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The Fourier Analysis in Transport Application Using Matlab

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Abstract

Fourier series arose during the eighteenth century as a formal solution to the classic wave equation. Later on, it was used to describe physical processes in which occurrences recur in a regular pattern. Fourier's theorem provides the mathematical language which enables us to exactly describe the periodical processes. The aim of this paper is to show the application of the Fourier series in transport problem - the emission of the noise in the tire/pavement contact. Based on the tire imprint, the profile function was created, which was applied to the calculation of the coefficients of the Fourier series. The amplitude and frequency spectrum of the noise were assembled and calculated its performance using this coefficients. This issue has been used in the teaching of the applied mathematics and numerical methods in transport at the Jan Perner Transport Faculty, University of Pardubice.

KEY WORDS: *Applied Mathematics in Transport, Fourier Series, Frequency Spectrum, Matlab, Noise Emission*

1. Introduction

Every driver knows that tires produce a relatively loud noise on the pavement. Tire sounds are produced primarily by the dynamic interaction of the tread elements with the road surface. As each tread element passes through the contact patch, it contributes a pulse of acoustic energy to the total sound field radiated by the tire [3]. We mainly proceed in this paper from the work of professors Duffy [3], also Sandberg [17], Arteaga [1], Berkee, Wijnant and De Boer [2] and Iwao and Yamazaki [7], further Hub [4, 5, 6] in informatics issues, Soušek [16, 18] in transport issues, Záhorová [22] in statistics issues and Trna [20, 21] in pedagogical issues.

In this application, we will deal with assembly of the amplitude and frequency spectrum of the noise which is produced in tire/pavement contact and calculation of its performance using the particular sample measured at Jan Perner Transport Faculty, University of Pardubice, in Educational and Research Centre in Transport laboratories.

2. Materials and Methods

We determine the amplitude and frequency spectrum and the noise emissions performance in the following steps. The manufacturing of the imprint tread, the measurement of the critical dimensions of the tire imprint to determine the profile of the sample, the determination of the mathematical function describing the profile of the sample, the general calculation of the relevant coefficients of the Fourier series, the programming of the script in Matlab for the calculation of the specific coefficients of the Fourier series and the calculation of the performance of the noise emissions.

3. Manufacturing the Imprint Tread

As the experimental tire, the ContiEcoContact EP tire produced by Continental was elected, sized 165/70R14 and speed category 81T. Because the tire on the vehicle is rolling in the loaded state, the imprint was implemented in this form. The tire was briefly used, therefore, the tread depth is less than that of a new tire. Tire pressure was 2.5 Bar. The load was carried on the static adhesor, see Figure 1, which was set load of 262 kg, corresponding to the real wheel load. Prior to the implementation of imprint of the tire, it was necessary to treat the surface with the anti-adhesion coating in the form of a silicone oil.

The imprint of the tire was made using gypsum and subsequently it was measured. We can see the imprint of the tire in the Figure 2.

Fig. 1: Tyre with the Anti-Adhesion Coating

Fig. 2: Imprint of the Tire

4. The Measurement of the Critical Dimensions of the Tire Imprint

The critical dimensions were measured and the results were written down to Figure 3. The simplified profile can also be seen in Figure 3.

