Simple Calibration of FMCIW 35 GHz Meteorological Radar "PCDR 35"

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***Abstract*—During first experiments with weather radar PCDR 35 a question of calibration arose. Comparing pixel measurement with the measurement of the same pixel by operational radar we estimated the radar constant to be $C_3 = 2.62 \cdot 10^8$, which is tuned to the target distance in [km] and to received power in [mW]. Also all main necessary equations from radar meteorology were cleared in this study and the installation of equivalent radar reflectivity factor was explained based on historical development base. The example of radar reflectivity factor measurement in actual rain event is demonstrated.**

Keywords—Meteorological radar, Radar measurements, Radar reflectivity factor, Terrestrial atmosphere, Clouds

I. Introduction

Radar (RAdio Detection And Ranging or Radio Direction And Ranging) transmits and receives the electromagnetic power reflected from target(s). At the Institute of Atmospheric Physics of the Czech Academy of Sciences the meteorological PCDR 35 radar in cooperation with other institutes was developed. This Doppler radar is working at frequency 35.4 GHz and is of the FMCW type. For detailed description of the PCDR 35 radar see [3], [4], [6] and [7]. General studies of radars in meteorology you can find in [1], [2], [5] and [8].

II. Simplified Radar Equation in Meteorology

To understand the meteorological radar data see brief evolution of meteorological radar equation. In the case of one target it holds ("classical" radar equation)

$$P_r = C_1 \cdot \frac{\sigma}{r^4}, \tag{1}$$

where P_r is the received radar power,
σ is the effective area of target,
r is the distance of target,
C_1 is the expanded radar constant involving transmitted radar power.

In the case of multiple target (rain drops, cloud droplets) the effective areas of particular targets (drops) must be summed $\sigma_{TOTAL}(V)=\sum\sigma(D)$ over the "radar volume V" (pixel) and is expressed through the "radar reflectivity η" (per unit volume):

$$\eta = \frac{\sum \sigma(D)}{V} \qquad [\mathrm{mm^2 \cdot m^{-3}}], \tag{2}$$

or in the case we know the drop size distribution (DSD)

$$\eta = \int_0^\infty \sigma(D) \cdot N(D) \cdot dD, \tag{3}$$

$$[\mathrm{mm^2 \cdot m^{-3}}],$$

where N(D) is the drop size distribution (DSD),
D is rain drop or cloud droplet diameter,
V is the radar volume (pixel).

Through the transmitted ray divergence the illuminated radar volume (pixel) V is proportional to r^2 (see Chapter V), thus the radar equation in meteorology is expressed as follows

$$P_r = C_2 \cdot \frac{\eta}{r^2}, \tag{4}$$

where C_2 is the modified radar constant of meteorological radar.

Through equation (4) the radar measures the radar reflectivity η. But this quantity is dependent on radar frequency. Therefore the meteorologists are used to work with the quantity z "radar reflectivity factor" being independent on frequency. The radar reflectivity (small letter z) factor is defined as

$$z = \int_0^\infty D^6 \cdot N(D) \cdot dD, \tag{5}$$

$$[\mathrm{mm^6 \cdot m^{-3}, mm, mm^{-1} \cdot m^{-3}, mm}].$$

Now there remain a question: how to measure the radar reflectivity factor z?

Beforehand we must express that it is not possible. But we can estimate the radar reflectivity factor through the "equivalent radar reflectivity factor z_e" being nearly frequency independent, i.e. z -> z_e, while z_e is estimated from next equation (for explanation see Chapter VI):

$$z_e \approx \frac{\lambda^4}{\pi^5 \cdot K^2} \cdot \eta \qquad [\mathrm{mm^6 \cdot m^{-3}}], \tag{6}$$

where λ is the radar wave length [mm],
K is an auxiliary parameter (K ~ 0.964), $K = \left|\frac{\boldsymbol{m}^2-1}{\boldsymbol{m}^2+2}\right|$,
$\boldsymbol{m}$ is the complex refractivity index of rain or cloud water (dependence on radar frequency and temperature) .

