THE PROBLEM OF DIFFERENT PAIRWISE COMPARISONS SCALES IN THE AHP FRAMEWORK

Jiří Mazurek

Abstract: *In pairwise comparisons, several scales are used to compare objects. Perhaps the most known is Saaty's fundamental scale for the AHP/ANP from 1 to 9 (with reciprocals), but other scales with 3, 5 or 10 items are also used in practice. Since the AHP/ANP is scale invariant, the following problem arises: for example, a preference of one object over other object expressed by the value 2 means something else for the scale from 1 to 3 (it expresses the medium preference) and the scale from 1 to 100 (in this case the preference is almost negligible). Therefore, the need of a normalization for pairwise comparison scales arises. The aim of the article is to propose a suitable transformation of a general linear scale for pairwise comparisons to a unit real interval that preserves several natural and desirable properties.*

Keywords: *AHP, Normalization, Pairwise Comparisons, Comparison Scale.*

JEL classification: *D81, D83.*

Introduction

Pairwise comparisons (PCs) belong among the most common tools for multiple criteria decision making since the introduction of the analytic hierarchy process (AHP) and the analytic network process (ANP) by T. L. Saaty in 1977 and 1980 respectively, see Saaty (1977 and 1980). The list of successful AHP/ANP applications is rapidly growing, see e. g. Vaidya and Kumar (2006), Subramanian and Ramanathan (2012), Kramulová and Jablonský (2016), or Lidinská and Jablonský (2018). It should be noted that PCs method has a long history predating the papers by T. L. Saaty, with the first mention of the PCs method dating back to the work of the Catalan scholar and monk Ramon Lull in the 13th century, while the first modern work on PCs can be attributed to L. L. Thurstone and his *Law of Comparative Judgments*, see Thurstone (1927).

In the AHP/ANP, the so called fundamental (linear) scale from 1 to 9 (with reciprocals) is used for pairwise comparisons. However, other linear scales with 3, 5, 7, 8 or 10 items were also proposed, see Koczkodaj (1993) or Koczkodaj et al. (2016). In particular, Fülop et al. (2010) and Koczkodaj et al. (2016) provide strong arguments to use the scale only up to three. Variety of other studies suggested the use of non-linear scales such as logarithmic scales, exponential scales, or scales based on a logistic function, see for example Lootsma (1993), Donegan et al. (1992), Ma and Zheng (1991) or Salo and Hämäläinen (1997). Ishizaka and Labib (2011) provide a comprehensive review of scales of pairwise comparisons up to year 2010.

Studies on scale comparisons are rather seldom, see Dong et al. (2008), Elliot (2010), Triantaphyllou et al. (1994), or Starczewski (2017). Dong et al. (2008) provided comparisons on several scales for pairwise comparisons, and concluded that with respect to their algorithms the best scale was the geometrical one. Elliot (2010) experimentally compared three different scales with the result that none of the scales captured accurately the preferences of all individuals. Triantaphyllou et al. (1994)

compared 78 scales to conclude that no single scale could outperform all the other scales. Starczewski (2017) examined the effect of a scale (he compared the fundamental scale, extension scale and geometric scale) on a priority vector, and found that scales with more options lead to a better (more precise) evaluation of a priority vector. Franek and Kresta (2014) compared Saaty's scale to other scales for both consistent and inconsistent pairwise comparison matrices. According to the authors, Saaty's scale is still favorable, but if a decision maker demands higher consistency, he/she should use root square or logarithmic scales.

The AHP/ANP is scale invariant, which means that the result (a priority vector) does not depend on a scale used. However, when using different scales for PCs, the same value from a given scale has a different meaning. This problem was explicitly expressed by Koczkodaj (2015) and dubbed the "pairwise comparisons rating scale paradox". Koczkodaj (2015) also offered a solution to this paradox, a normalization of a rating scale via linear transform. However, the proposed linear transformation in Koczkodaj (2015) has a main drawback as it does not preserve consistency of PCs.

Therefore, the aim of this paper is to introduce a new (power) transform of a (linear) rating scale to a unit interval that has several desirable properties, namely it preserves consistency of PCs, the ranking of objects and the most inconsistent triad.

