Actuarial model for pricing disability insurance policy

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Abstract: The main objective of our contribution is to apply stochastic processes for disability policy that gives annuity benefit in case of temporary or permanent disability. We apply durational effect to the disable state, by splitting it into several states. Using the data supplied by the Continuous Mortality Investigation (CMI) we calculate the single and annual premiums for that policy.

Key words: disability insurance, Markov process, semi-Markov process, splitting of states,

JEL Classifications: C51, C52, G22, J11,

1 Introduction

The main goal of this paper is to apply Markov process for disability income insurance benefits. The data we used in our contribution were supplied by the Continuous Mortality Investigation (CMI). The CMI is a research organisation established by UK actuarial profession.

Disability insurance, long-term care insurance and critical illness cover are becoming increasingly important in developed countries as it is mentioned in Pacáková, V., Jindrová, P. (2014). The private sector insurance industry is providing solutions to problems resulting from these pressures and other demands of better educated and more prosperous populations.

Most of the disability policies in UK are accelerated policies (88%) and they are attached to life insurance, term insurance or endowments. Typically, regular premiums are payable throughout the term while the policy is in force.

We describe the actuarial structure of disability insurance. Actuarial problems such as pricing and reserving are considered within the context of multiple state modelling, providing a strong and sound framework for analysing personal insurances.

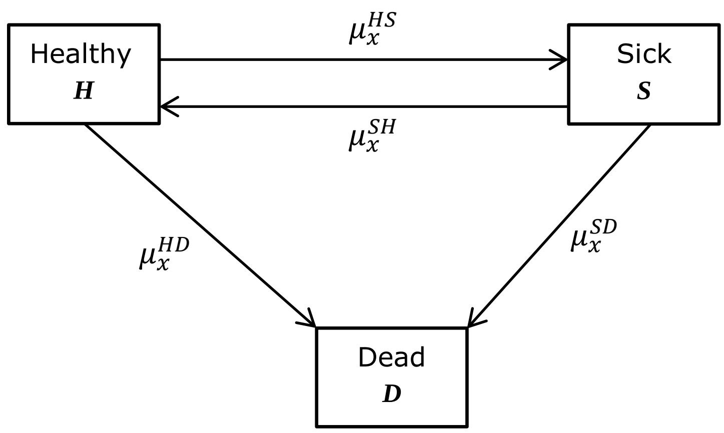
Our contribution is based on Markov process that can be used to develop a general, unified and rigorous approach for describing and analysing disability and related insurance benefits. The use of Markov process or Markov chain in life contingencies and their extensions has been proposed by several authors; for example Dickson, D. C., Hardy, M. R., & Waters, H. R. (2013), Haberman, S., & Pitacco, E. (1998).

2 Methodology and Data

Multiple state models are one of the most exciting developments in actuarial science nowadays. They are a natural tool for many important areas of practical interest to actuaries. They provide solid foundation for pricing and valuing complex insurance contracts. Many actuarial applications are modelled as time inhomogeneous Markov processes. Markov process assumes that probabilities of transitions at any time t depend only on the current state and not on the past. A Markov model for disability insurance has state space $\mathscr{S} = \{H, S, D\}$, where '*H*' means healthy, '*S*' sick (or ill) and '*D*' dead. Transitions rates at each age x are illustrated in the Figure 1. An individual is, at any time t, in one of three states, "Healthy", "Sick" or "Dead". We can use this simple three state model to define a random variable $Y(t)$ which takes one of the three values '*H*', '*S*' and

'*D*'. Suppose we have an individual aged x years at time t=0. The event $Y(t) = H$ means that an individual is healthy at age $x+t$, and $Y(t) = D$ means that an individual died before age $x+t$. The set of random variables $\{Y(t)\}_{t \geq 0}$ is an example of a continuous time stochastic process. We will assume that $\{Y(t)\}_{t \geq 0}$ is a Markov process. A policyholder is supposed to be healthy at the time of the commencement of the policy and he/she stays in this state until at some time he/she transits to one of the 2 possible states, that means a death or an illness occurred.

