

## Terminal falling velocity of spherical particles moving through a Carreau purely viscous fluid

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*The paper deals with the determination of the terminal falling velocity of solid spherical particles moving slowly through an unbounded time-independent purely viscous Carreau liquid. The relationships based on the Carreau four-parametric viscosity model are given for calculation of a sphere terminal velocity falling in the creeping flow region. The comparison of terminal velocities calculated according to these relationships with those obtained experimentally is presented. In experiments, the terminal velocity of spheres in the aqueous solutions of polymers was measured. A good agreement between the calculated and experimental values of terminal falling velocities was found.*

**Keywords:** Sphere free fall; Non-Newtonian fluids; Creeping flow region; Carreau viscosity model

### Introduction

The knowledge of the terminal falling velocity of a sphere moving through a fluid under the influence of gravity is needed for solutions of numerous engineering problems (design calculations of fluidised beds equipment, thickeners, pipeline transport systems, falling particle viscometry, etc).

For calculation of the terminal falling velocity of a sphere settled in a purely viscous non-Newtonian liquid, different rheological models of the liquid shear viscosity are used [1].

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Quite frequent and simple model is the two-parameter power-law (Ostwald–de Waele) model:

$$\eta = K \dot{\gamma}^{n-1} \quad (1)$$

where  $\eta$  is the shear viscosity,  $\dot{\gamma}$  the shear rate,  $K$  the fluid consistency coefficient, and  $n$  the power-law index. The disadvantage of this model is that it describes the course of the dependence of the viscosity on the shear rate only in a limited interval of the shear rate. Therefore, when calculating the sphere terminal falling velocity, it is first necessary to estimate the appropriate viscosity function interval in order to determine the parameters  $K$  and  $n$ . For this purpose, a simple iterative method was used in our previous work [2].

Beside other, the power-law predicts unreal high values of the shear viscosity at sufficiently low shear rates. For this reason, the models containing the zero-shear rate viscosity as a parameter are preferred when calculating the terminal falling velocity. Such a widely used viscosity model is the four-parameter Carreau model

$$\eta - \eta_\infty = \frac{\eta_0 - \eta_\infty}{\left[1 + (\lambda \dot{\gamma})^2\right]^{\frac{1-m}{2}}} \quad (2)$$

with the parameters  $\eta_0$  (zero-shear rate viscosity),  $\eta_\infty$  (infinite-shear rate viscosity), and fitting parameters  $\lambda$  and  $m$ .

In this paper, the results are presented as the values of terminal velocity of spheres falling in unbounded purely viscous polymer solutions and calculated using the Carreau viscosity model (2), making comparison with those obtained experimentally by Strnadel [3,4].

## Fundamentals

Analogously to Newtonian flow, the terminal falling velocity  $u_t$  for the fall in an unbounded Carreau liquid in the creeping flow around a sphere can be expressed as

$$u_t = \frac{g d^2 (\rho_s - \rho)}{18 \eta_0 X(\Lambda, \eta_r, m)} \quad (3)$$

Here,  $g$  is the gravity acceleration,  $d$  the sphere diameter,  $\rho_s$  the sphere density,  $\rho$  the liquid density, and  $X$  is the drag coefficient as a corrective factor depending on the dimensionless time parameter

$$\Lambda = \frac{2u_t \lambda}{d} \quad (4)$$

the dimensionless viscosity

$$\eta_r = \frac{\eta_0 - \eta_\infty}{\eta_0} \quad (5)$$

and the Carreau model parameter  $m$ .

The functional dependence of  $X$  on  $\Lambda$ ,  $\eta_r$ , and  $m$  was numerically solved by Strnadel [3]. The resulting dependence can be approximated by the following relationship:

$$X = \eta_r \left[ 1 + (k_1 \Lambda)^{k_2} \right]^{\frac{k_3-1}{k_4}} + (1 - \eta_r) \quad (6)$$

where

$$k_1 = -0.1284 m (\eta_r + 1) + 0.4237 \quad (7)$$

$$k_2 = 0.3584 m (\eta_r - 1) + 1.3791 \quad (7a)$$

$$k_3 = -0.3584 \eta_r (m - 1) + 0.4237 m + 0.5763 \quad (7b)$$

$$k_4 = -0.5763 m (\eta_r - 1) - 1.0120 \eta_r + 1.1057 \quad (7c)$$

In a next step, the values of terminal velocities calculated according to the relationships (5)–(7c) are compared with those obtained experimentally by Strnadel for purely viscous polymer solutions of different degree of shear-thinning [3,4].

