

# Interval-Valued Fuzzy Cognitive Maps with Genetic Learning for Predicting Corporate Financial Distress

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**Abstract.** Fuzzy cognitive maps (FCMs) integrate neural networks and fuzzy logic to model complex nonlinear problems through causal reasoning. Interval-valued FCMs (IVFCMs) have recently been proposed to model additional uncertainty in decision-making tasks with complex causal relationships. In traditional FCMs, optimization algorithms are used to learn the strengths of the relationships from the data. Here, we propose a novel IVFCM with real-coded genetic learning. We demonstrate that the proposed method is effective for predicting corporate financial distress based on causally connected financial concepts. Specifically, we show that this method outperforms FCMs, fuzzy grey cognitive maps and adaptive neuro-fuzzy systems in terms of root mean squared error.

## 1. Introduction

Fuzzy cognitive maps (FCMs) are employed for knowledge representation through signed fuzzy weighted digraphs [13]. The nodes of the graphs stand for descriptive concepts usually expressed in terms of fuzzy sets. The concepts are causally connected by directed edges labelled with fuzzy weights. Thus, a high uncertainty can be incorporated. In addition, FCMs have the capacity to effectively model nonlinear problems since a nonlinear activation function is used to transform the impact of the concepts. These characteristics have made FCMs appealing for various economic and financial applications [7].

The development of FCMs is based on either expert knowledge or experimental data. Expert knowledge is associated with subjectivity and therefore it fails to develop highly complex models. Therefore, various learning approaches have been used to automatically design FCMs [11]. More precisely, the concepts are usually proposed by an expert and a learning algorithm is then used to compute the parameters of FCM that best fits the data. Evolutionary approaches such as genetic algorithms (GAs) [16] have been particularly effective in learning FCMs. Alternatively, evolutionary approaches have also been employed to aggregate the opinions of multiple experts [8].

To overcome the problems of traditional FCMs, several extensions of FCMs have been proposed. A major issue to be addressed is determining the precise values of a weight matrix under strong uncertainty

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in dynamic and unstructured environments. This uncertainty can be handled by generalizing the concept of fuzzy sets. As a result, intuitionistic FCMs [12], interval-valued FCMs (IVFCMs) [7], fuzzy grey cognitive maps (FGCMs) [15] and interval-valued intuitionistic FCMs [14] have recently been introduced. The most striking quality of these generalizations is the higher level of uncertainty that can be used to represent concepts and relationships. The main differences between these extensions lie in their motivation and their target application tasks, resulting in different inference mechanisms. However, little attention has been focused on learning these FCM generalizations, with the exception of learning FGCMs for time series forecasting [4].

In economic and financial decision making, intervals are used to represent the uncertainty associated with an insufficient model accuracy and knowledge of context [14]. Moreover, causal interaction effects exist between financial concepts such as the prediction of corporate financial distress [10]. Despite this interest, no scholars have thus far applied FCMs or their generalizations to predict corporate financial distress. This study bridges this gap by developing an IVFCM with genetic learning for predicting corporate financial distress. In the proposed model, financial concepts are extracted from the corporate annual reports of U.S. companies and causal relationships are estimated by using real-coded GAs. Financial distress is represented by Altman's Z-score, the most widely used financial distress measure [1]. In the presented case study, we show that the proposed approach can serve as an appropriate decision support system for financial problems with a strong uncertainty. To understand how financial concepts affect corporate financial distress is of particular interest because corporate financial distress reduces information asymmetry in creditor-debtor relationships. The early-warning systems for corporate financial distress are attracting widespread interest also due to their importance for the stakeholders of firms, affecting financing and strategic decisions. The proposed prediction model is aimed to provide an accurate and interpretable early-warning system to the stakeholders.

This paper is organized as follows. Section 2 presents the theoretical background on IVFCMs. In Section 3, we demonstrate the effectiveness of this method for predicting corporate financial distress. Finally, this paper is concluded with possible research directions.

