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WALL EFFECTS ON A SINGLE SPHERICAL PARTICLE MOVING THROUGH A CAREAU MODEL FLUID

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The steady slow motion of solid spheres through a Carreau model fluid contained in a cylindrical tube has been solved numerically using a finite element method by means of the COMSOL Multiphysics software package for steady non-Newtonian flows. From the resulting stress fields, the drag force on the sphere, drag coefficient, drag coefficient corrective factor, and wall correction factor have been evaluated in dependence on the Carreau model parameters and the sphere-to-tube diameter ratio. The results of the wall correction factor calculations are presented herein and compared with our new experimental data.

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Introduction

It is known that the confining walls or bounding surfaces cause an extra retardation effect on a falling particle due to an upward flux of the fluid displaced by the particle. The particle retardation is customarily quantified using the wall correction factor F_w, which can be defined as the ratio of the terminal falling velocity of a particle in a bounded fluid to that in an unbounded one

$$F_w = \frac{U}{U_\infty} \tag{1}$$

Great deal of information on wall effects available in literature, especially for non-Newtonian fluids, concerns spherical particles and is based mainly on experiments. Only little theoretical and numerical work has been carried out on the effect of containing walls on sphere motion in purely viscous fluids without a yield stress [1]. Missirlis *et al.* [1, 2] presented a numerical study of the wall effects on the terminal velocity of a sphere falling freely through a power-law fluid at the axis of a cylindrical tube in the creeping flow regime. In order to test the possibility to exploit the COMSOL Multiphysics software package for steady non-Newtonian flows to the solution of the flow of purely viscous fluids around a solid obstacle, we have recalculated the effect of containing walls on sphere motion in a power-law fluid using this software package [3]. The results obtained were in very good agreement with the data published by Missirlis *et al.* [1,2], which documents the suitability of the COMSOL Multiphysics software for numerical calculations of the flow of purely viscous fluids about solid obstacles.

However, the fluid viscosity models containing zero shear viscosity as a parameter are preferred for describing the flow of non-Newtonian fluids around a sphere [1]. Such a widely used viscosity model, especially for polymeric liquids, is the Carreau model. In this paper, the results are reported of our numerical calculation of the wall correction factor in the creeping flow of a Carreau model fluid over a solid sphere in a cylindrical tube. The numerical results are compared with the results of our experimental investigation of the wall effects on the fall of spherical particles moving through inelastic aqueous solutions of polyalkylene glycol Emkarox with small amounts of carboxymethylcellulose, which are differing by the measure of their pseudo-plasticity.

Mathematical Model

We consider the flow of a Carreau model fluid around a sphere falling in an unbounded fluid and along the axis of a cylindrical vessel. The schematic representation of the domain used for the solution of the flow is shown in Fig. 1. For convenience, it is assumed that the sphere is held fixed and the cylinder walls

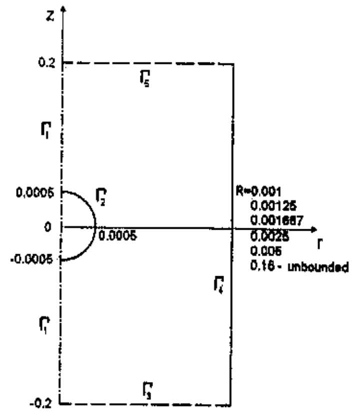

Fig. 1 Schematic representation of solution domain

are moving with the fluid velocity U.

The field equations governing the fluid motion are:

continuity equation	$\nabla \cdot \vec{u} = 0$	(2)
equation of motion	$\rho \vec{u} \cdot \nabla \vec{u} = -\nabla P + \nabla \cdot \vec{\vec{\tau}}$	(3)
constitutive equation	$\vec{\vec{\tau}} = \eta(\dot{\gamma}) \vec{\vec{\gamma}}$	(4)

where the viscosity function is given by the four-parameter Carreau model

$$\frac{\eta - \eta_\infty}{\eta_o - \eta_\infty} = \left[1 + (\lambda \dot{\gamma})^2\right]^{\frac{m-1}{2}} \tag{5}$$

Here $\vec{u}$ is the velocity vector, ρ the fluid density, P the pressure, $\vec{\vec{\tau}}$ the extra stress tensor, $\vec{\vec{\gamma}} = \nabla \vec{u} + \nabla \vec{u}^{\,T}$ the shear rate tensor, $\dot{\gamma} = \sqrt{\frac{1}{2} \vec{\vec{\gamma}} : \vec{\vec{\gamma}}}$ the shear rate, η_0 is zero shear rate viscosity, η_∞ is the infinite shear rate viscosity, λ is the time parameter, and m is the index.