## Materials and methods

The relevant falling sphere experiments were carried out in six types of cylindrical Perspex columns filled with aqueous solutions of carboxymethyl cellulose, hydroxyethyl cellulose, methyl ethyl cellulose, and mixtures of aqueous solution of polyalkylene glycol Emkarox HV45 with a small addition of carboxymethyl cellulose. The composition along with the density of the test liquids is summarized in Tab. 1. The polymer solutions L1–L3 were prepared by dissolution of powdered polymers in demineralised water, the remaining ones were made as the solutions of Emkarox. The measurements of liquid flow curves, primary normal stress differences, oscillatory, creep & recovery, stress relaxation and stress growth tests were carried out on a rheometer (model Haake MARS II; Thermo Scientific, Karlsruhe, Germany). The diameters of the columns were 16, 21, 26, 34, 40, and 90 mm which lead to ratio  $d/D \in (0.011; 0.499)$ .

**Table 1** Characteristics of the test polymer solutions

Liquid	Polymer used	Concentration [wt. %]	Density [kg m <sup>-3</sup> ]
L1	Carboxymethyl cellulose (CMC)	1.2	1002
L2	Hydroxyethyl cellulose (Natrosol 250 HHX)	1.0	1000
L3	Methyl ethyl cellulose (Tylose)	3.0	1006
L4	Emkarox/CMC	35/0.04	1054
L5	Emkarox/CMC	35/0.08	1054

Seventeen types of spherical particles made of glass, ceramics, steel, lead, and tungsten carbide were used for the drop tests. Typical characteristics of the test particles are given in Tab. 2.

**Table 2** Specifications of the spherical particles used

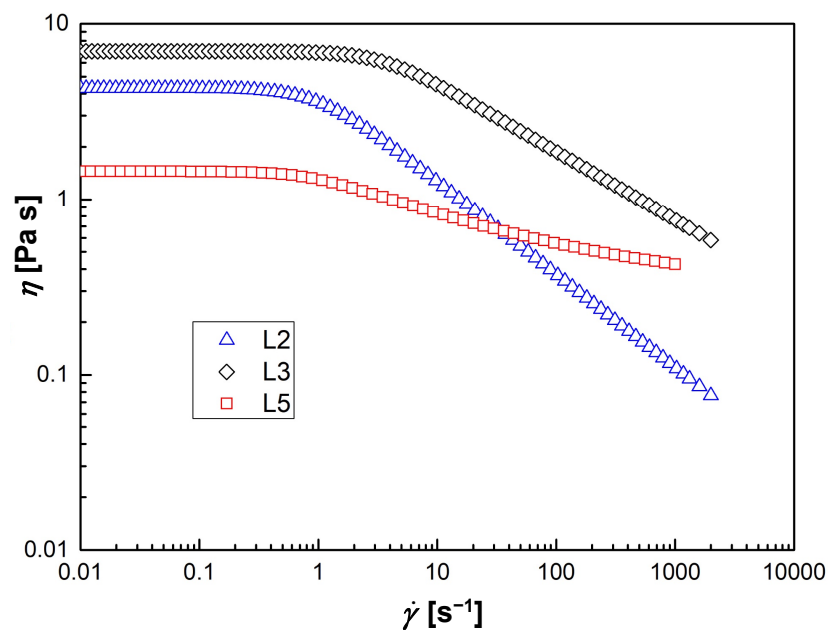
Particle	Material	$d$ [mm]	$\rho_s$ [kg m <sup>-3</sup> ]	Particle	Material	$d$ [mm]	$\rho_s$ [kg m <sup>-3</sup> ]
S1	glass	1.93	2525	S10	ceramics	7.99	3908
S2	glass	3.13	2486	S11	carbide	0.99	15119
S3	glass	4.12	2597	S12	carbide	1.49	15119
S4	glass	4.93	2508	S13	carbide	1.99	15119
S5	glass	6.12	2495	S14	carbide	2.99	15119
S6	ceramics	1.99	3908	S15	steel	0.99	7526
S7	ceramics	2.99	3908	S16	steel	3.17	7789
S8	ceramics	3.99	3908	S17	lead	2.00	11118
S9	ceramics	5.99	3908	–	–	–	–

The values of the terminal falling velocities  $u_{t,\text{exp}}$  in unbounded fluid were determined by a linear extrapolation of the experimental dependences of the terminal falling velocities measured in the individual test columns on the ratio  $d/D$  to the value  $d/D \rightarrow 0$ . The ranges of the measured data of the terminal falling velocity  $u_{t,\text{exp}}$  and the corresponding values of the effective shear rate  $u_{t,\text{exp}}/d$  are for the test liquids given in the Tab. 3. At the same time, the achieved values of the Reynolds number  $Re_C$  ranged from  $1.19 \cdot 10^{-4}$  to 0.36. All the experiments are described in more detail in reports listed as [3,4].