The paper is organized as follows: section 1 provides brief introduction to pairwise comparisons and the problem with different scales, in section 2 the normalization is proposed along with several of its properties, section 3 provides a numerical example and several aspects of the paper are discussed in section 4. Conclusions close the article.

1 Statement of a problem

1.1 Preliminaries

Let $C = \{c_1, ..., c_n\}$, $n \in N, n \geq 2$, be a set of compared objects (concepts, entities). Let $a_{ij} \in [1/m, m]$ denotes the relative importance of an object c_i over object c_j. Then the (square) matrix $A(a_{ij})_{n \times n}$, $i, j \in \{1, ..., n\}$ is called a pairwise comparison matrix (PCM).

The matrix A is reciprocal if and only if:

$$a_{ij} = 1/a_{ji}, \ \forall i, j \in \{1, ..., n\} \tag{1}$$

The matrix A is consistent (or, alternatively, pairwise comparisons are said to be consistent) if and only if:

$$a_{ij} \cdot a_{jl} = a_{il} \ \forall i, j, l \in \{1, ..., n\} \tag{2}$$

The final weights of objects (priority vector) w in the AHP/ANP, which were pairwise compared, is derived by the eigenvalue method, and satisfies the following condition:

$$Aw = \lambda_{\max} w$$

where $\lambda_{\max}$ is the maximum (positive) eigenvalue of the matrix A.

As pairwise comparison matrices are often inconsistent, various inconsistency indices were proposed. Perhaps the most known are Saaty's consistency index C.I. and consistency ratio C.R., see Saaty (1980, 2008). For other inconsistency indices proposed in the literature see e.g. Alonso and Lamata (2006), Brunelli and Fedrizzi (2015) or Brunelli (2017).

In this paper Koczkodaj's inconsistency index (KII) is used to measure PCM inconsistency, which is defined as follows:

Definition 1. *Koczkodaj's inconsistency index (KII)*, Koczkodaj (1993, 2014): Let $\mathrm{T}(n)$ be the set of all ordered triples ("triads") $\left(a_{ij}, a_{jk}, a_{ik}\right)$ satisfying (2) for $\forall i, j, k \in \{1, 2, ..., n\}$. Then:

$$KII = \max_{\mathrm{T}(n)}\left(\min\left(\left|1-\frac{a_{ik}}{a_{ij}a_{jk}}\right|, \left|1-\frac{a_{ij}a_{jk}}{a_{ik}}\right|\right)\right) \tag{3}$$

The KII expresses inconsistency of a pairwise comparison matrix in terms of the most inconsistent triad, and $KII \in [0,1[$.

Let $[1, m]$ with reciprocals (which will be further omitted in the text) denote alternative scales for pairwise comparisons, where $m \in R, m \geq 2$. In the context of this study whether the scale is discrete or continuous is not important. For Saaty's scale $m = 9$.

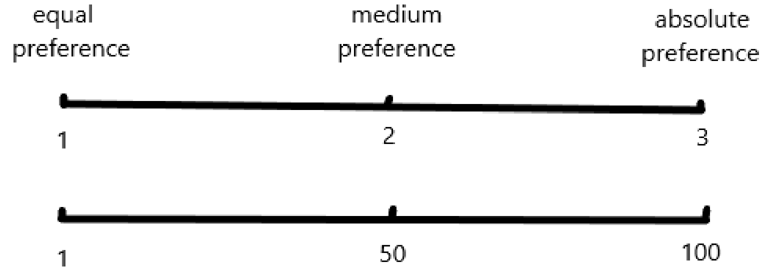
1.2 The problem of different scales

Originally, T. L. Saaty proposed to use the linear scale from 1 to 9 for the pairwise comparisons, see Saaty (1977). The strength of preference or importance for the Saaty's scale is shown in Tab. 1.

However, when at least two different scales are used for pairwise comparisons, the following problem emerges.