Finally, the meteorological radars and meteorologists evaluate the equivalent radar reflectivity factor z_e from following equation:

$$z_e \approx \frac{\lambda^4}{\pi^5 \cdot K^2} \cdot \frac{r^2}{C_2} \cdot P_r. \tag{7}$$

The distance r is, of course, measured by radar, too. In practice, in meteorology we work with logarithmic form of equation (7) (in this case "Z" is expressed by capital letter):

$$Z_e = 10 \cdot \log(z_e) \qquad [\mathrm{dBZ}]. \tag{8}$$

For Z_e expressed in [dBZ] the "small" z_e in (8) must be in [mm^6m^{-3}] units.

III. Radar Constant Estimation

In order to estimate the radar constant we used the known radar reflectivity factor z_{et} taken from the Czech operational weather radar "Brdy" measurement (Fig. 1), where we found Z_{et} = 36 dBZ at the same pixel which was measured by our radar.

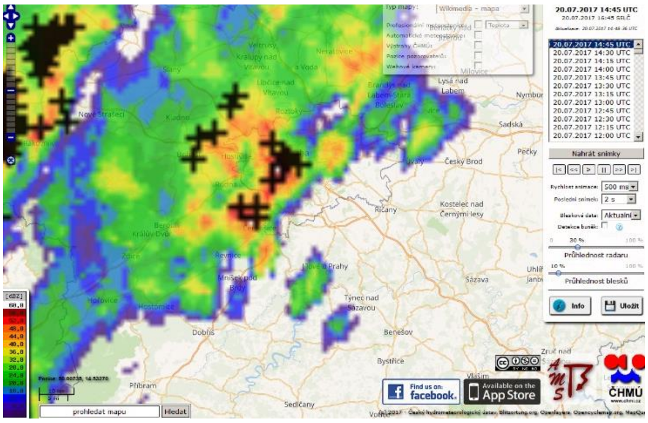

Fig. 1. CAPPI (constant altitude plan position indicator) from operational radar „Brdy."

Radar reflectivity value north to Prague was used to calibrate our radar measuring the same radar volume.

This 36 dBZ value is corresponding to z_{et} = 3 981 mm^6m^{-3}, see equation (8), measured distance from our radar was r = 5.7 km and P_r = -63.3 dBm, i.e. $4.69 \cdot 10^{-7}$ mW).

We integrated all constants in equation (7) into a new constant C_3:

$$z_e = C_3 \cdot P_r \cdot r^2, \tag{9}$$

while

$$C_3 = \frac{\lambda^4}{\pi^5 \cdot K^2} \cdot \frac{1}{C_2}. \tag{10}$$

From equation (9) and measured values we obtain the numerical value for the new radar constant C_3:

$$C_3 = 2.62 \cdot 10^8, \tag{11}$$

which is tuned to the target distance in [km] and to received power in [mW].

It is very practical to express equation (9) in the logarithmic [dB] form:

$$\begin{aligned} Z_e &= 10 \cdot \log(z_e) = \\ &= 10 \cdot \log(C_3) + 20 \cdot \log(r[\mathrm{km}]) + 10 \cdot \log(P_r[\mathrm{mW}]) = \\ &= 84.2 + 20 \cdot \log(r[\mathrm{km}]) + P_r[\mathrm{dBm}] \qquad [\mathrm{dBZ}], \end{aligned} \tag{13}$$

while it should be emphasized that this equation was derived for our PCDR 35 radar.

To estimate the rain rate R corresponding to measured radar reflectivity factor the meteorologists are using the well known Marshall-Palmer Z – R relation ($z = 300 \cdot R^{1.5}$) or its modified form used in EuRAD network

$$R = \left(\frac{z_e}{200}\right)^{0.63}, \tag{14}$$

where R is the rain intensity [mm/h].