## Results and discussion

### Rheological measurements

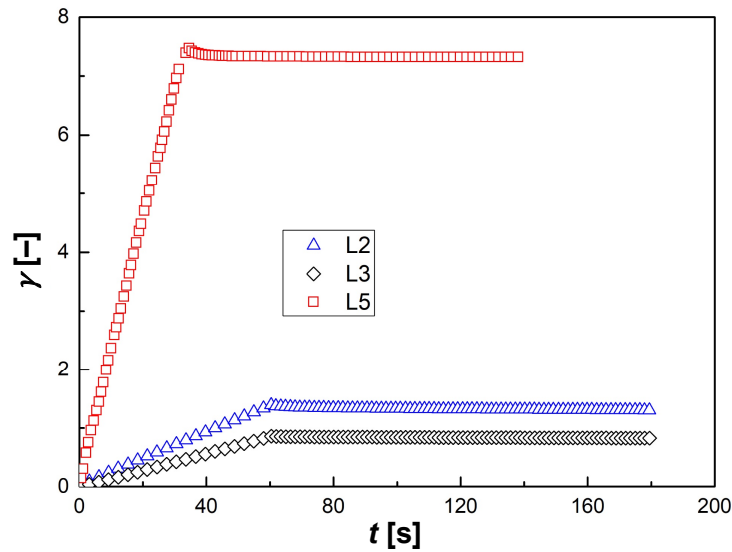
The viscosity functions of the test polymer solutions, of which the examples are displayed in Fig. 1, were evaluated from the experimentally obtained flow curves. The course of these functions was approximated by the four-parameter Carreau model (2), when the respective parameters are summarized in Tab. 3. It followed from the creep & recovery tests and normal stress measurements [3,4] that the test polymer solutions have exhibited a negligible elastic behaviour and can be considered as purely viscous liquids. Typical examples of the creep & recovery tests are shown in Fig. 2.



**Fig. 1** Viscosity functions of the polymer solutions L2, L3, and L5

**Table 3** Carreau model parameters of the polymer solutions used

Liquid	$\eta_0$ [Pa s]	$\eta_\infty$ [Pa s]	$\lambda$ [s]	$m$ [-]	$\eta_R$ [-]
L1	6.103	0.001	2.160	0.509	1.000
L2	4.301	0.001	0.999	0.470	1.000
L3	6.965	0.001	0.290	0.610	1.000
L4	1.070	0.289	3.091	0.805	0.730
L5	1.447	0.296	1.177	0.691	0.795



**Fig. 2** Creep & recovery tests of polymer solutions L2, L3, and L5

### Terminal falling velocity

The theoretical values of terminal falling velocities  $u_t$  for the fall of spheres S1–S17 in liquids L1–L5 were calculated by solving the non-linear equation (3) using the Solver in Excel programme.

The suitability of the proposed corrective factor (6) with coefficients given by Eqs. (7)–(7c) for prediction of terminal falling velocity of a sphere according to Eq. (3) was evaluated by comparing the experimental values  $u_{t,exp}$  with those calculated making use of equations tested.

The agreement between the individual experimental data,  $u_{t,exp}$ , and the calculated ones,  $u_t$ , was evaluated according to the relative deviations

$$\delta_i = \left| \frac{u_{t,i} - u_{t,exp,i}}{u_{t,exp,i}} \right| \cdot 100 \% \quad (8)$$