For calculation, the two dimensional axial symmetric geometry with cylindrical coordinates (r, z) has been used. We have postulated the following boundary conditions for the flow solution in an unbounded fluid:

boundary Γ_1 symmetry condition

$$\vec{i}_r \cdot (-P\vec{\vec{I}} + \vec{\vec{\tau}}) \cdot \vec{i}_z = \vec{0} \tag{6a}$$

$$\vec{i}_z \cdot \vec{u} = 0 \tag{6b}$$

boundary Γ_2 – no slip condition

$$\vec{u} = \vec{0} \tag{6c}$$

boundary Γ_3 – velocity condition

$$u_r = 0, \quad u_z = U \tag{6d}$$

boundaries Γ_4, Γ_5 – normal stress condition

$$(-P\vec{\vec{I}} + \vec{\vec{\tau}}) \cdot \vec{i}_z = \vec{0} \tag{6e}$$

For the flow solution in a bounded fluid, the boundary conditions on boundaries Γ_1, Γ_2 and Γ_5 remain the same. On the boundary Γ_3, the normal stress condition (Eq. (6e)) and on the boundary Γ_4 the velocity condition (Eq. (6d)) are valid. The results of the numerical solution of the given mathematical model are the velocity, pressure, and stress fields. The quantities of interest are the drag force on the sphere

$$F_D = 2\pi \int_0^R \int_{-R}^R (P + \tau_{rz} + \tau_{zz}) \mathrm{d}r \, \mathrm{d}z \tag{7}$$

and the drag coefficient

$$c_D = \frac{|F_D|}{\pi R^2 \frac{1}{2} \rho U^2} \tag{8}$$

The drag coefficient for the creeping flow of a Carreau model fluid around a sphere is commonly expressed as

$$c_D = \frac{24}{Re_0} X(\eta_r, \Lambda, m) \tag{9}$$

where

$$Re_0 = \frac{U(2R)\rho}{\eta_0} \tag{10}$$

is the generalised Reynolds number and X is a drag coefficient corrective factor depending on the Carreau model parameters. From the comparison of Eq. (8) with Eq. (9), it follows that

$$X(\eta_r, \Lambda, m) = \frac{|F_D|}{6\pi \eta_0 R U} \tag{11}$$

Solution Procedure

The governing Eqs. (2)-(5) together with the boundary conditions (Eqs. (6)) have been solved by a finite-element method based on the Galerkin formulation of the conservation equations. The following values of entering quantities were used: $d = 0.001$ m, $\rho = 1000$ kg.m^{-3}, $\eta_0 = 30$ Pa s. The infinite shear viscosity ranged in the interval $0 < \eta_\infty < 15$, which led to the relative viscosity $\eta_r \in \langle 0.5; 1.0 \rangle$. The values of parameter m ranged in the interval $0.3 \leq m \leq 0.9$ and the time parameter λ in the interval $0.0025 \leq \lambda \leq 50$, which led to the dimensionless time parameter $\Lambda \in \langle 0.1, 2000 \rangle$. The calculations were performed for the ratio d/D varying in the interval $0.5 \leq d/D \leq 1$.

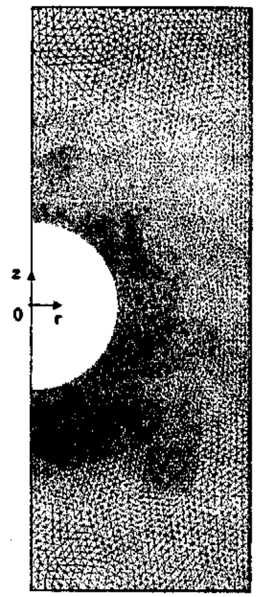

Fig. 2 Example of grid used for ratio d/D = 0.4; number of elements 90 624

For calculation of the drag forces $F_{D\infty}$ in an unbounded fluid, the value $U_\infty = 0.02$ m s^{-1} was chosen, which led to Reynolds number $Re_0 = 6.67\times10^{-4}$. Using the same value $U = 0.02$ m s^{-1} for calculation in a bounded fluid, the magnitude of the drag force F_D raised in comparison with the value $F_{D\infty}$. For calculation of the factor F_W according to Eq. (1), an iterative solution procedure had to be used for searching such velocity $U < U_\infty$ at which the drag force $F_D = F_{D\infty}$. This procedure was based on the Newton's tangent method.