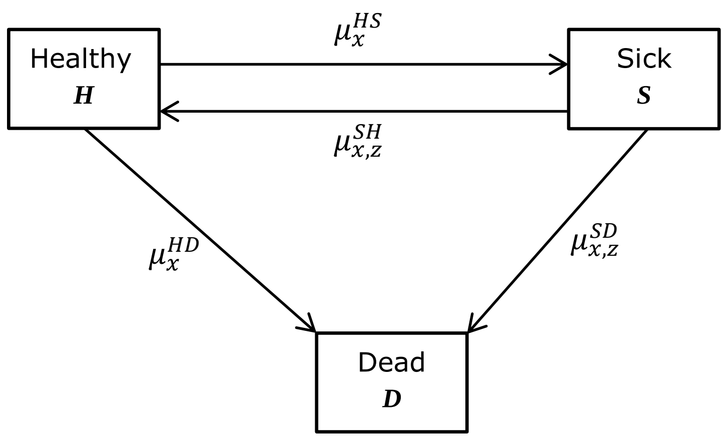
Figure 1 The disability income model – Markov model



Source: Own processing

It is perhaps more realistic to modify the previous model so as rates from "Sick" to "Healthy" and from "Sick" to "Dead" depend on the length of time z already spent in the "Sick" state, as well as on the age x of the individual. But then the Markovian property of the process is lost. This approach gives more complicated insurance policies which require more sophisticated model – semi-Markov model.

Figure 2 The disability income model – Semi-Markov model



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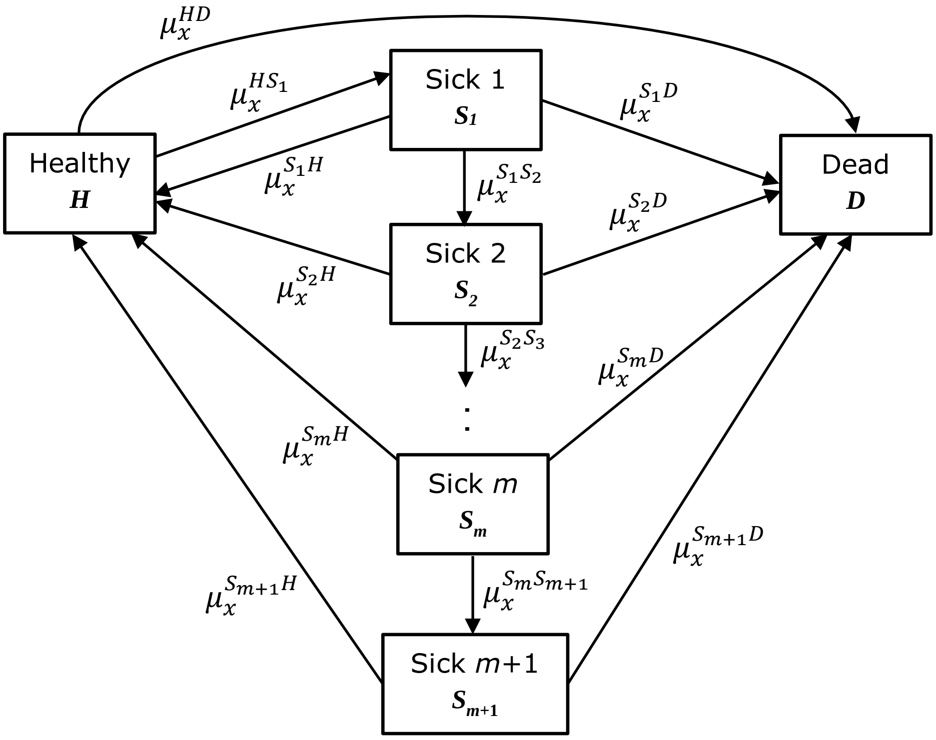
The disability income insurance pays a benefit during periods of sickness, the benefit ceases on recovery. Figure 2 shows a model suitable for policy which provides an annuity benefit while person is sick, with premiums payable while the person is healthy. The model represented by Figure 2 differs from that in Figure 1 in one important aspect: the dependence of transition rates (and probabilities) on the time spent in the state "Sick" since the latest transition to that state.

