Modelling and Identification of Magnetic Levitation Model CE 152 / Revised

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Abstract. Paper describes procedure of first principle modelling and experimental identification of Magnetic Levitation Model CE 152. Author optimized and simplified dynamical model to a minimum what is needed to characterize given system for the simulation and control design purposes. Only few experiments are needed to estimate the unknown parameters. Model quality is verified in the feedback control loop where the real and simulated data are compared.

Keywords: magnetic levitation, maglev, first principle model, experimental identification, verification, feedback control

1 Introduction

Magnetic levitation, maglev or magnetic suspension is a method when the object is suspended with no other support than the magnetic fields. Magnetic force counteract effect of the gravitational force or other forces. Maglev is used e. g. in the transportation for trains, magnetic bearings, vibration isolation or contactless melting. All applications are inherently open-loop unstable and rely on the feedback control for producing the desired levitation action.

In case of Model CE 152 the steel ball is levitated in the air by the electromagnetic force generated by an electromagnet [1]. The single-input single-output (SISO), strongly nonlinear, unstable system is a nice object to study the system dynamics and experiment with different control algorithms based on classical or modern control theory. PID controllers, polynomial, robust or model predictive controllers including nonlinear case are applied in the literature [2-10]. Dynamical mathematical model is required for most of the controller design methods. Usually linear model is needed but for more realistic control simulations or control design methods the nonlinear model can be used as well. Modelling and identification problematics of the magnetic levitation process can be found e.g. in [11-20]. Usually first principle model is derived and the unknown parameters are estimated from experimental data. Black box identification can be used as well – parameters of external or internal mathematical representation are estimated from measured process responses. Author prefers first principle approach to get model with physical meanings and to identify parameters of the subsystems by separate experiments. Model complexity is reduced to a minimal structure with few estimated parameters only. The nonlinear model can be used directly by the simulation or can be analytically linearized in given working point for the controller design method. Paper is structured as follows. Process is described in chapter 2, model is derived in chapter 3, unknown parameters are estimated in chapter 4, model is verified in chapter 5 and conclusions are given in chapter 6.

2 CE 152 Magnetic Levitation Model

Magnetic Levitation Process consists of a the base with coil, electronics and metal ball (see Fig. 1) and PC with Data Acquisition (DAQ) Card. Ball levitates in the magnetic field. The magnetic field of the coil is driven by a power amplifier connected to D/A output of DAQ card. Position of the ball is sensed by an inductive linear position sensor connected to A/D input of DAQ card.

Fig. 1. Magnetic Levitation Model CE 152

3 First Principle Mathematical Model

Process is decomposed to individual subsystems which are modelled and identified separately. One subsystem is the power amplifier connected do D/A DAQ output. Coil and ball is another subsystem – this is the only subsystem with dynamics. The last subsystem is a position sensor connected to A/D DAQ input.

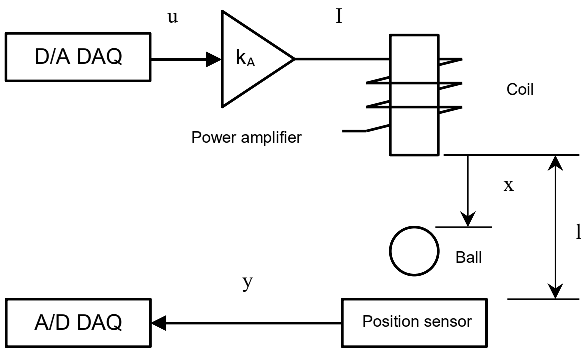

Fig. 2. Magnetic Levitation Model block scheme

3.1 Power Amplifier

Power amplifier is stabilized source of a current I which is proportional to the input voltage u generated by DAQ card

$$I = k_A u \tag{1}$$

The voltage u is in the range from 0 to 5 V and the current is in the range from 0 to approx. 1.5 A. Precisely the gain of the amplifier k_A is 0.297. This can be derived and calculated from parameters of the used electric components [14]. Time constant of the amplifier is very small and can be neglected. The amplifier gain has not to be estimated very precisely because the coil constant k can compensate the error.

