Scientific Papers of the University of Pardubice,
Series A; Faculty of Chemical Technology
24 (2018) 197–210.

University
of Pardubice
Faculty
of Chemical Technology

Cake filtration in viscoelastic liquids

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Received: April 19, 2018; Accepted: May 29, 2018

The paper deals with the modeling of the constant-pressure cake filtration of suspensions in viscoelastic liquids. The filtration equations are presented based on the capillary hybrid model for power law fluid flow and comprising correction functions of the influence of dispersing liquid elasticity. The validity of these equations has been verified experimentally. The tests with suspensions of nearly spherical polystyrene particles Krasten in viscoelastic aqueous solutions of polyacrylamides Kerafloc and Praestol were performed at constant pressure in a laboratory cylindrical filtration unit. The rheological properties of liquids have been measured on a rotational rheometer Mars II and an extensional rheometer Caber 1. By analysing the experimental data, the filtration characteristics have been evaluated. The elastic effects manifest themselves predominantly at the early period of the filtration due to a higher filtration velocity which results in an evidently increased filtration resistance. The satisfactory agreement of the experimental dependences of the cumulative filtration volume on the time of filtration has been achieved when compared to those obtained by the numerical solution of the proposed differential equation of filtration. This confirms that the cake filtration from viscoelastic liquids can be predicted with a satisfactory accuracy, thus making the suggested filtration model applicable in practice.

Keywords: Cake filtration; Constant pressure; Viscoelastic fluid; Filtration equation

Introduction

Filtration of non-Newtonian suspensions is encountered in various industrial areas such as, for example, biotechnology, polymer and food industry, or petroleum product processing.

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Most of rheologically complex fluids are shear thinning and they also frequently exhibit the viscoelastic behaviour. In porous media flows, viscoelasticity manifests itself in the form of a pressure drop excess above that accountable by shear viscosity of the flowing liquid (see e.g. [1]). Therefore, the liquid elasticity can play an important role in the cake filtration performance with respect to the increased filtration resistance.

The cake filtration in viscoelastic liquids has been, for example, studied by Kozicki and Kuang [2], Kozicki and Slegr [3], Hwang et al. [4], Surý and Machač [5], and Surý [6]. At the same time, the pressure drop excess occurs only when the flow rate exceeds a limit value and increases with the increasing fluid velocity. Thus, Hwang et al. [4] expect that the elastic effect is more pronounced at the beginning of filtration and due to the decrease of filtration rate, the flow becomes progressively pure viscous. However, the results of our experimental investigation on the constant pressure cake filtration of Krasten polystyrene particles from viscoelastic aqueous solutions of polymers [5,6] show that the contribution of elastic effect to the liquid pressure drop is also not negligible in the advanced period of filtration.

In this contribution, a modified form of the filtration equation for a cake filtration in viscoelastic liquids is presented. The validity of the proposed filtration equation and influence of elasticity on the filtration process are documented by means of representative filtration experiments performed by Surý [6].

Fundamentals

In development of a cake filtration equation in viscoelastic liquids, the equations proposed for the filtration in purely viscous power-law liquids are modified by inclusion of terms expressing the resistance associated with the liquid elasticity [2,4,5].

Based on the analysis by Kozicki and Kuang [2], we have shown [5] that the total pressure drop Δp_{ve} for the flow of viscoelastic filtrate through the filter cake and filter screen can be given by the sum

$$\Delta p_{\mathrm{ve}} = \Delta p_{\mathrm{ve,c}} + \Delta p_{\mathrm{ve,fs}} = \Delta p_{\mathrm{pv,c}} \left(1 + \frac{E_{\mathrm{c}}}{W \gamma_{\mathrm{ef}} x' K}\right) + \Delta p_{\mathrm{pv,fs}} \left(1 + \frac{E_{\mathrm{fs}}}{R_{\mathrm{fs}} K}\right) \tag{1}$$

where the pressure drop in the flow of purely viscous power-law filtrate through the filter cake is defined as

$$\Delta p_{\mathrm{pv,c}} = u_1^{\mathrm{n}} K m_{\mathrm{c}} \gamma_{\mathrm{ef}} \tag{2}$$

whereas the pressure drop in the flow of power law filtrate through the filter screen by similar Eq

$$\Delta p_{\mathrm{pv,fs}} = u_1^{\mathrm{n}} K R_{\mathrm{fs}} \tag{3}$$

Here, E_c is the coefficient characterizing the filter cake resistance associated with the liquid elasticity, E_fs – coefficient characterizing the filter screen resistance associated with the liquid elasticity, K and n – parameters of the power law viscosity model, m_c – total mass of filter cake solids per filter screen unit area, W – cumulative volume of filtrate per the filter screen unit area, R_fs – filter screen resistance, u_1 – superficial filtrate velocity, x' – suspension concentration (kg solid per m^3 of filtrate), and γ_ef – effective specific filter cake resistance.

When substituting $\Delta p_\mathrm{pv,c}$ and $\Delta p_\mathrm{pv,fs}$ according to Eqs. (2) and (3) into Eq. (1), we get after rearrangement

$$\frac{\Delta p_{\mathrm{ve}}}{K\gamma_{\mathrm{ef}} x'} = u_1^{\mathrm{n}} \left[W \left(1 + \frac{E_\mathrm{c}}{W\gamma_{\mathrm{ef}} x' K} \right) + \frac{R_\mathrm{fs}}{\gamma_\mathrm{ef} x'} \left(1 + \frac{E_\mathrm{fs}}{R_\mathrm{fs} K} \right) \right] \tag{4}$$

The cake filtration Eq. (4) can be solved if only the dependences of E_c and of E_fs on u_1 or W are known.

