Accepted Manuscript

Fully graph solution of the sensitivity of switched circuits

Bohumil Brtník

PII: S0026-2692(17)31000-5

DOI: 10.1016/j.mejo.2018.09.008

Reference: MEJ 4420

To appear in: *Microelectronics Journal*

Received Date: 22 December 2017

Revised Date: 13 June 2018

Accepted Date: 21 September 2018

Fully Graph Solution of the Sensitivity of Switched Circuits

Bohumil Brtník

Abstract— **The most general parameter of the electronic circuit is its sensitivity. Sensitivity analysis helps circuit designers to determine boundaries to predict the variations that a particular design variable will generate in a target specifications, if it differs from what is previously assumed. There are two basic methods for calculating the sensitivity: matrix methods and graph methods. The method described in this article is based on a graph, that contains separate input ad output nodes for each phase. This makes it possible to determine the transmission sensitivity even between partial switching phases.**
The described fully-graph method is suitable for switched current circuits and switched capacitors circuits, too.

Keywords — **Sensitivity, switched capacitors, switched currents, graph method.**

I. Introduction

THE basic parameter of the electronic circuit is its sensitivity. Sensitivity analysis is useful for identifying tolerances of circuit elements. Some graph methods for sensitivity analysis were described in [1], [2], [3]. Sensitivity analysis is very important in switched circuit design, because it helps us to optimize the behavior of a given switched circuit by showing us which components of the entire systems are more sensitive. The most general element for switched circuits is a transconductance with input and output branches in different phases [4]. The main disadvantage of the previously published graphical method for switched circuits (for example [5] for SC circuits) is the impossibility to determine the sensitivity between the two different switching phases. The reason is that these graphs so far published do not include separate output and input nodes for different switching phases.

For switched current circuits the expression that gives the sensitivity of the transfer $F_{Ob,Ea}$ relative to a conductance G (control current in phase E at branch a and output current in phase O at branch b) is

$$S_G^{F_{Ob,Ea}} = \frac{G}{F}\frac{\partial F}{\partial G}. \qquad (1)$$

For switched capacitors circuits, the sensitivity of the transfer $F_{Ob,Ea}$ relative to a capacitance C (the control voltage is in phase E at node a and output voltage in phase O at node b) is

$$S_C^{F_{Ob,Ea}} = \frac{C}{F}\frac{\partial F}{\partial C}. \qquad (2)$$

B. Brtník is with University of Pardubice, Faculty of the Electronics and Informatics Czech Republic. (phone: 420-466-037-065, e-mail: bohumil.brtnik@upce.cz).

II. The Transfer Function and the Sensitivity Determinations

For example, the relative sensitivity S_C^F of the circuit is given as (3)

$$S_C^F = \frac{C}{F}\frac{\partial F}{\partial C} \qquad (3)$$

where F is the transfer function, C is the circuit element. The derivation $\frac{\partial F}{\partial C}$ for the circuit, which is described by simplest general MC-graph in Fig. 1 taken from [6], for example $\frac{\partial F}{\partial C_{23}}$, is given as follows (4).

$$\frac{\partial F}{\partial C_{23}} = \frac{\partial}{\partial C_{23}}\frac{C_{21}.C_{32}}{C_{22}.C_{33} - C_{32}.C_{23}} =$$

$$= \frac{\frac{\partial C_{21}.C_{32}}{\partial C_{23}}(C_{22}.C_{33} - C_{32}.C_{23}) - C_{21}.C_{32}.\frac{\partial(C_{22}.C_{33} - C_{32}.C_{23})}{\partial y_{23}}}{(C_{22}.C_{33} - C_{32}.C_{23})^2} =$$

$$= \frac{0\cdot(C_{22}.C_{33} - C_{32}.C_{23}) - C_{21}.C_{32}.(0 - C_{32})}{(C_{22}.C_{33} - C_{32}.C_{23})^2} =$$

$$= \frac{C_{21}.C_{32}}{C_{22}.C_{33} - C_{32}.C_{23}}\cdot\frac{C_{32}}{C_{22}.C_{33} - C_{32}.C_{23}} = T_{31}\cdot T_{32} \qquad (4)$$

where first member $\frac{C_{21}.C_{32}}{C_{22}.C_{33} - C_{32}.C_{23}} = T_{31}$ is a partial transfer function from node 1 to the node 3, second one member $\frac{C_{32}}{C_{22}.C_{33} - C_{32}.C_{23}} = T_{32}$ is a partial transfer function from node 2 to the node 3 after application Mason´s formula (5)