## 2. IVFCMs with Genetic Learning

An FCM is a signed fuzzy weighted digraph with  $N$  nodes (concepts). The  $i^{\text{th}}$  concept in the  $k^{\text{th}}$  iteration is labelled with fuzzy value  $c_i^k$ . Fuzzy weight  $w_{ji}$  in the range on  $[-1,1]$  denotes the sign and strength of the causal relationship from the  $j^{\text{th}}$  to the  $i^{\text{th}}$  concept. Thus, positive (negative) fuzzy weight  $w_{ji}$ , an increase in  $c_j^k$  will lead to an increase (decrease) in  $c_i^{k+1}$ . To calculate  $c_i^{k+1}$ , multiple edges usually have to be considered. In addition, activation function  $f$  (usually a nonlinear function) can be employed to transform the linear values of  $c_i^{k+1}$ .

**Definition 1.** *Inference in FCMs is expressed as:*

$$c_i^{k+1} = f\left(c_i^k + \sum_{\substack{j=1 \\ j \neq i}}^N c_j^k \times w_{ji}\right), \quad (1)$$

Inference in conventional FCMs must be reformulated to accommodate the additional level of uncertainty. In IVFCMs, interval-valued fuzzy sets (IVFSs) are used instead of fuzzy sets. In IVFS  $A$ , the degree of membership is defined by an interval  $[\mu_A^L(x), \mu_A^U(x)]$ , where  $\mu_A^L(x)$  denotes the lower extreme and  $\mu_A^U(x)$  represents the upper extreme, respectively,  $0 \leq \mu_A^L(x) \leq 1$ ,  $0 \leq \mu_A^U(x) \leq 1$ . Note that the higher is the length of the interval, the higher is the level of uncertainty.

**Definition 2.** *For two IVFSs  $A$  and  $B$ , the addition, subtraction and multiplication operators are defined as follows [3]:*

$$A \oplus B = \left\{ \left\langle x, [\min(\mu_A^L(x) + \mu_B^U(x), \mu_A^U(x) + \mu_B^L(x)), \mu_A^U(x) + \mu_B^U(x)] \right\rangle \mid x \in X \right\}, \quad (2)$$

$$A \ominus B = \left\{ \left\langle x, [\mu_A^L(x) - \mu_B^U(x), \max(\mu_A^L(x) - \mu_B^L(x), \mu_A^U(x) - \mu_B^U(x))] \right\rangle \mid x \in X \right\}, \quad (3)$$

$$A \otimes B = \left\{ \left\langle x, [\mu_A^L(x) \times \mu_B^L(x), \max(\mu_A^L(x) \times \mu_B^U(x), \mu_A^U(x) \times \mu_B^L(x))] \right\rangle \mid x \in X \right\}. \quad (4)$$

As demonstrated by [3], the operator of multiplication is defined on [0,1] and the remaining operators represent the extensions of traditional addition and subtraction operators on fuzzy sets. Specifically, if  $x^L = x^U$ ,  $y^L = y^U$  (the case of traditional fuzzy sets,  $\pi_A(x) = 0$ ), then  $x^L + y^L$  for the addition and  $x^L - y^L$  for the subtraction operator. As reported by [3], the reasoning in IVFCMs can benefit from several important properties, such as associativity and commutativity. By using these operators, reasoning in IVFCMs can be expressed as [7]:

$$c_i^{k+1} = \{[\mu_A^L(c), \mu_A^U(c)]_i^{k+1} = f(\{[\mu_A^L(c), \mu_A^U(c)]_i^k \oplus (\bigoplus_{\substack{j=1 \\ j \neq i}}^N \{[\mu_A^L(c), \mu_A^U(c)]_j^k \otimes \{[\mu_A^L(w), \mu_A^U(w)]_{ji}\})\})\}. \quad (5)$$

In other words, the lower and upper bounds  $[\mu_A^L(c), \mu_A^U(c)]$  of the  $i$ -th concept are calculated using an activation function  $f$ , similarly as in eq. (1), but the addition, subtraction and multiplication operators are replaced by those defined in eq. (2-4).