Figure 3 Splitting of "Sick" state



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We restrict the influence of the durational effect to some specified states. Let us replace state "Sick" by m+1 states S_1, S_2,…, S_m, S_{m+1}, where state

S_1 means that an insured is disabled with duration of disability between 0 and τ_1 units of time,

S_2 means that an insured is disabled with duration of disability between τ_1 and τ_2 units of time, …

S_m means that an insured is disabled with duration of disability between τ_{m-1} and τ_m units of time,

S_{m+1} means that an insured is disabled with duration of disability longer than τ_m units of time.

Thus, we have made a splitting of state "Sick" of the state space $\mathscr{S} = \{H, S, D\}$. New state space is as follows: $\mathscr{S}^* = \{H, S_1, S_2,…, S_m, S_{m+1}, D\}$. Hence, we formally revert to the Markov process $\{X(t); t \geq 0\}$ based on the state space $\mathscr{S}^*$.

Thus, the introduction of more states representing the durational effect is a notational tool for treating semi-Markov model within the simpler Markov framework.

The splitting allows us to consider select intensities (and probabilities) without formally introducing a semi-Markov model (leading to major difficulties).

In general case, with states H, S_1, S_2,…, S_m, S_{m+1}, D we refer to μ_x^{ij} as the force of transition or transition intensity between states i and j at age x. The transition intensities are fundamental quantities which determine everything we need to know about a multiple state model.

The model requires the following transition intensities:

$$\mu_x^{HD}, \mu_x^{HS_1}, \mu_x^{S_1H}, \mu_x^{S_1S_2}, \mu_x^{S_1D}, \mu_x^{S_2H}, \mu_x^{S_2S_3}, \mu_x^{S_2D}, \dots, \mu_x^{S_mH}, \mu_x^{S_mS_{m+1}}, \mu_x^{S_mD}, \mu_x^{S_{m+1}H}, \mu_x^{S_{m+1}D}.$$

As far as recovery is concerned, morbidity experience suggests:

$$\mu_x^{S_1H} > \mu_x^{S_2H} > \cdots > \mu_x^{S_mH} > \mu_x^{S_{m+1}H}.$$

In particular it is possible put $\mu_x^{S_{m+1}H} = 0$, in the case of no recovery is possible after τ_m units of time.

Let us assume that a time-continuous Markov model has been assigned. Thus, the transition intensities have been specified and the transition probabilities have been derived. It is self-evident that the implied time discrete probabilistic structure can be immediately derived. To do this, we simply have to restrict our attention to transition probabilities ${}_tp_x^{ij}$ only, where x, t now denote integer values, so we get time-discrete Markov model.

Our splitting states model requires the following transition probabilities

$$p_x^{HD}, p_x^{HS_1}, p_x^{S_1H}, p_x^{S_1S_2}, p_x^{S_1D}, p_x^{S_2H}, p_x^{S_2S_3}, p_x^{S_2D}, \dots, p_x^{S_mH}, p_x^{S_mS_{m+1}}, p_x^{S_mD}, p_x^{S_{m+1}H}, p_x^{S_{m+1}D}.$$

3 Results

Consider the model (Figure 3) for a disability income insurance. The disability state is split into six states (due to available data). For states S_1, S_2,…, S_5 recovery is possible, whilst S_6 is assumed to represent permanent disability. These assumptions lead to the one-year transition matrix (or the transition matrix of one-year probabilities) of Table 1.

Table 1 One-year transition probabilities M_x

	H	***S*₁**	***S*₂**	***S*₃**	***S*₄**	***S*₅**	***S*₆**	***D***
H	p_x^{HH}	$p_x^{HS_1}$	0	0	0	0	0	p_x^{HD}
S_1	$p_x^{S_1H}$	0	$p_x^{S_1S_2}$	0	0	0	0	$p_x^{S_1D}$
S_2	$p_x^{S_2H}$	0	0	$p_x^{S_2S_3}$	0	0	0	$p_x^{S_2D}$
S_3	$p_x^{S_3H}$	0	0	0	$p_x^{S_3S_4}$	0	0	$p_x^{S_3D}$
S_4	$p_x^{S_4H}$	0	0	0	0	$p_x^{S_4S_5}$	0	$p_x^{S_4D}$
S_5	$p_x^{S_5H}$	0	0	0	0	0	$p_x^{S_5S_6}$	$p_x^{S_5D}$
S_6	0	0	0	0	0	0	$p_x^{S_6S_6}$	$p_x^{S_6D}$
D	0	0	0	0	0	0	0	1

Source: Own processing

For calculation we use transition intensities from the CMI Working paper 12. From these data we apply our model for the probabilities of disablement, i.e. $p_x^{HS_1}$,

$$p_x^{HS_1} = e^{-6,11+0,0458\cdot x}, \tag{1}$$

and for the probabilities of recovery as a function of attained age x. For each state S_1, S_2,…, S_6, the following functions are used

$$p_x^{S_jH} = \alpha_j - \beta_j \cdot x, \tag{2}$$

for j =1, 2,…, 6.

