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CONTROL OF HAMMERSTEIN NONLINEAR SYSTEM

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The paper deals with the control of a nonlinear plant with the quadratic steady - state characteristic. The plant is described by Hammerstein model and controlled by a deadbeat controller. The controller is designed by the classical complex area method. The choice of a sampling period T in the dependence of the plant character (with minimum or nonminimum phase) and of the model plant parameters is presented. The choice of T is based on an admitted actuator range and on the keeping the deadbeat - strong version criterion condition. Simulation examples are presented.

Introduction

The plant described by Hammerstein model is often included in the nonlinear systems group[1]. Many different proposals for the nonlinear systems control were presented in the past. The survey of the various methods can be found in Ref.[2] As the Hammerstein model in fact represents a linear dynamic system with quadratic input signal, a method well-known in the field of the linear discrete system can be used for the controller design. The modified procedure described in monograph[3] for the stochastic system control and applied for the deterministic control system[4] is used.

The digital control process quality depends first of all on the choice of suitable sampling period. In connection with solving this task the Shannon theorem is quoted most often[5,6]. The problematic appointment of the highest spectrum frequency gave rise to a lot of methods based e.g. on transient response evaluation[7,8], maximum admitted manipulated variable[7] etc. The automatic tuning of the sampling period based on the periodic oscillation obtained by a feedback connection of a linear system with a relay having hysteresis can be found in Refs[9,10]. From plenty of the further works let us introduce the recent papers on the choice of sampling period for linear stochastic control[11] and on the estimation of the control period for self-tuners[12].

It results from the above brief survey that in most publications the choice of T is tightly connected with controlled plant character, with type of the controller used, with the control criterion etc. The relations for the sampling interval choice in dependence on the Hammerstein model plant parameters are presented. The plant is controlled by deadbeat controller. The starting point for the sampling interval choice is the maximum admitted actuator range and the deadbeat (strong version) criterion fulfilled only for the manipulated variable $U > 0$.

The Control Plant Description and the Control Aim

Let us suppose the dynamic plant (an electrical oven, a boiler or a distillation column with electrical heating etc.) described by Hammerstein model - Fig. 1. The nonlinear part with parabolic shape of the steady-state characteristic is described by equation

$$X(k) = s_0 + s_1 U(k) + s_2 U^2(k) \tag{1}$$

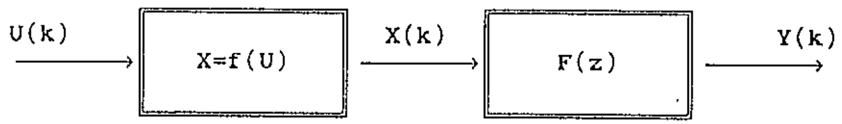


Fig. 1 Hammerstein model of the nonlinear system

Further, let suppose the linear part in form of the linear difference second order equation

$$Y(k) + \sum_{i=1}^{2} a_i Y(k-i) = \sum_{i=1}^{2} b_i X(k-i) \tag{2}$$

The continuous equivalent of Eq. (2) corresponds to the continuous transfer function

$$F(p) = \frac{1 + pT_3}{(1 + pT_1)(1 + pT_2)} \tag{3}$$

Let us assume the plant to be stable and consequently the roots $z_{1,2}$ of the characteristic equation

$$z^2 + a_1 z + a_2 = 0 \tag{4}$$

to lie within the unit circle of the plane z.

Further, the plant is with the minimum or nonminimum phase, i.e. the root of the equation

$$b_1 z + b_2 = 0 \tag{5}$$

lies within $(b_2/b_1 < 1)$ or outside $(b_2/b_1 > 1)$ the unit circle of the plane z.

Introducing Eq. (1) into (2), we obtain the complet model in the form

$$Y(k) + \sum_{i=1}^{2} a_i Y(k-i) = s_0 \sum_{i=1}^{2} b_i + s_1 \sum_{i=1}^{2} b_i U(k-i) + s_2 \sum_{i=1}^{2} b_i U^2(k-i) \tag{6}$$

This equation represents a model in the form of the linear difference equation with the right side which is nonlinear quadratic function of $U(k)$.