In this study, a real-coded GA was used to optimize the weight matrix  $\mathbf{W}=\{w_{ji}\}$ ,  $j \neq i$ , of an IVFCM with respect to the RMSE (root mean squared error). The weight matrix  $\mathbf{W}$  can be characterized by  $N(N-1)$  variables. Since both lower and upper bounds  $[\mu_A^L(w_{ji}), \mu_A^U(w_{ji})]$  have to be determined, each chromosome in the GA comprises  $2N(N-1)$  genes. Thus, each chromosome represents a candidate IVFCM.

Input training data vectors represent the initial values of the concepts ( $k=0$ ). Note that  $\mu_A^L(c)=\mu_A^U(c)$  at  $k=0$ . The RMSE was used as the fitness function in genetic learning. This was calculated as the difference between the outputs of training data  $y_m$ ,  $k=1, 2, \dots, M$  (again note that  $y_k^L=y_k^U$ ) and output  $\hat{y}_k$  predicted by the IVFCM. Sigmoid function  $f$  was employed for the concept values' transformation. To avoid overfitting, the number of iterations in the IVFCM reasoning was fixed and set to 10 [15]. To obtain defuzzified output  $\hat{y}_k$ , we calculated the average of the output IVFS as:

$$\hat{y}_k = \frac{\hat{y}_k^L + \hat{y}_k^U}{2}. \quad (6)$$

As a result, the fitness function can be expressed in the following way:

$$RMSE = \sqrt{\frac{1}{M} \times \sum_{m=1}^M (\hat{y}_m - y_m)^2}. \quad (7)$$

The genetic learning of IVFCMs can be defined as:

**Input:** training data vectors:  $\{c(1), c(2), \dots, c(M)\}$

**Output:** learnt weight matrix:  $\mathbf{W}$

Randomly initialize population  $H_i$ ,  $i=1$ ;

Evaluate population( $H_i$ );

**While** (stopping condition is not satisfied) **do**

$H_{i+1} \leftarrow$  Select the best-fit individuals ( $H_i$ );

Crossover( $H_{i+1}$ );

Mutation( $H_{i+1}$ );

$i \leftarrow i+1$ ;

**return**  $h_{best} \in H_i$  – chromosome with the best fitness value;

Table 1: Description of the data

Category	Variable	Mean±St.Dev.	Category	Variable	Mean±St.Dev.
Linguistic	Optimism	0.0200±0.0041	Financial	WC/TA <sup>2013</sup>	0.1349±0.0931
	Realism	0.2992±0.0287		RE/TA <sup>2013</sup>	0.2810±2.3179
	Profitability	0.0072±0.0016		EBIT/TA <sup>2013</sup>	0.2191±0.5931
	Leverage	0.0299±0.0054		E/TL <sup>2013</sup>	1.8348±4.4409
	Liquidity	0.0055±0.0045		S/TA <sup>2013</sup>	0.6970±4.3588
Predicted output	Z-score <sup>2015</sup>	2.1226±1.9363			

### 3. Predicting Corporate Financial Distress

Recently, it has been reported that corporate financial distress can be predicted based on concepts extracted from the documents (usually annual reports) related to assessed firms [6]. To develop an easy-to-interpret model, we used the approach based on predefined rules (dictionaries). However, this approach may be context-sensitive, requiring specific dictionaries for the financial domain. To address this issue, we extracted the concepts from two perspectives, namely a sentiment and a topic perspective. The sentiment perspective was based on the dictionaries reported as the most discriminative in previous studies on financial performance prediction, namely optimism and realism [6]. Diction 7.0 was used as the source of these two dictionaries. The topic perspective was extracted by using the dictionaries for financial ratio categories defined by [9]. In terms of predictive accuracy, profitability, liquidity and leverage dictionaries were reported to be strongly correlated with financial performance. Moreover, strong relationships were also observed between the financial ratio categories, which corresponds to the theoretical considerations of the causal interaction effects between financial concepts [10].