IV. Radar Volume Estimation

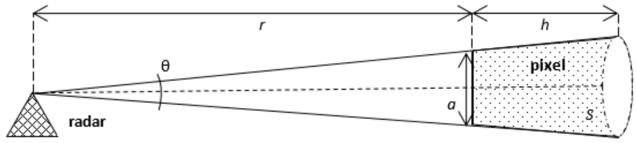

Fig. 2. Illustration for the radar pixel (radar volume) value derivation [8].

The radar volume (pixel) is a volume at distance <r; $r+h$> while h is the length of the radar volume (it corresponds to the radar distance resolution). In the case of pulse radar

$$h = \frac{c \cdot \tau}{2}, \tag{15}$$

where c is speed of light [3·108 m/s] and τ is the radar pulse length [s].

From figure (2):

$$a = 2 \cdot r \cdot \tan\frac{\theta}{2} \approx r \cdot \theta \tag{16}$$

and

$$S = \frac{\pi \cdot a^2}{4} = \frac{\pi \cdot r^2 \cdot \theta^2}{4}, \tag{17}$$

where θ is the antenna diagram width while we consider that $r >> h$ and $\tan(\theta) \approx \theta$ for small θ (θ in radians).

Finally we obtain:

$$V \approx S \cdot h \approx \frac{\pi \cdot r^2 \cdot \theta^2}{4} \cdot h. \tag{18}$$

V. Brief Conventional Meteorological Radar Equation Illation

Power density in one point of radar pixel for given radar antenna diagram pattern and for antenna direction angles φ and ϑ is given by following equation:

$$S_t(r, \vartheta, \varphi) = \frac{P_t \cdot G \cdot f(\vartheta, \varphi)}{4 \cdot \pi \cdot r^2}, \tag{19}$$

$$[\mathrm{W} \cdot \mathrm{m}^{-2}],$$

where G is the radar antenna gain,
$f(\vartheta, \varphi)$ is the value of the antenna diagram pattern for given angles φ and ϑ.

Reflected power from differential volume (which is one point of radar pixel in our derivation) is expressed by

$$dP = S_t(r,\vartheta,\varphi)\cdot\eta\cdot dV = = \frac{P_t\cdot G\cdot f(\vartheta,\varphi)\cdot\eta}{4\cdot\pi\cdot r^2}\cdot dV \quad [W], \qquad (19)$$

where $S_t(r,\vartheta,\varphi)$ is the power density in differential volume dV is differential volume.

The reflected power density back at radar antenna is

$$dS_r(r,\vartheta,\varphi) = \frac{dP}{4\cdot\pi\cdot r^2} = = \frac{P_t\cdot G\cdot f(\vartheta,\varphi)\cdot\eta}{(4\cdot\pi\cdot r^2)^2}\cdot dV \quad [W\cdot m^{-2}], \qquad (20)$$

where dP is reflected power from differential volume dV.

Then the received power is obtainable from

$$dP_r = dS_r(r,\vartheta,\varphi)\cdot A_{ef} = = dS_r(r,\vartheta,\varphi)\cdot\frac{G\cdot\lambda^2}{4\cdot\pi}\cdot f(\vartheta,\varphi) \quad [W], \qquad (21)$$

where $dS_r(r,\vartheta,\varphi)$ is power density at radar antenna, A_{ef} is the effective radar antenna area.

The received power reflected from differential radar volume dV is

$$dP_r = \frac{\lambda^2\cdot G^2\cdot P_t\cdot f^2(\vartheta,\varphi)\cdot\eta}{(4\cdot\pi)^3\cdot r^4}\cdot dV \quad [W]. \qquad (22)$$

Supposing common antenna property that $\vartheta = \varphi = \theta$ and η = const. in whole radar volume RV (pixel) we obtain for the received power by radar:

$$P_r = \frac{\lambda^2\cdot G^2\cdot P_t\cdot\eta}{(4\cdot\pi)^3\cdot r^4}\cdot\int_{RV} f^2(\vartheta,\varphi)\cdot dV \quad [W], \qquad (23)$$

where RV is radar volume being a space volume scanned by meteorological radar.