3.2 Coil and Ball

We are using Lagrange's method for modelling of coil and ball subsystem. Motion equation is derived from the equilibrium of acting forces – gravitational force F_g and electromagnetic force F_m. Air resistance is neglected – the speed of the ball is not so high that this force would play a role. Accelerating force F_a is

$$F_a = F_g - F_m \tag{2}$$

$$m\frac{d^2x}{dt^2} = mg - k\left(\frac{I}{x+x_0}\right)^2 \tag{3}$$

where m is ball mass (kg),
g is acceleration of gravity ($m.s^{-2}$),
k is coil constant (-),
I is coil current (A),
x is ball position (m) and
x_0 is coil offset (A).

Two unknown parameters k and x_0 must be estimated experimentally. Remaining parameters are listed in Table 1.

Table 1. Coil and Ball parameters

Symbol	*Units*	*Value*	*Meaning*
m	kg	$8.28 \cdot 10^{-3}$	ball mass
g	$m.s^{-2}$	9.81	acceleration of gravity
d	m	$12.7 \cdot 10^{-3}$	ball diameter
l	m	$18.4 \cdot 10^{-3}$	distance between sensor and coil core

3.3 Position Sensor

Position sensor has a linear characteristic with two unknown parameters a and b

$$y = ax + b \tag{4}$$

Sensor senses ball position and outputs voltage y approximately in the range from 0 to 5 V.

4 Estimation of unknown parameters

4.1 Position Sensor

Position sensor is linear - two points for calibration are enough. Practically the simplest method is to hold the ball down at position sensor and measure the voltage and then place the ball to the coil core and measure the voltage again. We must take care only that the ball is placed in the centre of the coil core. The origin of the position axis x is placed at the end of the coil core and points down to the sensor.

Table 2. Data for position sensor parameters identification

x (m)	y (V)
0	4.920
l-d	0.047

Estimated parameters of equation (4) by using the data in Table 2 are a = -855 V/m and b = 4.92 V.

4.2 Coil and Ball

Because the system is unstable, closed loop control experiment must be carried out to estimate unknown coil constant k and offset x_0. Ball position y is controlled to a constant set-point w and used control action u is read out. We have measured four points (two would be enough) – see Table 3.

Table 3. Data for Coil and Ball parameters identification

y (V)	u (V)
1	2.87
2	2.34
3	1.89
4	1.44

In every steady-state point the electromagnetic force must equal to gravitational force which is constant

$$k\left(\frac{I}{x+x_0}\right)^2 = mg \tag{5}$$

Numerical optimization method can be used to estimate the unknown parameters. The problem can be transformed to a linear problem and Least Square Method can be applied also. Current I is calculated from the voltage input u according to equation (1). Position x is calculated from the output voltage y according to equation (4). Set of the position estimations (6) in matrix form is

$$\underbrace{\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\\ \vdots\\ \hat{x}_4\end{bmatrix}}_{\hat{\boldsymbol{X}}} = \underbrace{\begin{bmatrix} I_1/\sqrt{mg} & -1\\ I_2/\sqrt{mg} & -1\\ \vdots & \vdots\\ I_n/\sqrt{mg} & -1\end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix}\sqrt{k}\\ x_0\end{bmatrix}}_{\boldsymbol{P}} \tag{6}$$

Cost function is sum of the squares of the position estimation errors which is

$$J = \sum_{i=1}^{n}(x_i - \hat{x}_i)^2 = \left(\boldsymbol{X} - \hat{\boldsymbol{X}}\right)^T\left(\boldsymbol{X} - \hat{\boldsymbol{X}}\right) \tag{7}$$

Optimal estimation of the vector $\boldsymbol{P}$ can be calculated as

$$\boldsymbol{X} = [x_1 \quad x_2 \quad \cdots \quad x_4]^T, \quad \boldsymbol{P} = (\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T\boldsymbol{X} \tag{8}$$

Estimated parameters are $k = \boldsymbol{P}(1)^2 = 5.59\cdot10^{-6}$ and $x_0 = \boldsymbol{P}(2) = 2.4\cdot10^{-3}$ m.