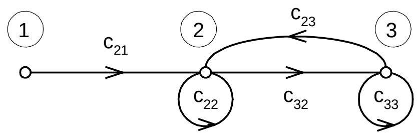

Fig. 1. The graph describing a general circuit and illustrating the members of the equations

$$T=\frac{\sum p_{(i)}\cdot\Delta_{(i)}}{V-\sum S^{(K)}\cdot V^{(K)}} \tag{5}$$

as is depicted in Fig. 1.
The derivation (6)

$$\frac{\partial F}{\partial C_{23}}=T_{31}\cdot T_{32} \tag{6}$$

can be generalized in form (7)

$$\frac{\partial F_{kq}}{\partial C_{ji}}=T_{iq}\cdot T_{kj} \tag{7}$$

Thus, relative sensitivity is (8)

$$S_{C_{ji}}^{F_{kq}}=\frac{C_{23}}{F_{kq}}\cdot T_{iq}\cdot T_{kj}\cdot \tag{8}$$

The second one is multiparameter relative symbolic sensitivity, which is calculated from (9).

$$S_{C_1C_2}^{F}=\frac{C_1\cdot C_2}{T_{ba}}\cdot\frac{\partial^2 T_{ba}}{\partial C_1\cdot\partial C_2}, \tag{9}$$

where the member $\frac{\partial^2 T_{ba}}{\partial C_1\cdot\partial C_2}$ can be replaced as follows

$$\frac{\partial^2 T_{ba}}{\partial C_1\cdot\partial C_2}=\frac{\partial^2 T_{ba}}{\partial C_1\cdot\partial C_2}\cdot\frac{\partial Y_{kl}}{\partial Y_{kl}}=\frac{\partial C_{kl}}{\partial C_2}\cdot\frac{\partial^2 T_{ba}}{\partial C_1\cdot\partial C_{kl}}=$$

$$=\frac{\partial C_{kl}}{\partial C_2}\cdot\frac{\partial^2 T_{ba}}{\partial C_{ji}\cdot\partial C_{kl}}=\frac{\partial C_{kl}}{\partial C_2}\cdot\frac{\partial^2 T_{ba}}{\partial C_{ji}}\cdot\frac{1}{\partial C_{kl}}=$$

$$=\frac{\partial C_{kl}}{\partial C_2}\cdot\frac{\partial}{\partial C_{kl}}\frac{\partial T_{ba}}{\partial C_{ji}}=\frac{\partial C_{kl}}{\partial C_2}\cdot\frac{\partial}{\partial C_{kl}}T_{ia}\cdot T_{bj}=$$

$$=\frac{\partial C_{kl}}{\partial C_2}\cdot\left(\frac{\partial T_{ia}}{\partial C_{kl}}\cdot T_{bj}+\frac{\partial T_{bj}}{\partial C_{kl}}\cdot T_{ia}\right)\cdot \tag{10}$$

Thus, the relative sensitivity can be rewritten to the form (11)

$$S_{C_1C_2}^{F}=\frac{C_1\cdot C_2}{T_{ba}}\frac{\partial C_{kl}}{\partial C_2}\cdot\left(\frac{\partial T_{ia}}{\partial C_{kl}}\cdot T_{bj}+\frac{\partial T_{bj}}{\partial C_{kl}}\cdot T_{ia}\right) \tag{11}$$

where members $C_{ji}=C_1+C_X$, $C_{kl}=C_2+C_Y$ are elements of the graphs, C_X, C_Y contain other network parameters between nodes i, j, and k, l respectively [7], as is depicted in Fig. 2.
The derivation given above shows that relations (7)

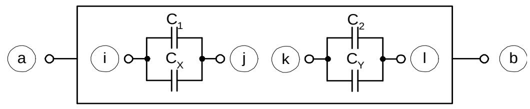

Fig. 2. The determination of the members Cji, Ckl.

and (11) are quite general. These relationships (7), (11) given for non-switched circuits in [7], [8] can be therefore used for switched circuits, as well.

However, the graph solution requires finding a graph that includes all switching phases. The circuit must be described by a graph showing the four-port. The construction of the graph using transformation graphs and two-graphs for switched capacitors and switched currents circuits of required properties was described in [9].

III. Example

In this section an example concerning the symbolic analysis of the switched capacitor circuit is presented to show that the proposed method is applicable to relative sensitivity analysis of the switched circuits.