The parameters α_j, β_j are given in Table 2.

Table 2 Parameters for probabilities of recovery

j	α_j	β_j
1	1,4161	0,0222
2	0,7400	0,0115
3	0,3739	0,0058
4	0,2855	0,0046
5	0,1918	0,0031

Source: CMI Report 12

We assume that the health mortality equals to general population mortality

$$p_x^{HD} = q_x = 1 - \exp\{-\int_0^1 \mu_{x+t}\, \mathrm{d}t\} \ , \qquad (3)$$

where μ_{x+t} is given by Gompertz-Makeham law of mortality $\mu_{x+t} = \mu_{x+t}^{HD} = a + b \cdot c^{x+t}$, with $a = 0.0005$, $b = 7{,}5858 \cdot 10^{-5}$ and $c = 1.09144$.

Probabilities of death for disabled insured, $p_x^{S_jD}$(for j =1, 2,…, 6), are assumed to be equal to $(1+\eta) \cdot p_x^{HD}$, where η is the extra level of mortality of disabled. The technical rate of interest i = 0.015 p.a. (or 1.5 % p.a.) has been assumed.

Let $\ddot{a}_{x:n}$ denote the actuarial value of an annuity

$$\ddot{a}_{x:n} = \sum_{k=0}^{n-1} {}_k p_x^{HH} \cdot v^k \ , \qquad (4)$$

where v is the discount factor ($v = \frac{1}{1+i}$).

The disability benefit (one monetary unit) is paid whatever disability state S_1, S_2,…, S_6 is occupied by insured.
Hence, the actuarial value of the disability benefit is defined, for a healthy individual, as follows

$$a_{x:n}^{HS} = \sum_{k=1}^{n} \sum_{j=1}^{6} {}_k p_x^{HS_j} \cdot v^k \ . \qquad (5)$$

The disability annuity is assumed to be payable up to the end of the policy term n.

The actuarial value of the disability benefit is defined, for a disabled individual aged $x+t$ who occupies the state S_j (j = 1, 2, …, 6) as follows

$$a_{x+t:n-t+1}^{S_jS} = \sum_{k=0}^{n-t} \sum_{h=1}^{6} {}_k p_{x+t}^{S_jS_h} \cdot v^k \ ; \ \ (t = 1, 2, \ldots, n), \qquad (6)$$

where ${}_0 p_{x+t}^{S_jS_h} = 1$ if $h = j$ and 0 otherwise.

The annual premium, $P_{x:n}$, payable for n years while the insured is healthy, is given by:

$$P_{x:n} = \frac{a_{x:n}^{HS}}{\ddot{a}_{x:n}}. \qquad (7)$$

The calculation of the actuarial values defined by equations (4), (5) and (6) and then the calculation of premium according to formulae (7) imply the use of the underlying probabilities, which can be derived from one-year transition matrix M_x (Table 1). The product of one-year transition matrices at successive ages is a two-year transition matrix $M_x \cdot M_{x+1} = {}_2M_x$. Continuing the matrix multiplication through subsequent ages, we can determine the k-year transition matrix ${}_kM_x$ for any integer k.

Table 3 presents numerical results for policy terms n = 10, 15, 20 years and η =0.20 and illustrate single and annual premiums calculation in a time-discrete Markov context.

Table 3 Single and annual premium, $\eta = 0.20$.