Let's consider two pairwise comparisons scales, $S_3 = \{1, 2, 3\}$, and $S_{100} = \{1, 2, ..., 100\}$. If, for instance $a_{ij} = 2$, then its meaning for the scale $S_3 = \{1, 2, 3\}$ and the scale $S_{100} = \{1, 2, ..., 100\}$ is different. In the former case, the preference $a_{ij} = 2$ means medium preference of the object *i* to the object *j*, while in the latter case the preference is almost negligible, see Fig. 1. Therefore, the scale for pairwise comparisons cannot be neglected. This is true especially in situations when different scales are used simultaneously, or, when results of pairwise comparisons of the same set of objects with different scales need to be compared. In such cases, scale normalization is necessary.

Fig. 1. A comparison of two different scales



Source: author

Tab. 1. Saaty's fundamental scale.

Intensity of importance	Definition
1	Equal importance
2	Weak or slight
3	Moderate importance
4	Moderate plus
5	Strong importance
6	Strong plus
7	Very strong importance
8	Very, very strong importance
9	Extreme importance

Source: Saaty (1977, 1980, 2008)

2 Methods

2.1 Normalization of a pairwise comparison scale

To avoid the problem described in the previous section, normalization of the comparison scale is applied.

Let the general pairwise comparison scale $GS = [1, m]$, $m \in R, m \geq 2$. Then, by the normalization, this scale is transformed into the $[1,2]$ unit interval of real numbers.

The transformation f (the normalization) should satisfy the following (obvious) conditions:

i) $f : [1, m] \to [1, 2]$,
ii) f is strictly increasing.
iii) $f(1) = 1$ and $f(\mathrm{m}) = 2$.

In Koczkodaj (2015), a linear transform f is proposed for normalization:

$$f(x) = \frac{1}{m-1} x + \frac{m-2}{m-1} \tag{4}$$

However, the linear transform (4) has a serious drawback: it does not preserve consistency of pairwise comparisons.

Consider for example the consistent triad (2, 3, 6), and let m = 9. Then, after the linear transform, the triad (9/8, 10/8, 13/8) is not consistent, because $\frac{9}{8} \cdot \frac{10}{8} \neq \frac{13}{8}$.

2.2 Properties of the proposed power normalization

Definition 2. Let $\mathrm{A}(\mathrm{a}_{ij})$ be a pairwise comparison matrix, $a_{ij} \in [1/m, m]$, $m \in R, m \geq 2$. Let f be the power transform:

$$f(a_{ij}) = a_{ij}^k, \; k = \frac{\ln 2}{\ln m} \tag{5}$$

Proposition 1: The power transform (5) satisfies conditions i)-iii).

Proof is obvious.

Proposition 2: The power transform (5) preserves consistency: if $a_{ij} \cdot a_{jl} = a_{il}$, then also $f(a_{ij}) \cdot f(a_{jl}) = f(a_{il})$.

Proof: Let $a_{ij} \cdot a_{jl} = a_{il}$. Then $f(a_{ij}) \cdot f(a_{jl}) = a_{ij}{}^{k} \cdot a_{jl}{}^{k} = \left(a_{ij} \cdot a_{jl}\right)^{k} = a_{il}{}^{k} = f\left(a_{il}\right)$.

Proposition 3: Let *A* be a PCM of the order *n*. Let $T_0(x_0, y_0, z_0)$ be a triad (from *A*) with maximum *KII*. Let A^* be the matrix *A* transformed by (5). Then the *KII* of the transformed triad $T_0^*(x_0, y_0, z_0)$ (from A^*) is also maximal.

Proof: For a triad $T_0(x_0, y_0, z_0)$ one of two possible cases holds: either $\frac{x_0 \cdot y_0}{z_0} \geq 1$, or $\frac{x_0 \cdot y_0}{z_0} \leq 1$. Without loss of generality suppose that $\frac{x_0 \cdot y_0}{z_0} \geq 1$. Also, because *KII* is maximal for T_0, $\frac{x_0 \cdot y_0}{z_0} \geq \frac{x \cdot y}{z}$ holds for all triads (x, y, z). After transformation (5), where $k > 0$, we get: $\frac{x_0^k \cdot y_0^k}{z_0^k} = \left(\frac{x_0 \cdot y_0}{z_0}\right)^k \geq \frac{x^k \cdot y^k}{z^k} = \left(\frac{x \cdot y}{z}\right)^k$, which is true for any positive *k*.