The aim is to control the plant so that the transition from one operating point to another one is as quick as possible. Of course, it offers a possibility to control in accordance with deadbeat criterion - strong version. The controlled variable reaches the required value in the second step and any further sampling interval.

The Incremental Deadbeat Algorithm

The incremental deadbeat algorithm is based on the nonlinear incremental model plant and derived in the following manner: let us rewrite Eq. (6) for k-1 and subtract this equation from Eq. (6). We obtain then

$$\Delta y(k) + a_1 \Delta y(k-1) + a_2 \Delta y(k-2) =$$

$$= c_1 \Delta u(k-1) + c_2 \Delta u(k-2) + c_3 \Delta u^2(k-1) + c_4 \Delta u^2(k-2) \tag{7}$$

where

$$\Delta y(k) = Y(k) - Y(k-1) \qquad \Delta u(k) = U(k) - U(k-1)$$

$$\Delta u^2(k) = U^2(k) - U^2(k-1) \tag{8a,b,c}$$

and

$$c_i = s_1 b_i \quad i = 1,2 \qquad c_i = s_2 b_{i-2} \quad i = 3,4 \tag{9a,b}$$

Further let us express *Y(k)* from Eq. (7) and put

$$Y(k) = \sum_{i=1}^{2} f_i W(k-i) \tag{10}$$

where $W(k)$ is the command variable. f_1, f_2 are calculated as the solution of the following algebraic equations

$$\frac{f_1}{f_2} = \frac{c_1}{c_2} = \frac{b_1}{b_2} \quad \text{or} \quad \frac{f_1}{f_2} = \frac{c_3}{c_4} = \frac{b_1}{b_2} \quad \text{and} \quad f_1 + f_2 = 1 \tag{11a,b,c}$$

It is apparent that the knowledge of the coefficients c_i, $i = 1,2$ or 3, 4 is sufficient to calculate f_1, f_2. We need not know the separated coefficients of the static and dynamic model part. In fact Eq. (10) is a quadratic equation in *U(k)*

$$U^2(k) + pU(k) + q = 0 \tag{12a}$$

where

$$p = \frac{s_1}{s_2} \qquad q = f\left[s_i, a_i, b_i, U(k-1), W(k-i+1), Y(k-j)\right] \tag{12b,c}$$

$$i = 1,2 \qquad j = 0, 1, 2$$

As in most cases we demand the manipulated variable not to drop in the region of negative values, the manipulated signal must be given by the positive root of Eq. (12a). Solving this equation with regard to this requirement, we obtain the controller equation in form

$$\begin{aligned} U(k) = -\frac{s_1}{2s_2} + \Bigg\{ \frac{s_1^2}{4s_2^2} - \frac{1}{s_2 b_1}\Big(& [1 - a_1]\, Y(k) + [a_1 - a_2]\, Y(k-1) + \\ & + a_2 Y(k-2) + s_1\big\{[b_2 - b_1]\, U(k-1) - b_2 U(k-2)\big\} + \\ & + s_2\big\{[b_2 - b_1]\, U^2(k-1) - b_2 U^2(k-2)\big\} - f_1 W(k) - f_2 W(k-1)\Big)\Bigg\}^{\frac{1}{2}} \end{aligned} \tag{13}$$

If the plant is controlled with the controller (13) the regulated variable reaches, for step command variable, the required value in the second control step and will stay on this level even within the third and any further sampling interval, if *T* has a certain value.

The Analysis of Conditions for Sampling Interval Choice

The control loop values U and Y must fulfil certain demands. The manipulated variable must not be larger in the first control step than a certain admitted positive value $U(0)_{max}$ and must fulfil the additional condition $U(k) \geq 0$, $k = 1, 2, \dots$. The time course of the controlled variable must correspond to the deadbeat criterion requirement. These demands will be fulfilled if the sampling interval value T does not decrease under certain minimum value T_{min}. The derivation of T_{min} will be shown below. The control loop behaviour will be analysed for a step command signal and, in the first case, the plant will have the minimum phase.