Let is find the relative sensitivity $S_{C_1}^{F_{5O,1E}}$ of the voltage transfer function $F_{5O,1E}$ (i.e. to node 5 in phase O from node 1 in phase E) of the SC circuit shown in Fig. 3.

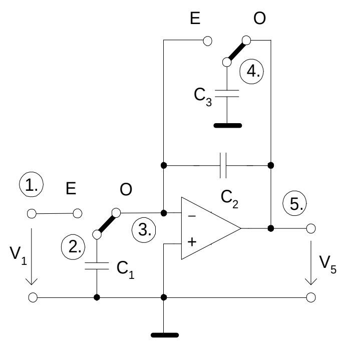

Fig.3. The circuit for the example

The transformation graph method will be used for solving of this example. Figure 4 shows the first step of the fully-graph sensitivity solutions.

The simplified graph of this circuit, its construction is described in detail in [7] in partial phases EE, OO, EO and OE, is in the upper part of this figure. In the middle of this figure, there are transformation graphs of the switches and of the operational amplifier, its construction is described in detail in [8]. The transformed graphs are constructed by using the algorithm described in [9] drawn at the bottom of this figure.

Described steps can be considered as simplification before generation of the summary graph, containing all these phases.

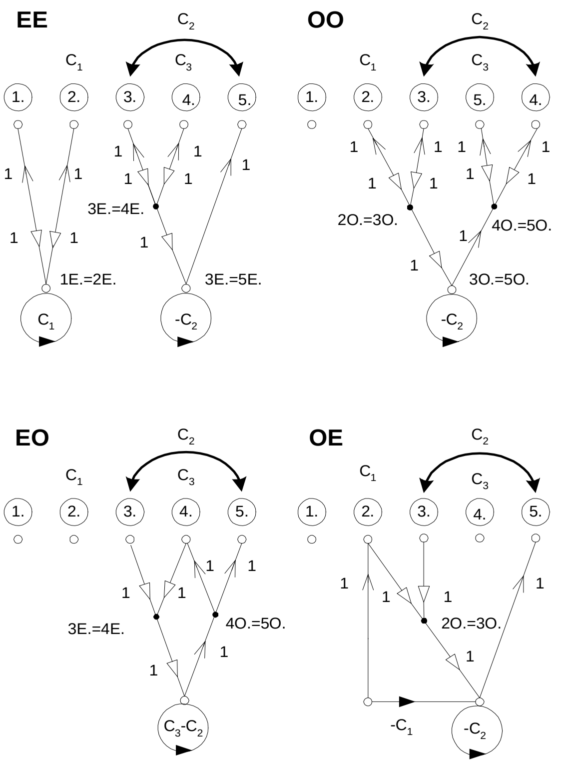

Fig.4. The first step of fully-graph solutions.

By using the simple algorithm described in detail in [10] a summary graph containing all the switching phases EE, OO, EO and OE are constructed from partially transformed graphs, as is shown in Fig. 5.

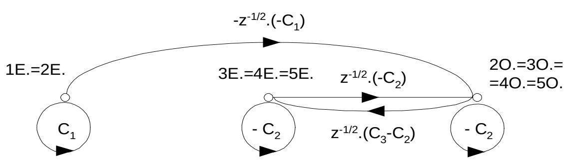

Fig. 5. The second one step of construction of the summa graph

It should be noted that the two-graph method leads to the same summary graph as it follows from the solution published in [9]. One of the easy-to-quantify ways to compare methods is the number of steps to be taken. The two-graph method requires drawing a partial schema, especially for each partial phase. Additional, voltage nodes and charge nodes must be marked differently, i.e. four graphs are needed. Oriented charts are drawn to them in next addition step, which are two extra add steps over the described method in Fig.4. Then, the four graphs are used to search for common skeletons and to draw a summary graph.

It is clear from this that the solution of the circuits using the two graphs is much more laborious than if the transformation graph is used and therefore is not discussed in detail, as well.

In second one step of the sensitivity solutions, the rule (8) is applied. The relative sensitivity $S_{C_1}^{F_{5O,1E}}$ thus will be calculated as (12).