Proposition 3 allows to show that if a pairwise comparison matrix *A* is more inconsistent than a pairwise comparison matrix *B* (with respect to *KII*), this relation is preserved by the power transform (5).

Proposition 4. Let $A(a_{ij})$ and $\mathrm{B}(\mathrm{b}_{ij})$ be inconsistent pairwise comparison matrices of the order *n*. Let *KII* be the Koczkodaj's inconsistency index (3) and let $KII(A) > KII(B)$. Let $A^*(a_{ij})$ and $\mathrm{B}^*(\mathrm{b}_{ij})$ be transformed pairwise comparison matrices by the transform (5), where $p = \frac{\ln 2}{\ln m}$. Then $KII(A^*) > KII(B^*)$.

Proof: Let the most inconsistent triad of the matrix A be $\left(a_{ik}, a_{ij}, a_{jk}\right)$ and the most inconsistent triad of $\mathrm{B}\left(b_{ik}, \mathrm{b}_{ij}, \mathrm{b}_{jk}\right)$. Then, either $\frac{a_{ik}}{a_{ij} \cdot a_{jk}} > 1$ and $\frac{b_{ik}}{b_{ij} \cdot b_{jk}} > 1$, or $\frac{a_{ik}}{a_{ij} \cdot a_{jk}} < 1$ and $\frac{b_{ik}}{b_{ij} \cdot b_{jk}} < 1$. Without loss of generality assume the latter.

Since $KII(A) \geq KII(B)$, we have $1 - \frac{a_{ik}}{a_{ij} \cdot a_{jk}} > 1 - \frac{b_{ik}}{b_{ij} \cdot b_{jk}}$, hence $\frac{a_{ik}}{a_{ij} \cdot a_{jk}} < \frac{b_{ik}}{b_{ij} \cdot b_{jk}}$.

From Proposition 3 it follows that the most inconsistent triad is preserved by the transform (5). Hence, for the transformed matrices A^* and B^* the most inconsistent triads are $\left(a^*{}_{ik}, a^*{}_{ij}, a^*{}_{jk}\right)$ and $\left(b^*{}_{ik}, \mathrm{b}^*{}_{ij}, \mathrm{b}^*{}_{jk}\right)$ respectively. As above, without loss of generality we assume $\frac{a^*{}_{ik}}{a^*{}_{ij} \cdot a^*{}_{jk}} = \frac{a^p{}_{ik}}{a^p{}_{ij} \cdot a^p{}_{jk}} < 1$ and $\frac{b^*{}_{ik}}{b^*{}_{ij} \cdot b^*{}_{jk}} = \frac{b^p{}_{ik}}{b^p{}_{ij} \cdot b^p{}_{jk}} < 1$.

Then we have: $\frac{a_{ik}}{a_{ij}\cdot a_{jk}} < \frac{b_{ik}}{b_{ij}\cdot b_{jk}} \Rightarrow \left(\frac{a_{ik}}{a_{ij}\cdot a_{jk}}\right)^p < \left(\frac{b_{ik}}{b_{ij}\cdot b_{jk}}\right)^p \Rightarrow \frac{a^p{}_{ik}}{a^p{}_{ij}\cdot a^p{}_{jk}} < \frac{b^p{}_{ik}}{b^p{}_{ij}\cdot b^p{}_{jk}} \Rightarrow$ $\frac{a^*{}_{ik}}{a^*{}_{ij}\cdot a^*{}_{jk}} < \frac{b^*{}_{ik}}{b^*{}_{ij}\cdot b^*{}_{jk}} \Rightarrow 1-\frac{a^*{}_{ik}}{a^*{}_{ij}\cdot a^*{}_{jk}} > 1-\frac{b^*{}_{ik}}{b^*{}_{ij}\cdot b^*{}_{jk}} \Rightarrow KII(A^*) > KII(B^*)$.