Typical radar antennas have Gaussian diagram pattern. Its integration over radar volume (RV) is

$$\int_{RV} f^2(\vartheta,\varphi)\cdot dV = h\cdot S\cdot\frac{1}{2\cdot\ln 2} \quad [m^3], \qquad (24)$$

where $\frac{1}{2\cdot\ln 2} = 0.72$ is a constant of typical „Gaussian“ antenna.

Its message is that power reflection from our radar volume RV is by 28 % lower in comparison with hypothetical radar antenna of uniform diagram pattern.

Radar equation in meteorology is

$$P_r = \frac{\lambda^2\cdot G^2\cdot P_t\cdot\theta^2\cdot h}{2^9\cdot\pi^2\cdot\ln 2}\cdot\frac{\eta}{r^2} = C_3\cdot\frac{\eta}{r^2} \quad [W]. \qquad (23)$$

VI. Explanation of Equation (6)

In history the radar people worked with weather radars operating on higher wave length (10 cm or even more). It corresponds to frequencies 3 GHz or less. Comparing these wavelengths with rain drop diameter we see that the condition for „Rayleigh region“:

$$\frac{\pi\cdot D}{\lambda} \ll 1, \qquad (24)$$

was met. For Rayleigh region and only for Rayleigh region John William Strutt (later“lord Rayleigh“) derived a formula for the back scattering cross section of rain drop:

$$\sigma(D) = \frac{\pi^5\cdot K^2}{\lambda^4}\cdot D^6, \qquad (25)$$

From this equation the equation (6) was derived, however the general validity is considered. But this is not true – the validity of (6) is only for condition (24). Therefore we introduced the „equivalent radar reflectivity factor z_e“ approaching the radar reflectivity factor z. By other words, equivalent radar reflectivity factor is approximately estimating the same frequency dependence of radar reflectivity like in the case of Rayleigh region.

Conclusion

Having solved many technological problems the radar PCDR 35 started to measure the meteorological targets. This contribution was dealing with the radar measurement interpretation (rain rate estimation) and the radar calibration. The approximate radar calibration was performed through comparison with radar pixel reflectivity measurement by other (operational weather CHMI) radar using pertinent equations. To understand the radar calibration we added derivation of meteorological radar equation derivation tuned to our radar. First measurement was performed through open window in an office in Prague (figures (2) and (3)) at time of rain and heavy cloudiness. The example of measured profile of received power is shown in figure (4). The received power of -50 dBm corresponds to labelled target. Then a correction of power limitation was performed [3]. Thus P_{cor} = -47 dBm. Through equation (13) the radar reflectivity factor was computed: Z_e = 51.4 dBZ. Using equation (14) we estimated the rain intensity R = 61 mm/h.

The first results are promising while the main persistent problem is the impedance matching of radar antenna as it is dis-matched after each radar transportation.

Fig. 2. Experimental radar workplace.

Fig. 3. Radar scanning sky from office window.

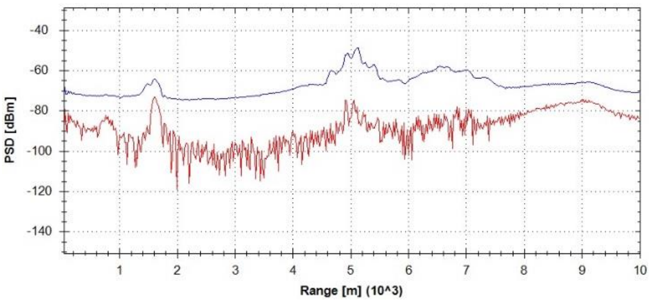

Fig. 4. Example of received power radar profile.

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