$$
\begin{aligned}
S_{C_1}^{F_{5O,1E}} &= \frac{C_1}{F_{5O,1E}}\cdot\frac{\partial F_{5O,1E}}{\partial C_1} = \frac{C_1}{F_{5O,1E}}\cdot\frac{\partial F_{5O,1E}}{\partial C_1}\cdot\frac{\partial C_{5O,1E}}{\partial C_{5O,1E}} = \\
&= \frac{C_1}{F_{5O,1E}}\cdot\frac{\partial F_{5O,1E}}{\partial C_{5O,1E}}\cdot\frac{\partial C_{5O,1E}}{\partial C_1} = \\
&= \frac{C_1}{F_{5O,1E}}\cdot T_{1E,1E}\cdot T_{5O,5O}\cdot\frac{\partial C_{5O,1E}}{\partial C_1} = \\
&= \frac{C_1}{\dfrac{-z^{-\frac{1}{2}}(-C_1)\cdot(-C_2)}{(-C_2)\cdot(-C_2)-z^{-\frac{1}{2}}(-C_2)\cdot z^{-\frac{1}{2}}(C_3-C_2)}}\cdot \\
&\cdot\frac{(-C_2)(-C_2)-z^{-\frac{1}{2}}(-C_2)z^{-\frac{1}{2}}(C_3-C_2)}{(-C_2)\cdot(-C_2)-z^{-\frac{1}{2}}(-C_2)\cdot z^{-\frac{1}{2}}(C_3-C_2)}\,\frac{\partial\left[-z^{-\frac{1}{2}}(-C_1)\right]}{\partial C_1} = \\
&= \frac{C_1}{-z^{-\frac{1}{2}}(-C_1)(-C_2)}\left[-(-z^{-\frac{1}{2}})\right](-C_2) = \frac{-z^{-\frac{1}{2}}C_1C_2}{-z^{-\frac{1}{2}}C_1C_2} = 1
\end{aligned}
\tag{12}
$$

where $T_{1E,1E}$, $T_{3O,3O}$ are partial transfer functions from summary graph (see Fig.5), there are calculated from Mason´s formula (5).

IV. Conclusion

The problem of sensitivity determination has been investigated in a great number of publications during the last decades [15], [16], [17], [18], [19], [20], [21] and many others. A typical problem here is the determination of the derivates in the sensitivity expressions and its decision had been searched in different ways.

A fully graph method for relative symbolic sensitivity analysis of the switched circuits is described.

This method requires no intermediate nullor models, which is an auxiliary step in the conventional solution, the method does not require the capacitance (and/or conductance) matrix assembling. The disadvantage of graph methods, which are described by prof. Mikula and prof. Dostál [10], [11], [12] and/or prof. Moschytz [22] and prof. Toumazou [4] and many others, is the impossibility to find the transfer and hence the sensitivity between different switching phases. Using the summary graph eliminates this disadvantage.

In addition to transformation graphs, the two-graph method can also be used to construct a summary graph. This construction is described in detail in [9], but this method is two steps more laborious, as shown in [13]. Therefore the abovementioned methods of solutions have been selected, because using them the graph solutions is simplest and least laborious.

The example is solved for the SC circuit, but this method is also applicable to SI circuits due to the similarity of relationships (1) and (2), too. The construction of a summary graph for the SI circuit is described in detail in [23], as well.

Since all other known graphical solutions of switching capacitor circuits do not differentiate the input and output nodes for each phase separately, these methods cannot be used to sensitivity solution.

Such a solution is only possible by matrix methods, not by known graph methods, thus the described graph method with the summa graph cannot be compared with other existing graphical methods, because this comparison is impracticable. If the matrix solution would be "by hand", it would include: 1) assembly a capacitance circuit matrix, 2) assembly matrix for all switching phases, 3) reducing matrix by the switch, 4) reducing by the op.amp, 5) calculation of the algebraic minors, 6) calculation of transfer, 7) calculation of its derivations, 8) sensitivity calculation. Another thing is the computational complexity of obtained matrix.

The computational complexity in calculating the matrix depends on the number of nodes in the circuit, while the graph evaluation depends on the number of branches in the circuit. Graph is suitable for a circuit with fewer branches. Thanks to its clarity, a graph shows the circuit more clearly than the matrix.

It is not known, that has yet been published graph method enabling to determine the sensitivity between the phases.

REFERENCES

[1] SHI, G. Graph-Pair Decision Diagram Construction for Topological Symbolic Circuit Analysis. *IEEE Transactions on Computer Aided Design of Integrated Circuits and Systems.* vol.32, no.2, pp.275-288. Feb.2013.

[2] PIERZCHALA, M., FAKHFAKH, M. *Transformation of LC filters to active RC circuits via the two-graph method*. Microelectronic Journal, vol.42, no.8, pp.999-10015, Aug.2011.