Computations were performed with the computer programme COMSOL Multiphysics using software package for steady non-Newtonian flows. Different triangular grids were used for computation at different ratios d/D. On the boundary Γ_2, the grid was substantially refined. The example of the grid used for calculation at $d/D = 0.4$ is shown in Fig. 2.

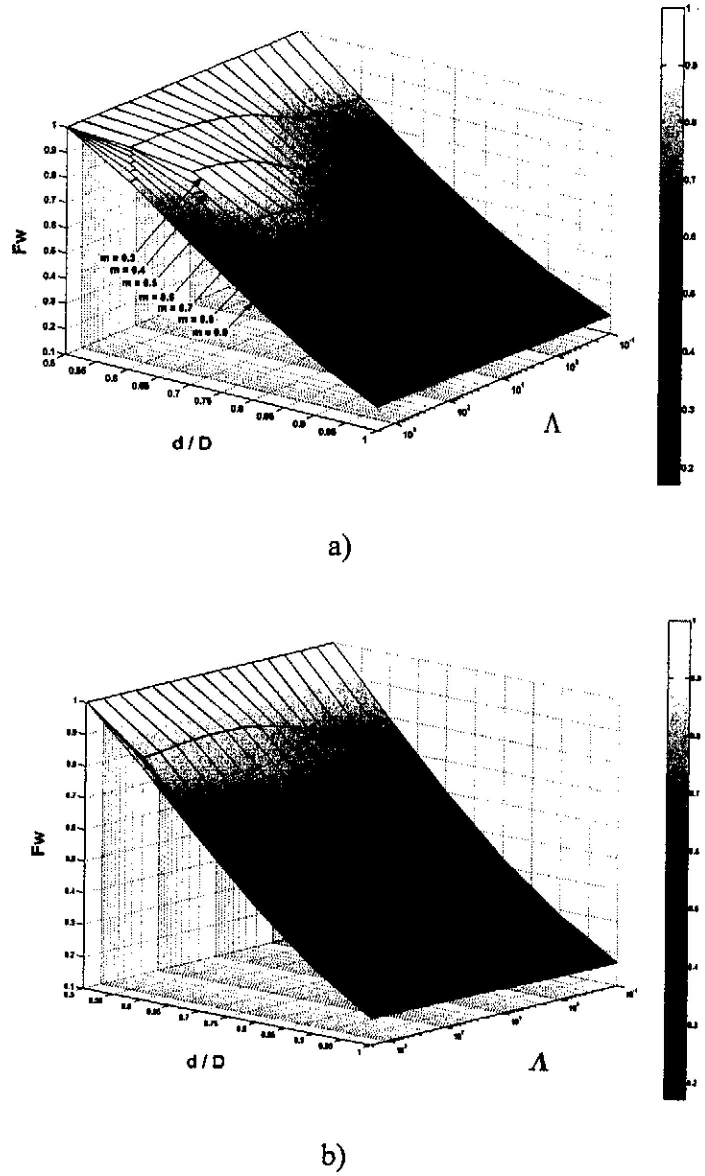

Fig. 3 Dependence of wall factor on d/D, m and Λ; a) $\eta_r = 1$, b) $\eta_r = 0.75$

Results of Wall Correction Factor Calculations

Examples of dependences $F = f(d/D, m, \Lambda)$ calculated for $\eta_r = 1$ and 0.75 are shown in Fig. 3. In accordance with expectation, the wall factor F_w is dominantly dependent on the ratio d/D. The measure of dependence of F_w on m and Λ varies according to the value of the viscosity parameter η_r. For $\eta_r \to 1$, the theoretical estimates of F_w significantly depend on m and Λ. At the same time, the wall effects are less severe with decreasing values m and increasing values Λ. With decreasing values η_r, the dependence of F_w on m and Λ is gradually decreasing and the dependence of F_w on Λ displays a local maximum.