Term\Age	x =30	x=40	x=50
n = 10	0.18053	0.39654	0.81965
n = 10	0.01977	0.04469	0.09831
n = 15	0.36988	0.84171	1.72679
n = 15	0.02840	0.06785	0.15502
n = 20	0.66287	1.49959	2.88050
n = 20	0.04030	0.09819	0.22109

Source: Own processing

4 Conclusions

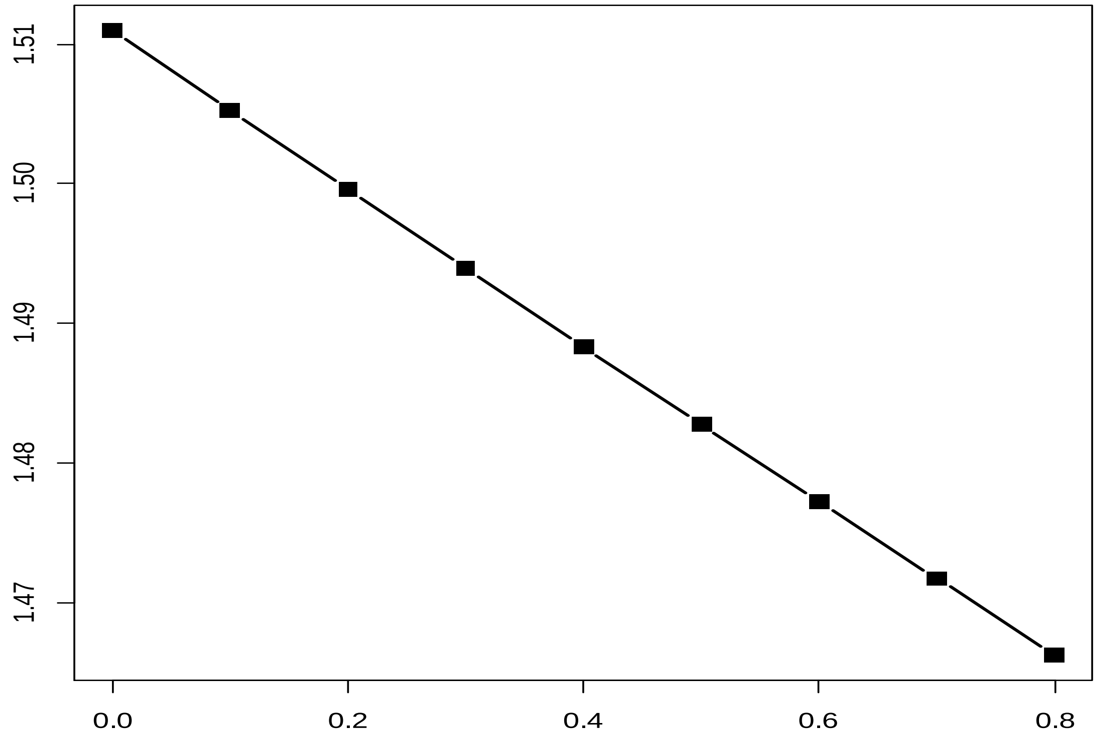
We have presented an application of multiple state models to problems in actuarial science. There are various extensions of multiple state models. One way is to allow the transition intensities out of a state to depend not only on individual's current age but also on how long they have been in current state. This breaks the Markov property assumption and leads to the new process known as a semi-Markov process. This could be appropriate for the disability income insurance process where the intensities of recovery and death from the sick state could be assumed to depend on how long the individual had been sick, as well as on current age.

We have to emphasize that no attempt has been made to discover a precise relationship between p_x^{HD} and $p_x^{S_jD}$ (for j = 1, 2, …, 6), i.e. probabilities of death for healthy and disabled insured. It does however suggest that an office which calculate disabled lives' reserves for claims in force, by assuming mortality according to a standard life table, may have an implicit margin of strength in its reserving basis. The Figure 4 illustrates the behaviour of single premium as a function of the extra level of mortality η.

The transition intensities are fundamental quantities which determine everything we need to know about a multiple state models. Therefore it would be useful to have data from domestic insurance industry. Our further research will focus on estimation of transition intensities for the Czech Republic (or other central European countries) in similar manner as in Pacáková, V., Jindrová, P., Seinerová, K. (2013).

There is a need for awareness of model risk when assessing a disability income insurance benefits and/or critical illness benefits, especially with long term. The fact that transition intensities can be estimated does not imply that they can sensibly describe future medical development.

Figure 4 Single premium as a function of the extra level of mortality η; x=40, n=20.



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