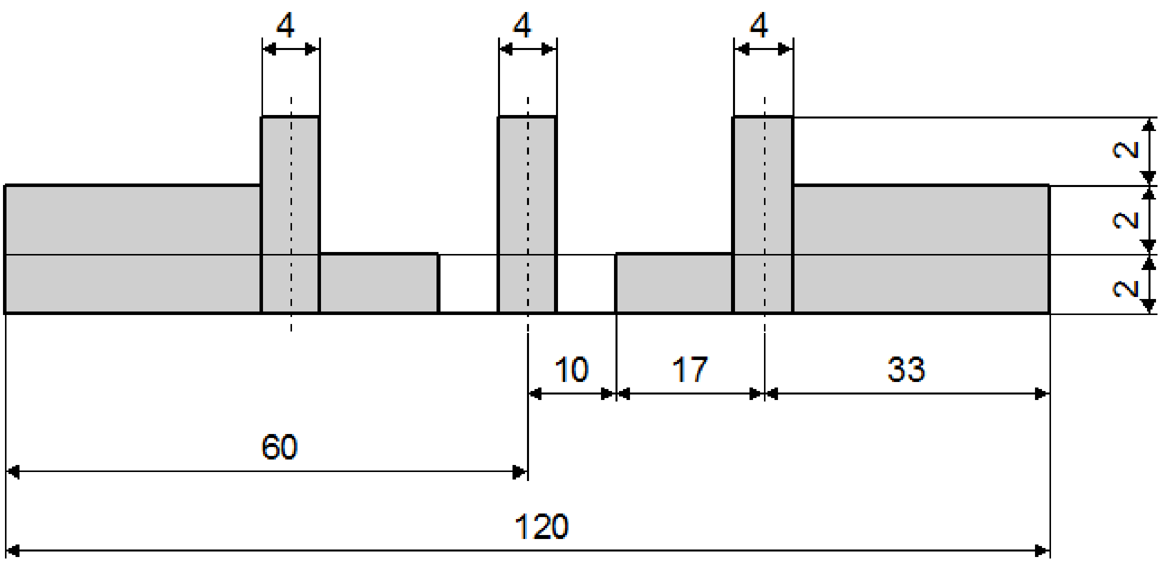

Fig. 3: The Approximate Profile with Dimensions in mm

5. The determination of the Mathematical Function Describing the Profile of the Sample

As can be seen in Figure 3, the profile function is symmetrical about the vertical axis in the middle of the imprint that implies it is even. The width of the tire contact with the road surface is 120 mm. Therefore, we just define the profile function over the interval x∈⟨0; 60⟩.

The mathematical function describes the profile of the tire given by equation (1):

$$f(x) = \begin{cases} 6; x \in \langle\ 0;\ \ 2) \\ 0; x \in \langle\ 2; 10) \\ 2; x \in \langle 10; 25). \\ 6; x \in \langle 25; 29) \\ 4; x \in \langle 29; 60\rangle \end{cases} \tag{1}$$

The period of this function is thus 120 mm and the angular frequency is:

$$T = 120;\ \omega = \frac{2\pi}{T} = \frac{\pi}{60}. \tag{2}$$

6. The General Calculation of the Relevant Coefficients of the Fourier series

We note [3], the function $f(x)$ defined over the range $-\frac{T}{2} < x < \frac{T}{2}$ has the Fourier series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega x) + b_n \sin(n\omega x). \tag{3}$$

For evenly spaced treads we envision that the release of acoustic energy resembles the profile in Figure 3. If we perform a Fourier analysis of this distribution, we find that coefficients $b_n = 0$ because $f(x)$ is an even function and

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(x)\, cos(n\omega x) dx. \tag{4}$$

Thus

$$a_0 = \frac{4}{120}\int_0^{60} f(x)\, dx = \frac{1}{30}\left\{\int_0^{2} 6\, dx + \int_{10}^{25} 2\, dx + \int_{25}^{29} 6\, dx + \int_{29}^{60} 4\, dx\right\} = \frac{19}{3},$$

and

$$a_n = \frac{4}{120}\int_0^{60} f(x) \cos\left(n\frac{\pi}{60}x\right) dx,$$

further

$$a_n = \frac{1}{30}\left\{\int_0^{2} 6\cos\left(n\frac{\pi}{60}x\right) dx + \int_{10}^{25} 2\cos\left(n\frac{\pi}{60}x\right) dx + \int_{25}^{29} 6\cos\left(n\frac{\pi}{60}x\right) dx + + \int_{29}^{60} 4\cos\left(n\frac{\pi}{60}x\right) dx\right\}.$$

After calculating the definite integrals and using the well-known trigonometric identities $\sin x + \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$ for simplification, we get:

$$a_n = \frac{12}{\pi n}\sin\frac{\pi n}{30} + \frac{8}{\pi n}\cos\frac{7\pi n}{24}\sin\frac{\pi n}{8} + \frac{24}{\pi n}\cos\frac{9\pi n}{20}\sin\frac{\pi n}{30} + \frac{16}{\pi n}\cos\frac{89\pi n}{120}\sin\frac{31\pi n}{120}.$$

After using the substitutions

$$a = \frac{12}{\pi n}; b = \frac{8}{\pi n}; c = \frac{24}{\pi n}; d = \frac{16}{\pi n};$$

and

$$arg1 = \frac{\pi n}{30}; arg2 = \frac{7\pi n}{24}; arg3 = \frac{\pi n}{8}; arg4 = \frac{9\pi n}{20}; arg5 = \frac{89\pi n}{120}; arg6 = \frac{31\pi n}{120},$$

for the easier writing and comprehension of the Matlab script, we get:

$$a_n = a \sin arg1 + b \cos arg2 \sin arg3 + c \cos arg4 \sin arg1 + d \cos arg5 \sin arg6. \tag{5}$$

7. The Script in Matlab for Calculating the Specific Coefficients of Fourier Series and the Performance of the Noise Emissions Calculating