Comparison of Calculated and Experimental Data

The numerically calculated dependences $F_w = f(d/D, m, \Lambda)$ were compared with our experimental data obtained for the fall of rigid spherical particles in aqueous solutions (liquids L1 – L4, Table I) of polyalkylene glycol Emkarox HV45 with small amounts (0.02-0.16 % wt.) of carboxymethylcellulose [4]. The measurements of liquid flow curves, primary normal stress differences, oscillatory, and creep & recovery tests, were carried out on rheometer Haake MARS (Thermo Scientific). The courses of the test liquid viscosity functions are shown in Fig. 4. These courses were approximated using the Carreau model. The corresponding parameters of the Carreau model are given in Table I. The measurements of the elastic behaviour confirmed that the test fluids can be considered as inelastic liquids. For illustration, the results of creep & recovery tests are displayed in Fig. 5. The obtained dependences of compliance J with time t are typical of inelastic fluids.

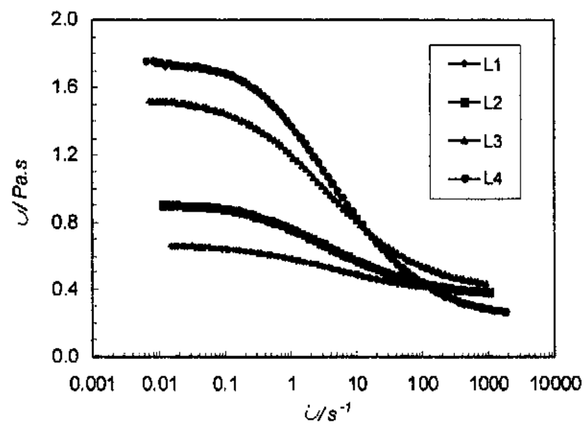

Fig. 4 Viscosity functions of test liquids

Seventeen types of spherical particles made of glass, ceramics, steel, lead, and tungsten carbide were used for the drop tests occurring in cylindrical columns of 90, 40, 34, 26, 21, and 16 mm in diameter. The diameter of spheres ranged from 1.0 mm to 8.0 mm. The range of the ratio d/D reached was from 0.01 to 0.5.

The experimental values $F_{w,exp}$ of the wall correction factor were calculated

Table I Rheological characteristics of test liquids

Liquid	Composition	Careau model parameters		
	Emkarox/CMC, % wt.	λ, s	m	η_r
L1	35 / 0.02	3.24	0.787	0.508
L2	35 / 0.04	2.04	0.752	0.676
L3	35 / 0.08	1.79	0.729	0.816
L4	30 / 0.16	1.57	0.691	0.931

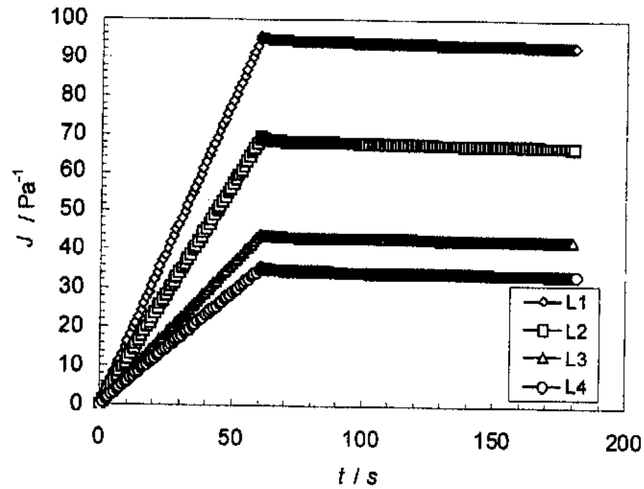

Fig. 5 Creep & recovery tests of liquids used

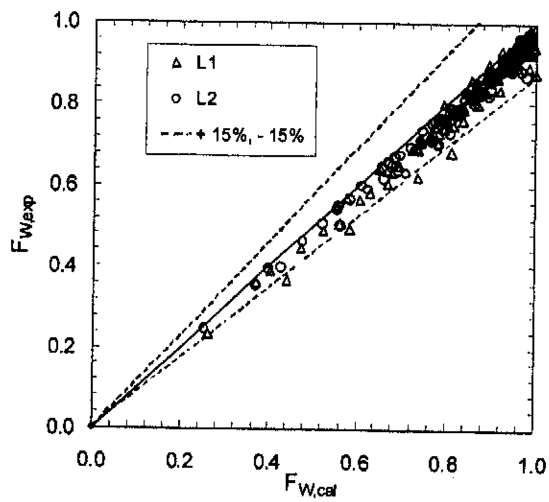

Fig. 6 Comparison of experimental data $F_{W,exp}$ with data $F_{W,cal}$ calculated for liquids L1 and L2: Liqid L1 – η_r = 0.508 Λ = 14-277; Liquid L2 – η_r = 0.676, Λ = 2-42