The Derivation of T_{min} from Admitted $U(0)_{max}$

The manipulated variable reaches the maximum value $U(0)_{max}$ in the first control step. For $k = 0$, given parameters s_0, s_1, s_2, $W(0)$ and for the initial conditions

$$U(-1) = U(-2) = 0 \qquad W(-1) = s_0 \qquad Y(0) = Y(-1) = Y(-2) = s_0$$

it is from Eq. (13)

$$U(0)_{max} = -\frac{s_1}{2s_2} + \sqrt{\left[\frac{s_1}{2s_2}\right]^2 + \frac{W(0) - s_0}{s_2 B}} \tag{14a}$$

where

$$B = b_1 + b_2 \tag{14b}$$

It is obvious that parameter B can be expressed too with using the equations for b_1 and b_2. Since the parameters b_1 and b_2 can be expressed as the functions of the rates T_1/T_2 and T_1/T, it is possible to construct graph the $T_1/T = f(B)$ for $T_1/T_2 = const$ - see Fig. 2. T_{min} is then determined in such a way that, for $U(0)_{max}$ the parameter B is calculated from Eq. (14a), and the corresponding value $M = T_1/T$ from Fig. 2 for the given rate of the time constants T_1/T_2. The minimum sampling period is

$$T_{min} = \frac{T_1}{M} \tag{15}$$

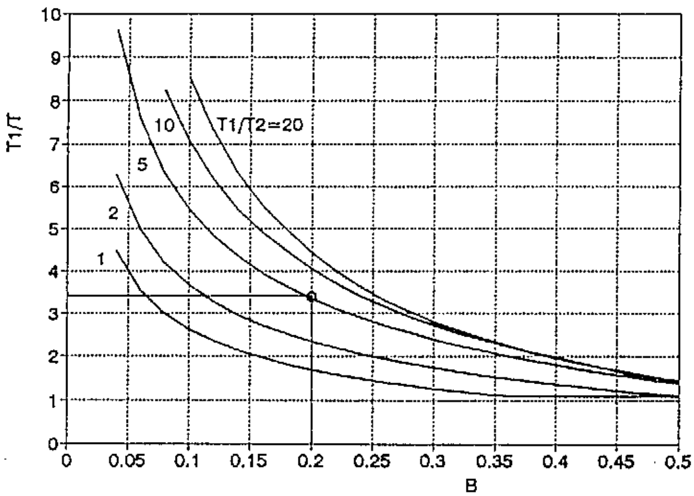


Fig. 2 Graph for determining T_{min} from $U(0)_{max}$ requirement

The Derivation of T_{min} from the Requirement $U(1) \geq 0$

The largest danger of transition of the manipulated variable in the negative value threatens in the second control step, when the manipulated variable passes from $U(0)_{max}$ to $U(1)$. Therefore, it is necessary to ensure that it is

$$U(1) \geq 0 \tag{16}$$

For $k = 1$, given parameters, and for the initial conditions

$$U(-1) = 0 \qquad W(1) = W(0) \qquad Y(0) = Y(-1) = s_0$$

the manipulated variable is in the second control step from Eq. (13)

$$U(1) = -\frac{s_1}{2s_2} + \left[\frac{s_1^2}{4s_2^2} - \frac{1}{s_2 b_1}\left(s_0 - W(0) + [b_2 - a_1 b_1][s_1 U(0) + s_2 U^2(0)]\right)\right]^{\frac{1}{2}} \tag{17}$$

Let us suppose the frequent case when $s_1 > 0$, $s_2 > 0$. The condition (16) will be fulfilled if in Eq. (17) it is

$$\frac{1}{s_2 b_1}\left(s_0 - W(0) + [b_2 - a_1 b_1][s_1 U(0) + s_2 U^2(0)]\right) \leq 0 \tag{18a}$$

and

$$(1+a_1)\frac{W(0)-s_0}{s_2 B} \geq 0 \qquad (18b)$$

As it is $0 < b_1 + b_2 < 1$ and for $W(0) - s_0 > 0$ we obtain after a short calculation

$$a_1 \geq -1 \qquad (19)$$

The functions $a_1 = f(T_1/T)$ for $T_1/T_2 = const$ are shown in Fig. 3. The function $T_1/T = f(T_1/T_2)$ can be formed from the intersections of these courses with line $-a_1 = 1$ - see solid line in Fig. 4.

