.

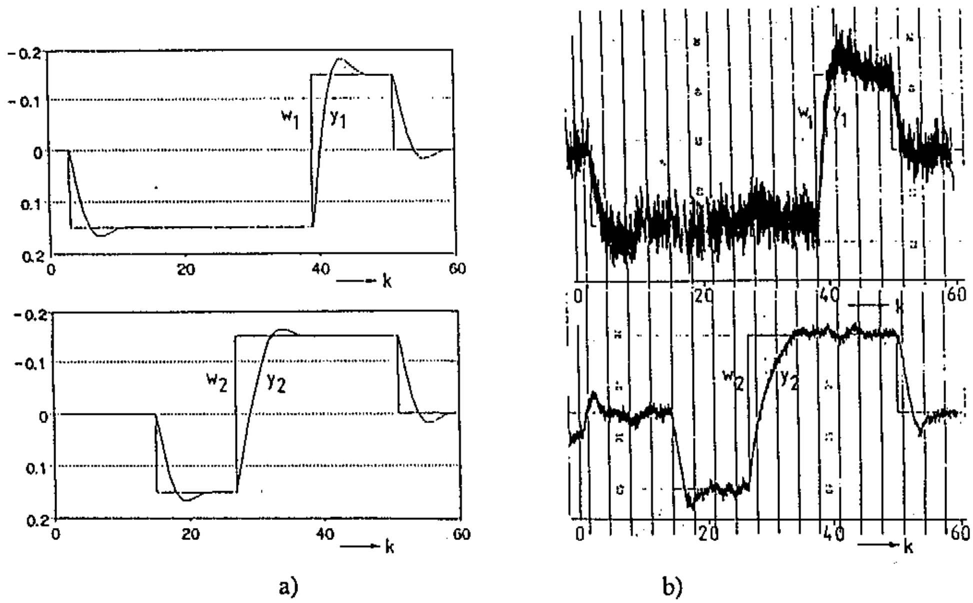


Fig. 8 Time responses of the destillation column with compensator for decoupling: a) simulation, b) real system

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