Proposition 5: Let A be a PCM of the order n. The transformation (5) does not change ranking of all alternatives if the weights of all alternatives are determined by the geometric mean method.

Proof: Let w_i, $i \in \{1,2,...,n\}$, be the weights of alternatives derived form a pairwise comparison matrix $A(a_{ij})$ by the geometric mean method:

$$w_i = \frac{\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} a_{ij}\right)^{1/n}} \tag{6}$$

Without loss of generality it suffices to show that the relation "to be less or equal than" is preserved for an arbitrary pair of weights. Hence, let $w_1 \ge w_2$, and let w_1^* and w_2^* be the transformed weights respectively, then $w_1^* \ge w_2^*$ should hold. When comparing weights given by relation (6), the denominator is the same, so it can be omitted. Also, n-th square can be omitted. Hence, $w_1 \ge w_2$ means that $\prod_{j=1}^{n} a_{1j} \ge \prod_{j=1}^{n} a_{2j}$.

After transform (5) we get:

$\prod_{j=1}^{n} a_{1j}^k = \left(\prod_{j=1}^{n} a_{1j}\right)^k \ge \left(\prod_{j=1}^{n} a_{2j}\right)^k = \prod_{j=1}^{n} a_{2j}^k$, hence $w_1^* \ge w_2^*$. Because the proof for any other pair of weights is analogous, the Proposition 5 is proved.

Remark 1. It is well-known that priority vectors derived from a pairwise comparison matrix of the order n = 3 by the eigenvalue method and the geometric mean method are identical. Therefore, Proposition 5 is also valid for the eigenvalue method and a pairwise comparison matrix of the order n = 3. However, whether Proposition 5 is valid for the eigenvalue method and n > 3 remains an open question.

3 Problem solving

3.1 Numerical example

In this section the use of the proposed normalization (5) is demonstrated on an example.

Example 1. Let the pairwise comparison matrix of four objects be given as follows:

Tab. 2. The input PCM.

1	2	3	5
0.5	1	2	4
0.33333	0.5	1	3
0.2	0.25	0.3333	1

Source: own.

By the GM method, the vector of weights (the priority vector) is: $w = (0.47, 0.29, 0.17, 0.07)$.

The first object has the highest weight which means it is the most preferred or important entity, the second object follows on the second place, etc. Further, there are four triads: (T1) $a_{12} \cdot a_{23} = a_{13}$, (T2) $a_{12} \cdot a_{24} = a_{14}$, (T3) $a_{13} \cdot a_{34} = a_{14}$ and T(4) $a_{23} \cdot a_{34} = a_{24}$. It can be easily checked that all four triads are inconsistent, and the most inconsistent triad is (T3).

Now suppose that *m* is 5, 7, 9 and 20 respectively, or in other words, consider four scales: [1,5], [1,7], [1,9], and [1,20] (with reciprocals).

Then, after the transformation (5) of the PCM given in Tab. 2, normalized pairwise comparison matrices are shown in Tab. 3 along with inconsistency of all four triads and weights of alternatives (priority vectors) on the right hand side of the table. The maximal values are highlighted in blue.

As can be seen, both the maximum inconsistency and objects' rankings are 'invariant' for all scales, the triad (T3) is still the most inconsistent one, and the ordering of objects remains unchanged.

Tab. 3. Four PCMs after transformation (5).

scale	PCM				KII	weights
$m = 5$	1	1.347866	1.6050366	2	0.11653	0.345465
	0.741914	1	1.34786552	1.816741	0.183248	0.278091
	0.623039	0.741914	1	1.605037	0.314114	0.222306
	0.5	0.550436	0.62303875	1	0.160228	0.154139
$m = 7$	1	1.280056	1.4789518	1.7741	0.097399	0.328601
	0.781216	1	1.28005623	1.638544	0.154155	0.274627
	0.676155	0.781216	1	1.478952	0.267907	0.228203
	0.563666	0.610298	0.67615456	1	0.134484	0.16857
$m = 9$	1	1.244413	1.41421356	1.661501	0.086757	0.319406
	0.803592	1	1.24441257	1.548563	0.137801	0.272477
	0.707107	0.803592	1	1.414214	0.241323	0.231264
	0.601866	0.64576	0.70710678	1	0.120067	0.176853
$m = 20$	1	1.173956	1.28942315	1.451197	0.064396	0.300545
	0.851821	1	1.1739559	1.378172	0.103044	0.26748
	0.775541	0.851821	1	1.289423	0.183366	0.237166
	0.689086	0.725599	0.7755406	1	0.08955	0.194809

Source: own

Also, it’s worth noting that with the growing upper boundary *m*, the weights of all objects are becoming more and more uniform (closer to each other).