To obtain the values of the above-mentioned five concepts, we first collected and preprocessed a set of 1329 U.S. firms' annual reports for 2013. As the source of annual reports, we used the EDGAR System freely available at [www.sec.gov/edgar.shtml](http://www.sec.gov/edgar.shtml). First, we used tokenization and lemmatization for the linguistic preprocessing of the documents. The tagged lemmas represented potential terms. Then, we compared them with the five dictionaries of financial concepts. The overall word (raw term) frequency count was used for each financial concept. This is words had the same weight within the category. The frequency shows how much attention the firm's management devoted to this financial concept in its communication with stakeholders. To take the different lengths of the texts into account, the counts were normalized by the length of the documents. The output variable was represented by Altman's Z-score for firms tradable on the stock market [1]:

$$Z = 1.2 \times \frac{WC}{TA} + 1.4 \times \frac{RE}{TA} + 3.3 \times \frac{EBIT}{TA} + 0.6 \times \frac{E}{TL} + 1.0 \times \frac{S}{TA}, \quad (8)$$

where WC is working capital, TA is total assets, RE is retained earnings, EBIT is earnings before interests and taxes, E is the market value of equity, TL is the book value of total liabilities, and S is sales. The selection of the profitability, leverage and liquidity ratios and their corresponding weights were based on empirical regression models using the sample of financially distressed/non-distressed firms [1]. The Z-score was calculated for the 1329 U.S. firms for 2015 (the prediction horizon was two years). Table 1 presents the descriptive statistics of the data. Since we also compare the performance of the concepts with the initial values of the financial indicators, we refer to variables  $x_1, x_2, \dots, x_5$  for 2013 as well.

To learn the weight matrix of the IVFCM, we used a GA with a population size of 40, roulette selection, crossover probability  $p_c=0.8$  and mutation probability  $p_m=0.05$ , 100 generations were used as the stopping condition. Each chromosome in the GA comprised 60 genes representing all lower and upper bounds of the weights  $\mu_A^L(w_{ji})$  and  $\mu_A^U(w_{ji})$ . 10-fold cross-validation was employed for the partition of training/testing data. Hereinafter, we report the results on testing data in terms of the RMSE, mean absolute error (MAE)

Table 2: Weight matrix of the IVFCM trained with GA (average over 10 experiments)

	Optimism	Realism	Profitability	Leverage	Liquidity	Z-score
Optimism		[0.03,0.61]	[0.00,0.03]	[0.06,0.76]	[-0.10,0.26]	[-0.03,0.28]
Realism	[0.07,0.12]		[0.06,0.26]	[0.41,0.54]	[0.01,0.50]	[-0.04,0.34]
Profitability	[0.05,0.31]	[0.10,0.57]		[-0.03,0.71]	[-0.01,0.03]	[0.01,0.69]
Leverage	[-0.10,0.42]	[0.00,0.57]	[0.01,0.31]		[-0.05,0.57]	[-0.24,0.93]
Liquidity	[-0.64,0.54]	[-0.30,0.43]	[-0.09,0.67]	[0.03,0.57]		[-0.69,0.52]
Z-score	[-0.25,0.47]	[-0.27,0.52]	[-0.12,0.77]	[-0.28,0.72]	[-0.20,0.50]	

and mean absolute percentage error (MAPE). Table 2 presents the average values of the weights in the trained IVFCMs. The large differences between the bounds of the weights denote strong uncertainty in the causal relationships. A strong positive effect on financial performance was observed for the profitability concept in particular. Both bounds of the target concept Z-score converged to the fixed equilibrium points after about five iterations.

To compare the performance of the IVFCM-GA when predicting corporate financial distress, we employed FCM-GA [16], FGCM-GA [4] and two neuro-fuzzy methods used for financial distress prediction in previous studies, namely adaptive neuro-fuzzy inference system (ANFIS-PSO) [2] and intuitionistic neuro-fuzzy network (INFN-PSO) [5]. In the traditional FCM-GA, each chromosome of the GA comprises  $N(N-1)$  genes, thus having lower computational complexity compared with the IVFCM-GA and FGCM-GA. In the experiments with the FCM-GA and FGCM-GA, we used the same setting of the GA parameters. In contrast to the operators defined for the IVFSs, FGCMs are based on the arithmetic of grey numbers. Also note that unlike [4], the gain parameters of the sigmoid functions were not optimized.