Positions estimation absolute and relative errors are in Table 4

Table 4. Positions estimation errors

x (m)	$\hat{x}$ (m)	$x - \hat{x}$ (m)	$(x - \hat{x})/l \cdot 100$ (%)
$4.59\cdot10^{-3}$	$4.64\cdot10^{-3}$	$-0.06\cdot10^{-3}$	-0.31
$3.42\cdot10^{-3}$	$3.34\cdot10^{-3}$	$0.08\cdot10^{-3}$	0.43
$2.25\cdot10^{-3}$	$2.23\cdot10^{-3}$	$0.02\cdot10^{-3}$	0.10
$1.08\cdot10^{-3}$	$1.12\cdot10^{-3}$	$-0.04\cdot10^{-3}$	-0.23

Relative position estimation error is less than 0.5 %. The model as Simulink block scheme is in Fig. 3.

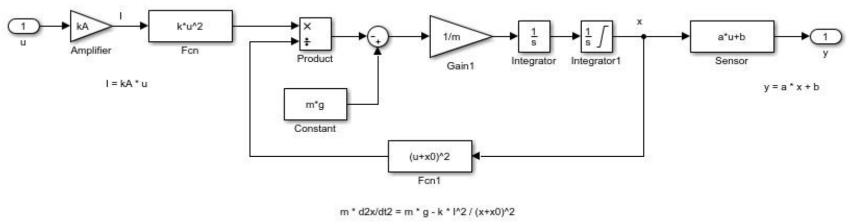

Fig. 3. Magnetic Levitation Simulink block scheme

5 Model verification

Model was verified in closed loop. Digital PID controller controls the process to a sequence of a step changes on the output voltage set-point. Identical controller is used to control the real system and the mathematical model. The set-point *w*, controlled variables *y* and manipulated variables *u* are plotted in Fig. 4. Real data are denoted as ‘r’ and simulated data with the mathematical model as ‘m’.