[3] RODRIGUES-CHAVEZ, S., PALMA-RODRIGUEZ, A., A., TLELO-CUAUTLE, E., TAN, S., X. *Graph-based symbolic and symbolic sensitivity analysis of analog integrated circuits.* Analog/RF and Mixed-Signal Circuit Systematic Design. Springer, Lecture Notes in Electrical Engineering, vol.233, pp. 101-122, 2013.

[4] TOUMAZOU, C., BATTERSBY, N. C., HUGHES, J. B. *Switched-currents an analogue technique for digital technology.* London (UK): Peter Peregrinus Ltd., 1993. ISBN: 0 86341 294 7.

[5] DOSTÁL, T., MIKULA, J. Signal-Flow Graph Network Analysis. *Radioengineering*, 1992, vol.1, no.1, p.14-20.

[6] ČAJKA, J., KVASIL, J. *Theory of circuits. Linear and linearized circuits.* Praha/Bratislava (CS): SNTL/ALFA, 1979.

[7] ASENOVA, I., BALIK, F. Multiparameter symbolic sensitivity analysis enhanced by nullor model and modified Coates flow graph. *Advanced in electrical and electronic engineering*, 2013, vol.11, no.2, p.108-115.

[8] ASENOVA, I. calculation of first-, second-order and multiparameter symbolic sensitivity of active circuits by using nullor model and modified Coates flow graph. *International journal of microelectronics and computer science*, 2011, vol.2, no.4, p.129-135.

[9] BRTNÍK, B. Comparison of the full-graph method switched circuit solution. *Przeglad Elektrotechniczny*, 2012, no.6, p.332-336.

[10] DOSTÁL, T. The analysis of active components containing switched capacitors by nodal voltage method and transformations. *Electronic horizon*, 1984, vol.45, no.1, p.21-27.

[11] MIKULA, J., DOSTÁL, T. A design approach to capacitor-switched networks, using MC graphs. Electronic horizon, 1985, vol.46, no.12, p. 560-565.

[12] POSPÍŠIL, J., DOSTÁL, T. *Electronic circuits and systems I.* Brno (CS): BUT, 1991.

[13] BRTNÍK, B. *Solving switched circuits by graph methods*. Praha (CZ): BEN, 2011.

[14] BRTNÍK, B. Graph only solution of SC circuits by means two-graph. In *Proceedings of the 15th WSEAS international conference on circuits (CSCC)*. 2011, Corfu Island, Greece, p.100-105.

[15] PALMA-RODRIGUEZ, A., A., TLELO-CUAUTLE, E., RODRIGUEZ-CHAVEZ, S. DDD-based symbolic sensitivity analysis of active filters. In *Proceedings of the 8th International Caribbean Conference on Devices, Circuits and Systems (ICCDCS)*, 2012, p. 48-51.

[16] NENOV, G., A. A comparative study on symbolic network sensitivity analysis by using some kinds of signal-flow graphs. In *Proceedings of the 5th International conference on circuits, system, control, signals (CSCS´14).* 2014, Salerno, Italy, p.54-63.

[17] OSOWSKI, S. Signal flow graphs for determination of higher order sensitivities of circuit functions. *Proceedings of the European Conference on Circuits Theory and Design*. 2011, pp.446-449.

[18] STENBAKKEN, G. N., STARZYK, J. A. Diakoptic and large change sensitivity analysis. *IEEE PROCEEDINGS-G. V*ol. 139. No. l., 1992, pp.114-118.

[19] IORDACHE, M., DUMITRIU, L., NICULAE, D. On the Sensitivity Analysis of Analog Circuits. Annals of the University of Craiova, Electrical Engineering series, 2008, No. 32, pp.11-16.

[20] DABROWSKI, A., MOSCHYTZ, G. S. Direct Analysis of Multiphase Switched-Capacitor Networks Using Signal-Flow Graphs. *IEEE Transactions on Circuits and Systems*, vol. 37, no.5, pp.594-607, May 1990.

[21] LI, P.M. Sensitivity Analysis of Large Linear Networks Using Symbolic Programs, *Proceedings of The IEEE International Symposium on Circuits and Systems ISCAS '92*, 10-13 May 1992, vol.3, pp.1145-1148.

[22] MAO, J.-F., KUH, E. S. Fast Simulation and Sensitivity Analysis of Lossy Transmission Lines by the Method of Characteristics, *IEEE Trans. Circuits and Systems I: Fundamental Theory and Applications*, vol. 44, issue 5, May 1997.

[23] BRTNÍK, B. Assembling a formula for current transferring by using a summary graph and transformation graphs. *Journal of electrical engineering.* 2013, vol.64, no.5, p. 334-336.