The calculation of coefficients of Fourier series was realized by the Matlab script. We can see it below (the comments follow after %):

```
clc; close; clear;
% the computing of the first 60 coefficients

n=1:60;

% the definition of the constants

a=(12/pi)./n; b=(8/pi)./n; c=(24/pi)./n; d=(16/pi)./n;
arg1=(pi/30).*n; arg2=(7*pi/24).*n; arg3=(pi/8).*n; arg4=(9*pi/20).*n;
arg5=(89*pi/120).*n; arg6=(31*pi/120).*n;

% the computing of the Fourier coefficients an – see (5)

an=a.*sin(arg1)+b.*cos(arg2).*sin(arg3)+c.*cos(arg4).*sin(arg1)+d.*cos(arg5).*sin(arg6);

% the computing of the magnitude

cn=0.5*sqrt(an.*an);

% the adding in the a0 term

cn=[19/6,cn];
n=[0,n];

% the clearing of any figures

clf
axes('FontSize',16)

% the plotting of the spectrum

stem(n,cn,'filled')

% the setting of aspect ratio

set (gca,'PlotBoxAspectRatio',[8 4 1])
xlabel('n');
ylabel('cn');

% the computing of the performance

cn_kvadrat=cn.*cn;
S=sum(cn_kvadrat);
m=size(n);
E=S/m(2)
```

8. Results and Discussion

The best illustrating of our Fourier coefficients is the amplitude or frequency spectrum $\frac{1}{2}\sqrt{a_n^2 + b_n^2}$ as a function of n. In our case it is $\frac{1}{2}\sqrt{a_n^2}$ because all coefficients b_n are equal to zero. The amplitude spectrum in Figure 4 shows that the spectrum for periodically placed tire treads has its largest amplitude at small n. This produced one loud tone plus weaker harmonic overtones because the fundamental and its´ overtones are the dominant terms in the Fourier series representation. The performance of the noise is 0.1868 W.

The tire produced the noise at more frequencies and the total noise would be comparable to that generated by other sources within the car.

9. Conclusion

On the basis of the tire tread imprint, which was created under simulating real operating conditions in Educational and Research Centre in Transport, Jan Perner Transport Faculty, University of Pardubice. Simple practical application of Fourier series in the transport practice was shown. The experiment was carried out in teaching of Applied Mathematics and Numerical Methods in transport at Jan Perner Transport Faculty. It turned out that this model provides a simple method for the experimentation and the possibly minimizing or optimizing the noise emission in the tire/pavement contact.

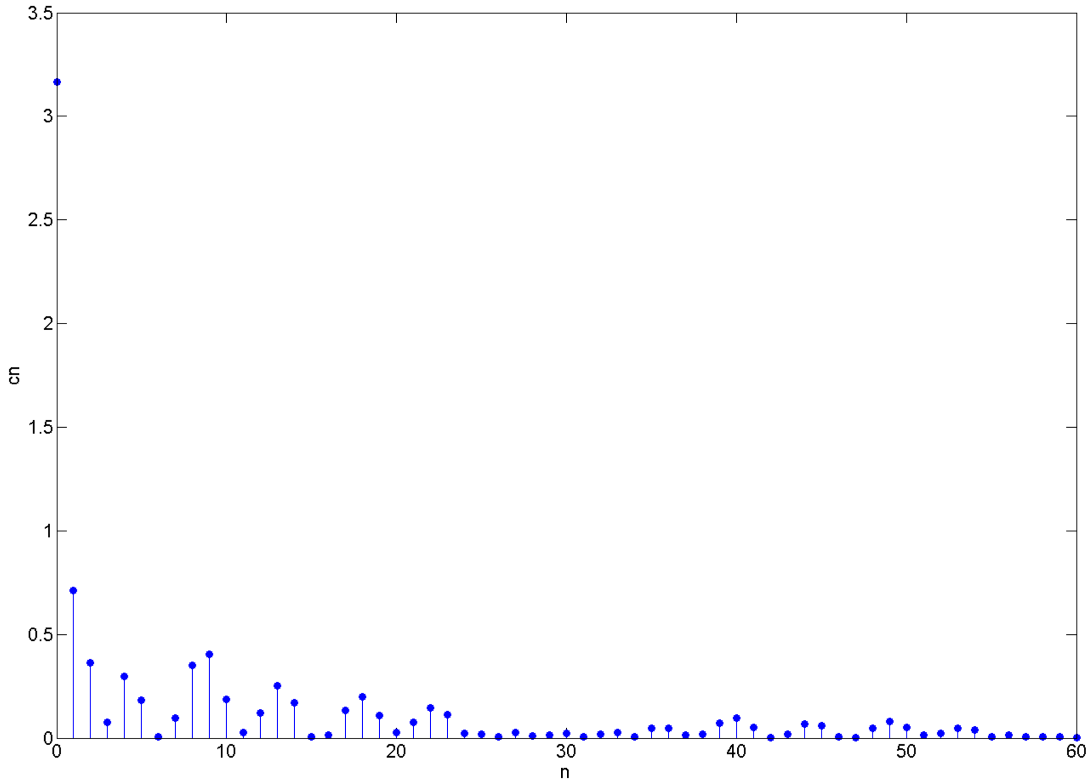

Fig. 4: The Frequency Spectrum of Measured Tire Treads

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