For the ANFIS-PSO and INFN-PSO, we adopted the Pittsburgh approach to evolutionary-based fuzzy systems, where each chromosome encodes a set of  $R$  if-then rules [5]. The PSO was used to tune the parameters of the functions in the consequents. First, subtractive clustering algorithm was employed to design membership/non-membership functions. To control complexity (granularity) and avoid the potential overfitting risk, we tested various numbers of if-then rules (from  $R=2$  to  $R=9$ ) for each training dataset. Thus, interpretability at the rule base and fuzzy partition levels were preserved. Second, the ANFIS and INFN were tuned by a PSO with population=40, cognitive param.=2, social param.=0.7, inertia weight=0.8, max. particle velocity=0.4, and 100 iterations as the stopping condition.

The results in Table 3 demonstrate that the IVFCM-GA with the linguistic concepts outperformed that with the financial indicators. The results also suggest the generalizations of FCMs performed better than the standard FCMs, indicating that strong uncertainty exists in the causal relationships. To test the statistical differences in the performance of the compared methods in terms of the RMSE, we performed the nonparametric Friedman test. First, the null hypothesis was tested that all the methods perform similarly. The Friedman  $p$ -value (0.130) does not indicate the existence of significant differences between the evaluated methods. However, the lowest average ranking was achieved by the IVFCM-GA. Second, we used the IVFCM-GA as a control method in the post-hoc procedures (Friedman and Holm) to compare its performance with those of the other methods. Significant differences at  $p<0.05$  were detected by using the Holm post-hoc procedure, whereas the results of the Friedman post-hoc procedure ( $p=0.157$ ) indicate that the FGCM-GA did not perform significantly worse than the IVFCM-GA. Overall, the results of the post-hoc tests indicate that the IVFCM-GA significantly outperformed the FCM-GA ( $p=0.066$ ), ANFIS-PSO ( $p=0.034$ ) and INFN-PSO ( $p=0.016$ ).

#### 4. Conclusion

In this paper, we develop a novel IVFCM with real-coded genetic learning. We show that this method can be effectively used to corporate financial distress prediction. This can be attributed to its capacity of

Table 3: Results for the IVFCM trained with GA vs. comparative methods

Method	Measure	Mean±Std.Dev.	Method	Measure	Mean±Std.Dev.
IVFCM-GA (linguistic)	RMSE	0.1421±0.0112	FGCM-GA [4]	RMSE	0.1467±0.0113
	MAE	0.0854±0.0075		MAE	0.0896±0.0058
	MAPE	19.91±3.72		MAPE	20.57±3.91
IVFCM-GA (financial)	RMSE	0.1706±0.0288	ANFIS-PSO [2]	RMSE	0.1527±0.0134
	MAE	0.1022±0.0146		MAE	0.1018±0.0084
	MAPE	21.66±3.40		MAPE	22.73±4.28
FCM-GA [16]	RMSE	0.1509±0.0139	INFN-PSO [5]	RMSE	0.1524±0.0139
	MAE	0.0925±0.0103		MAE	0.1012±0.0098
	MAPE	20.87±3.97		MAPE	22.72±4.82

modelling the complex relationships among financial concepts. The fact that this method outperforms traditional FCMs suggests that this application domain is associated with a highly uncertain environment. We also demonstrate that the reasoning based on IVFSs can be for this task more effective than that based on grey numbers or adaptive fuzzy rule-based systems used in previous financial distress prediction studies.

The current findings add to a growing body of the research on the optimization of FCMs. However, as mentioned above, alternative evolutionary approaches can be used to learn FCMs. Therefore, this study can serve as a base for future research on IVFCM learning. Moreover, since the current study only examined the learning of an IVFCM weight matrix, further research should focus on the optimization of the slope parameter of sigmoid activation functions.

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