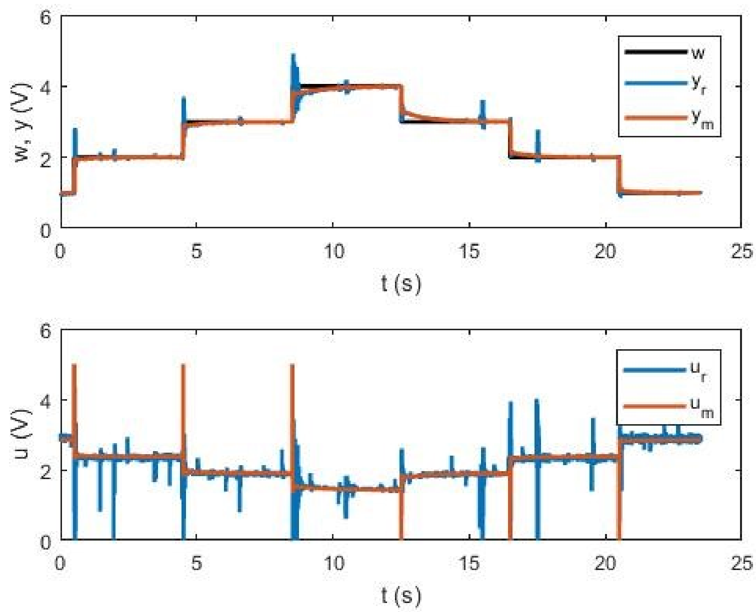

Fig. 4. Real and simulated control responses

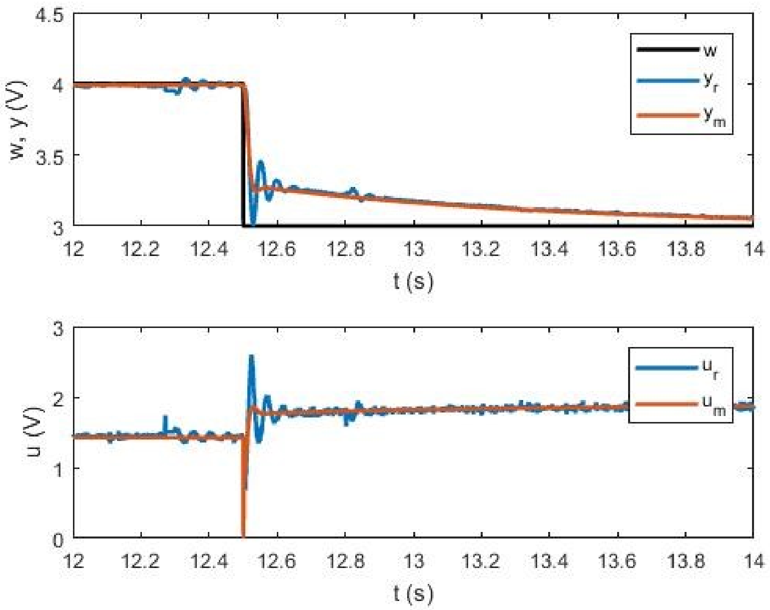

Fig. 5. Real and simulated control responses (from 4 to 3 V)

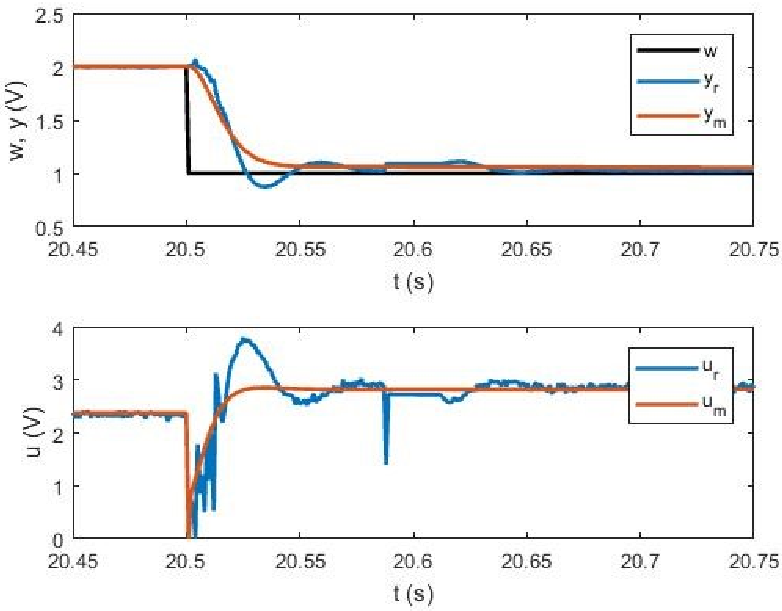

Fig. 6. Real and simulated control responses (from 2 to 1 V)

From the control responses it can be seen that not only static behaviour of the model corresponds to the real system but also the transient responses are very similar (see zoomed data in Fig. 5 and 6). The dynamics for bigger ball distances is slower as a fact of the nonlinearity of the process.

6 Conclusion

Magnetic levitation is an example of unstable real system and hence without the feedback control impossible to operate. Good mathematical model is a key point for the design and testing of different control strategies. Modern control methods are often model-based so the model is a part of the controller or at least used by the control design procedure.

Revised approach to the modelling and identification of magnetic levitation process is presented in the paper. For a specific system some phenomena like amplifier dynamics, air resistance or bouncing are neglected with practically no influence to the model quality. Only four unknown parameters are estimated from the experimental data – two parameters of the position sensor a and b and two parameters of coil and ball subsystem – coil constant k and offset x_0. Whole procedure is designed in such a way that only the position sensor bust be calibrated manually but the rest can be automated.

Offset parameter x_0 for the coil was identified as 2.4 mm. This means that the force generated by the coil slightly differs from the ideal case. It can't be higher than the force corresponding to the ball placed 2.4 mm far from the coil core in theoretical case. The real system tends to oscillate unfortunately. This is caused by the horizontal movement of the ball. At the same time the position sensor variable oscillates and the control action reacts to this – even if the vertical position of the ball changes only slightly. Guide bars would help but the visual effect of the levitation would not be so interesting. The solution is to operate the system carefully and time to time manually stabilize the ball if oscillations occur.

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