4 Discussion

In the previous two sections the normalization (the power transform) of the scale for pairwise comparisons was introduced, and several natural and desirable properties of this transformations with respect to the geometrical mean (GM) method and Koczkodaj’s inconsistency index (*KII*) were shown and proved. The simplicity of the proposed normalization and its nice properties might provide incentive for its practical use.

Nevertheless, the use of the GM method and *KII* could be considered limitations of this study. The eigenvalue method for the derivation of a priority vector can be used instead of GM method, and there are many other inconsistency indices than *KII* proposed in the literature, such as Pelaez-Lamata *PLI* index, Golden-Wang *GWI* index, Aguaron and Moreno-Jimenez *GCI* index, and so on, see e.g. Brunelli and Fedrizzi (2015), which might be examined with regard to the proposed normalization. If pursued, this direction of research might prove to be interesting as well, though there is no certainty that similar results can be obtained for other inconsistency indices than *KII*, since *KII* is a maximum-based index of inconsistency unlike other, rather mean-based inconsistency indices. Yet, this research direction certainly deserves attention of experts in the field.

Conclusions

The aim of the paper was to propose a normalization of the scale for pairwise comparisons in the multiplicative AHP/ANP framework, as the AHP/ANP is scale invariant. This leads to undesired effects regarding the intensity of preference, which is, actually, dependent on the upper bound of an applied scale.

The proposed solution to the problem is a normalization in the form of a simple power transforms. The transform has several virtues, see Propositions 1-5 in section 2, namely it preserves consistency of pairwise comparisons, the most inconsistent triad, relation of inconsistency between two arbitrary inconsistent matrices, and last, but not least, it also preserves objects’ rankings when the inconsistency is expressed in terms of Koczkodaj’s inconsistency index.

The proposed approach, the scale normalization, is recommended as an additional step of AHP/ANP when different scales for pairwise comparisons are employed simultaneously, or, when results of pairwise comparisons obtained with different scales are to be compared.

Further research might focus on a scale problem in the context of additive AHP/ANP or fuzzy AHP/ANP. Also, the research of the effect of the proposed normalization on inconsistent pairwise comparison matrices with respect to other inconsistency indices would be desirable.

Acknowledgement

This paper was supported by the Grant Agency of the Czech Republic no. 18-01246S.

References

Alonso, J. A., Lamata M. T. (2006). Consistency in the analytic hierarchy process: a new approach, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, 14(4), pp. 445-459.

Brunelli M., Fedrizzi M. (2015). Axiomatic properties of inconsistency indices for pairwise comparisons, *Journal of the Operational Research So*ciety, 66(1), pp. 1-15. doi: 10.1057/jors.2013.135

Brunelli M. (2017). Studying a set of properties of inconsistency indices for pairwise comparisons. *Annals of Operations Research*, 248(1-2), pp. 143-161.

Donegan, H. A., Dodd, F. J., McMaster, T.B M. (1992). A new approach to AHP decision-making. *The Statistician*, pp. 295-302.

Dong, Y., Xu, Y., Li, H., Dai, M. (2008). A comparative study of the numerical scales and the prioritization methods in AHP. *European Journal of Operational Research* 186: pp. 229–242. doi: 10.1016/j.ejor.2007.01.044.

Elliott, M. A. (2010). Selecting numerical scales for pairwise comparisons, *Reliability Engineering & System Safety*, 95(7), pp. 750-763

Fülop J., Koczkodaj, W. W., Szarek, S. J. (2010). A Different Perspective on a Scale for Pairwise Comparisons. N.T. Nguyen and R. Kowalczyk (Eds.): *Transactions on CCI I, LNCS 6220*, pp. 71–84.