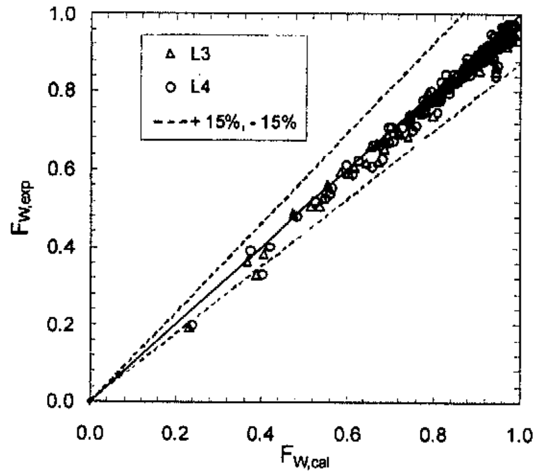

Fig. 7 Comparison of experimental data $F_{W,exp}$ with data $F_{W,cal}$ calculated for liquids L3 and L4: Liqid L3 – $\eta_r = 0.816$ $\Lambda = 3\text{-}86$; Liquid L4 – $\eta_r = 0.931$, $\Lambda = 3\text{-}79$

from Eq. (1) using experimental values U and values U_∞ obtained by extrapolation of experimental dependences $U = f(d/D)$ to the $d/D \to 0$. Then, the experimental data $F_{w,exp}$ were compared with their predictions $F_{w,cal}$ calculated using above described numerical procedure. In these calculations, the velocity $U_{\infty,cal}$, corresponding to the condition $d/D = 0$, was searched to be fulfilled the condition $F_{D,\infty,cal} = F_{D,cal}$ corresponding to the velocity U in the bounded fluid.The results of comparison are shown for liquids L1 and L2 in Fig. 6 and for liquids L3 and L4 in Fig. 7. Figures 6 and 7 show that the agreement between experimental and calculated data of F_W is satisfactory. The mean relative deviation δ_m between experimental and calculated data of F_W is 3.5 % and maximum individual deviation is 17 %. From Fig. 6, it is evident that for lower values of η_r the calculated values of F_W are a little undervalued.

Conclusion

The results of numerical calculations of the wall correction factor F_w have been presented for the slow free fall of spherical particles in a Carreau model fluid contained in a cylindrical tube. For viscosity parameter $\eta_r \to 1$, the theoretical estimates of F_w significantly depend on index m and dimensionless time Λ for a given value d/D. At the same time, the wall effects are less severe with decreasing values m and increasing values Λ. Ascertained dependence of the factor F_w on the parameters Λ, m, η_R and d/D was confirmed by experiments with satisfactory differences.

Acknowledgements

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Symbols

c_D drag coefficient
d sphere diameter, m
D tube diameter, m
F_w wall correction factor (Eq. (1))
F_D drag force magnitude, N
$\vec{i}$ unit vector
$\vec{\vec{I}}$ unit tensor
m Careau model parameter, Pa
P pressure, Pa
r radial cylindrical coordinate, m
R sphere radius, m
Re_0 Reynolds number (Eq. (10))
u velocity vector component, m s^{-1}
$\vec{u}$ velocity vector, m s^{-1}
U particle terminal falling velocity, m s^{-1}
X drag coefficient corrective factor (Eq. (9))
z axial cylindical coordinate, m

Greek Letters

$\dot{\gamma}$ shear rate, s^{-1}
$\vec{\vec{\dot{\gamma}}}$ shear rate tensor, s^{-1}
η shear viscosity, Pa s
η_0 Careau model parameter (zero/shear rate viscosity), Pa s
η_r $\left(= \dfrac{\eta_0 - \eta_\infty}{\eta_0}\right)$ viscosity ratio
η_∞ Careau model parameter (infinity shear rate viscosity), Pa s
λ Careau model time parameter, s
Λ $\left(= \dfrac{2\lambda U}{d}\right)$ dimensionless time parameter
ρ liquid density, kg m^{-3}
τ extra stress tensor component, Pa
$\vec{\vec{\tau}}$ extra stress tensor, Pa

Subscripts

r	related to radial cylindrical component
z	related to axial cylindrical component
∞	related to unbounded fluid
cal	calculated value
exp	experimental value

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