For the requirement (16) to be fulfilled it is necessary (when T_1 and T_2 are given) to choose T in such a way as to keep the rate value T_1/T under the solid boundary line in Fig. 4. This line then determines the minimum value of the sampling interval T_{min}. As this value does not suit the requirement of $U(0)_{max}$, it is necessary to accept the complicated solution as the result of the procedures described in chapters above.

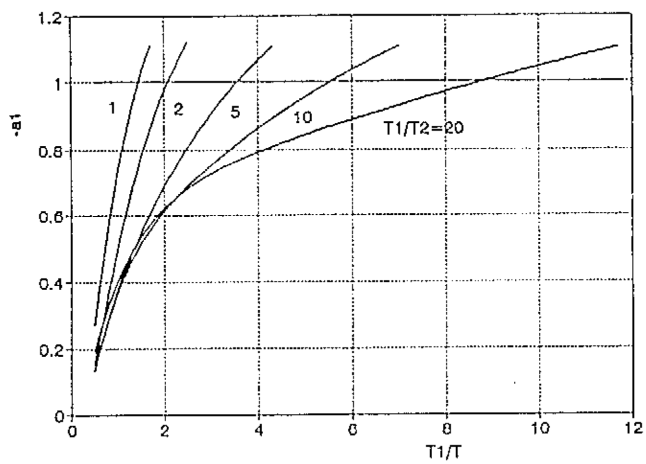


Fig. 3 The functions $a_1 = f(T_1/T)$, $T_1/T_2 = const$

The Derivation T_{min} for Nonminimum Phase Plant

Till now we have devoted attention to the control of the minimum phase plant $(b_2/b_1 < 1)$. The plant with this behaviour is described by the transfer function (3), where $T_3 \geq 0$ (Ref[7].). For $T_3 < 0$ and at improper T the ratio b_2/b_1 can be great than unity, system is getting nonminimum phase and the control loop is unstable. Like in the case of the graph construction in Fig. 3. we can find the functions $T_1/T = f(T_1/T_2)$ for $T_3/T_1 = const$ and $b_2/b_1 = 1$ - see the set of dotted curves in Fig. 4. The individual courses for different rates T_3/T_1 are the boundary lines giving T_{min}

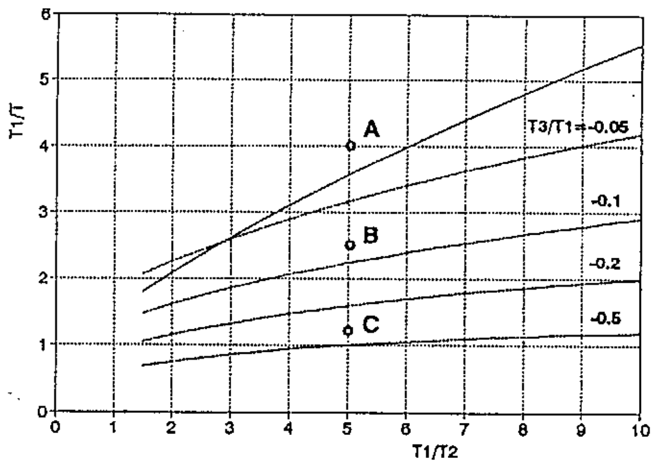


Fig. 4 Graph for determining T_{min} from $U(1) \geq 0$ requirement for minimum phase plant (solid curve) and for nonminimum phase plant (set of curves T_3/T_1 = *const*)