Franek, J., Kresta, K. (2014). Judgment Scales and Consistency Measure in AHP, *Procedia Economics and Finance*, 12, pp. 164-173.

Ishizaka, A., Labib, A. (2011). Review of the main developments in the analytic hierarchy process. *Expert Syst. Appl.,* 38, pp. 14336-14345.

Koczkodaj, W. W. (2015). Pairwise comparison scale rating paradox, arXiv:1511.07540v2. Retreived: Dec 2015.

Koczkodaj, W. W. (1993). A new definition of consistency of pairwise comparisons, *Mathematical and Computer Modeling*, 18(7), pp. 79–84.

Koczkodaj, W., W., Mikhailov, L., Redlarski, G., Soltys, M., Szybowski, J., Wajch, E., Kam Fung Yuen, K., Tamazian, G. (2016). Important Facts and Observations about Pairwise Comparisons. *Fundamenta Informaticae*, 144(3-4), pp. 291-307.

Kramulová, J., Jablonský, J. (2016). AHP model for competitiveness analysis of selected countries. *Central European Journal of Operations Research,* 24(2), pp. 335-351. doi: 10.1007/s10100-015-0394-7.

Lidinska, L., Jablonský, J. (2018). AHP model for performance evaluation of employees in a Czech management consulting company. *Central European Journal of Operations Research*, 26(1), pp. 239-258. doi: 10.1007/s10100-017-0486-7

Lootsma, F. A. (1993) Scale sensitivity in the multiplicative AHP and SMART. *Journal of Multi-Criteria Decision Analysis*, 2(2), pp. 87-110.

Ma, D., Zheng, X. (1991). 9/9-9/1 scale method of AHP. In: *Proceedings of the 2nd International Symposium on the AHP*, vol. 1, pp. 197-202.

Salo, A. A, Hämäläinen, R. P. (1997). On the measurement of preferences in the analytic hierarchy process. *Journal of Multi-Criteria Decision Analysis*, 6(6), pp. 309-319.

Saaty T. L. (1977). A scaling method for priorities in hierarchical structures, *Journal of Mathematical Psychology*, 15(3), pp. 234-281.

Saaty, T. L. (1980). *Analytic Hierarchy Process*. McGraw-Hill, New York.

Saaty, T. L. (2004). Decision making – The analytic hierarchy and network processes (AHP/ANP), *Journal of Systems Science and Systems Engineering*, 13(1), pp. 1–34.

Saaty, T. L. (2008). Decision making with the analytic hierarchy process, *Int. J. Services Sciences*, 1, pp. 83–98.

Thurstone, L. L. (1927). A law of comparative judgments, *Psychological Reviews*, 34, pp. 273-286.

Triantaphyllou, E., Lootsma, F. A., Pardalos, P. M., Mann, S. H. (1994). On the Evaluation and Application of Different Scales For Quantifying Pairwise Comparisons in Fuzzy Sets, *Journal of Multi-Criteria Decision Analysis*, 3(3), pp. 133-155.

Starczewski, T. (2017). Remarks on the impact of the adopted scale on the priority estimation quality. Journal of Applied Mathematics and Computational Mechanics, 16(3), pp. 105-116. doi: 10.17512/jamcm.2017.3.10

Subramanian, N., Ramanathan, R. (2012). A review of applications of Analytic Hierarchy Process in operations management, *International Journal of Production Economics*, 138(2), pp. 215-241, doi: 10.1016/j.ijpe.2012.03.036.

Vaidya, O. S., Kumar, S. (2006). Analytic hierarchy process: An overview of applications, *European Journal of Operational Research*, 169, pp. 1-29.

Contact Address

Mgr. Jiří Mazurek, Ph.D.
Silesian University in Opava, School of Business Administration in Karvina
Univerzitní náměstí 1934/3, 733 40 Karviná, Czech Republic
Email: mazurek@opf.slu.cz

Received: 17. 08. 2018, reviewed: 25. 01. 2019
Approved for publication: 19. 4. 2019