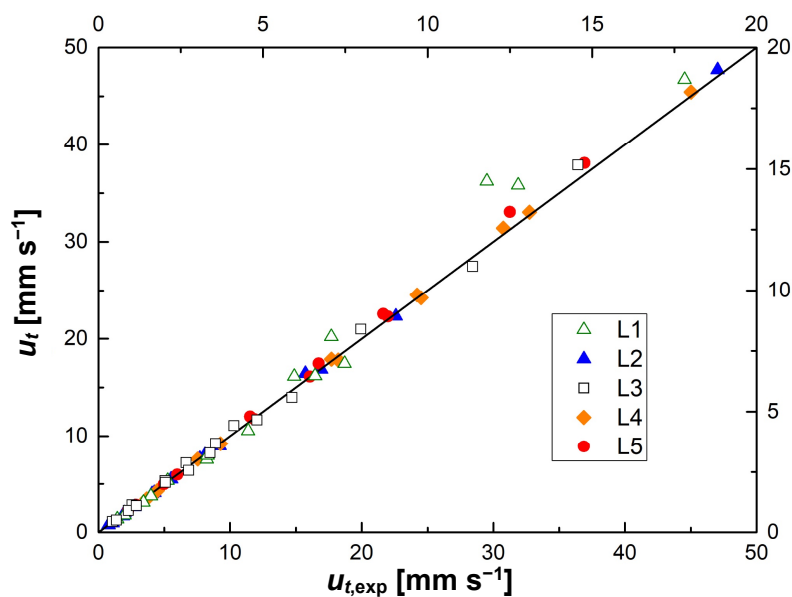
and for the set of measurements in an individual test polymer solution according to the mean relative deviation

$$\delta_m = \frac{1}{N} \sum_{i=1}^N \delta_i \cdot 100 \% \quad (9)$$

The obtained values  $\delta_m$  are summarized in Tab. 4 along with the maximum deviations  $\delta_{max}$  of the individual measurements. The agreement between the predicted and experimental terminal velocity data is also evident from Fig. 3, where the experimental values  $u_{t,exp}$  are compared with the calculated data  $u_t$  for the liquids tested. It is seen that the agreement between experimental and calculated data is entirely satisfactory.

**Table 4** Summary of the results obtained

Liquid	$u_{t,exp}$ [mm s <sup>-1</sup> ]	$\dot{\gamma}$ [s <sup>-1</sup> ]	$\delta_m$ [%]	$\delta_{max}$ [%]
L1	0.572–12.8	0.297–12.7	9.06	22.8
L2	0.770–49.7	0.400–16.6	3.60	10.6
L3	0.429–14.6	0.223–3.80	4.94	11.4
L4	3.68–53.2	1.90–22.6	2.87	6.18
L5	2.26–36.9	1.17–15.7	2.24	5.82



**Fig. 3** Comparison of the calculated terminal falling velocities  $u_t$  with experimental velocities  $u_{t,exp}$  for the test polymer solutions  
Open symbol = liquids L1 and L3: see secondary axes

### Conclusions

In this article, the relationships have been presented dealing with the prediction of terminal falling velocity of a sphere moving through a Carreau model fluid in the creeping flow region.

The suitability of the given relationships has been verified by comparing the calculated values of terminal velocity with those measured by Strnadel [3]. Good agreement of the calculated and experimentally obtained data confirms that these relationships can be successfully used for the predictive calculation of a sphere terminal falling velocity moving slowly through a purely viscous fluid.

## Nomenclature

$d$	sphere diameter, m
$g$	gravity acceleration, $\text{m s}^{-2}$
$K$	power law parameter, $\text{Pa s}^n$
$k_1-k_4$	coefficients in Eq. (6)
$m$	Carreau model parameter
$n$	power law parameter
$Re_C$	Carreau model Reynolds number $\left( = \frac{du_t \rho}{\eta_0} \left[ 1 + \frac{1}{4} \Lambda^2 \right]^{\frac{m-1}{2}} \right)$
$u_t$	terminal falling velocity, $\text{m s}^{-1}$
$X$	drag coefficient correction function (Eq. (3))

## Greek letters

$\delta$	individual relative deviation defined by Eq. (8)
$\delta_m$	mean relative deviation defined by Eq. (9)
$\dot{\gamma}$	shear rate, $\text{s}^{-1}$
$\eta$	non-Newtonian viscosity, Pa s
$\eta_0$	zero shear rate viscosity, Pa s
$\eta_r$	dimensionless viscosity parameter defined by Eq. (5)
$\eta_\infty$	infinity shear rate viscosity, Pa s
$\lambda$	Carreau model time parameter, s
$\rho_s$	particle density, $\text{kg m}^{-3}$
$\rho$	liquid density, $\text{kg m}^{-3}$
$\Lambda$	dimensionless time parameter defined by Eq. (4)

## Subscripts

exp	experimental
max	maximum



## References

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