Example

For illustration results of a control simulation are shown. Let us consider four plants which have the same parameters

$$T_1 = 5 \quad T_2 = 1 \quad s_0 = 10 \quad s_1 = 0 \quad s_2 = 0.05 \quad W = 50$$

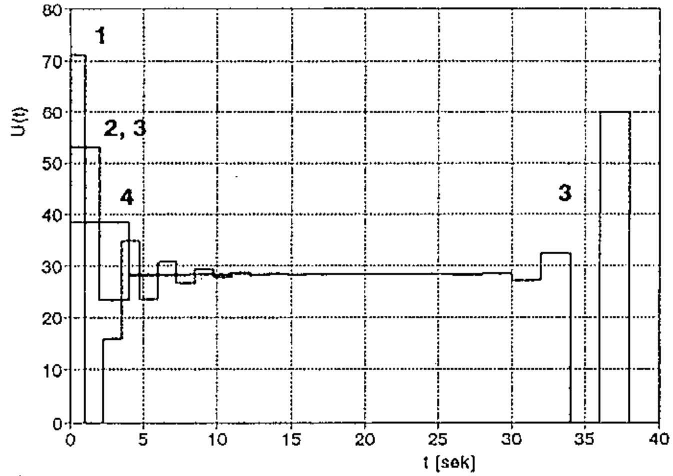
They differ in the parameters T_3 and T according to Table I. It is demanded that the admitted manipulated variable be $U(0)_{max} \leq 80$.

From the simulation results it is apparent that if the conditions are fulfilled for the choice of T, the quality of the control processes is on the demand level.

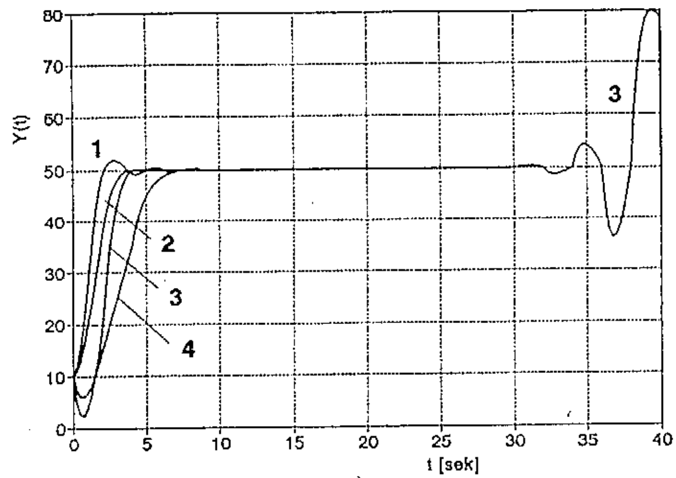
Table I

Number of the plant	T_3	T	The point in graph in Fig. 4	Simulation results		
				Contr. proc. in Fig. 5	Criterion	$U(0)_{max}$
1	0	1.25	A	10	not fulfilled	71
2	0	2	B	11	fulfilled	53
3	- 1	2	B	12	not fulfilled*)	53
4	- 1	4	C	13	fulfilled	38.5

*) The control process is unstable from the 14th control step



a)



b)

Fig. 5 Time courses of the manipulated and controlled variables

Conclusion

We have shown utilization of a formaly analogous method for synthesis of a control loop with the linear and nonlinear system controlled in accordance with the deadbeat criterion - strong version. The nfluence of the sampling interval size on the starting value of the manipulated signal and especially on the fulfilling of criterion was quantitatively investigated. The analysis was carried out not only for minimum phase plant but for a nonmimimum one too. We have constructed graphs which facilitate the choice of sampling interval for the control of the plant described by the Hammerstein model.

It can be easily shown that the derived relations can be used in the case of the time delay plant control as well. The unknown controlled variables must only be replaced by their predictions with the horizontal q - control steps ($q = T_d/T$, T_d is time delay in time units). The regulated variable reaches the required value in (q +2) control steps.

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