Analogous to porous media flows, the relative elastic pressure drop excess $\Delta p_\mathrm{ve}/\Delta p_\mathrm{pv}$ can be expressed as a function f(De) of Deborah number

$$De = \frac{\lambda_\mathrm{f}}{t_\mathrm{p}} \tag{5}$$

defined as the ratio of the liquid characteristic time λ_f and the process characteristic time t_p.

For the flow through porous media, the corrective Deborah number function is frequently considered to have a simple form (e.g. [1])

$$\frac{\Delta p_{\mathrm{ve}}}{\Delta p_{\mathrm{pv}}} = \mathrm{f}(De) = 1 + A De^{\mathrm{B}} \tag{6}$$

with variable parameters A and B depending on the porous medium and liquid properties. Due to the uncertainty in determining the fluid relaxation time from rheological measurements [1], it is useful to modify the correction function (6) into the form

$$\mathrm{f}(De_\mathrm{m}) = 1 + A_\mathrm{c} D_\mathrm{w}^{\mathrm{B}} \tag{7}$$

for the flow through filter cake and into the form

$$\mathrm{f}(De_\mathrm{m}) = 1 + A_\mathrm{fs} u_1^{\mathrm{B}} \tag{8}$$

for the flow through filter screen [6].

Using modified Deborah number corrective functions defined by Eqs. (7) and (8) for expression of relative elastic pressure drop excess in the viscoelastic liquid flow through the filter cake and filter screen, the filtration equation (4) can be rewritten as

$$\frac{\Delta p_{\mathrm{ve}}}{u_1^{\mathrm{n}} K} = \gamma_{\mathrm{ef}} x' W \left(1 + A_{\mathrm{c}} D_{\mathrm{w}}^{\mathrm{B}}\right) + R_{\mathrm{fs}} \left(1 + A_{\mathrm{fs}} u_1^{\mathrm{B}}\right) \tag{9}$$

From comparison of Eqs. (4) and (9), the following relationships result for the coefficients E_{c} and E_{fs}.

$$E_{\mathrm{c}} = A_{\mathrm{c}} D_{\mathrm{w}}^{\mathrm{B}} \gamma_{\mathrm{ef}} x' W K \tag{10}$$

$$E_{\mathrm{fs}} = A_{\mathrm{fs}} u_1^{\mathrm{B}} R_{\mathrm{fs}} K \tag{11}$$

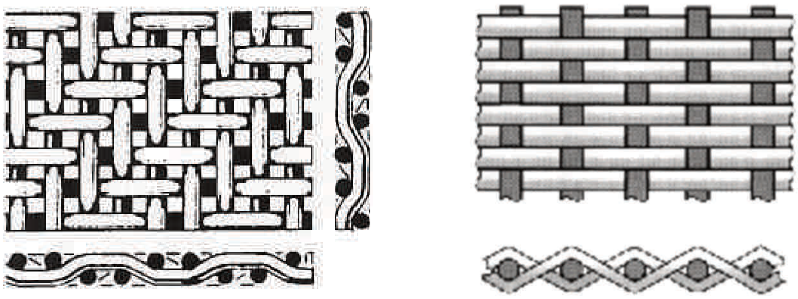
The characteristics of filtration, including parameters A_{c}, A_{fs}, and B, must be determined experimentally.

Materials and methods

Filtration experiments with suspensions of nearly spherical particles of Krasten of diameter d = 0.379 mm (hardened polystyrene; density, ρ = 1039 kg m^{-3}) in aqueous solutions of polyacrylamide Preastol and Kerafloc were carried out at a constant pressure in a cylindrical filtration unit (50 mm in diameter and 152 mm in length). For filtration in the presented experiments, two woven wire filter screen was used. Basic characteristics of the filter screens are summarized in Tab. 1. The screen dimensions were determined using a micrometre and an image analysis of microscopic photographs. The scheme of the screens knitting is shown in Fig. 1.

Table 1 *Basic characteristics of filter screen*

Symbol	Type of weaving	d_{s} [mm]	d_{w} [mm]	H [mm]	ε [–]
I	plain-twill	0.037	0.037	0.091	0.796
II	Dutch	0.036	0.053	0.136	0.418



(a) (b)

Fig. 1 *Weave of two filter screens used*

(a) – screen I: plain twilled weave; (b) – screen II: Betamesh Dutch weave

Powdered polyacrylamide was dissolved in deionized water to prepare viscoelastic liquids with polymer concentration of 0.8 % (wt.) Praestol or 0.6 % (wt.) Kerafloc. The liquid rheological properties were measured using a rotational rheometer Haake MARS II. Additional measurements of capillary thinning and the break-up process of liquids were performed using a CABER 1 rheometer.

Krasten particles (with a mean particle diameter of 0.376 mm) were added into the solution of polymer to obtain the desired form of a slurry for filtration. The solid concentrations used were 5 %, 7 %, and 9 % (wt.). In experiments, a pressured reservoir (montejus) was used for drawing the slurries. The pressure differences employed were 15, 30, and 45 kPa; the requested value of pressure being adjusted using a regulating valve. The volume flow rate of filtrate was measured by weighing on a digital balance. The values of the filtration pressure and filtrate flow rate were recorded by a personal computer. The scheme of filtration equipment is illustrated in Fig. 2.

The additional pressure drop measurements, aimed at the evaluation of the pressure drop excess in the flow of viscoelastic polymer solutions through filter cake or filter screen, were carried out in a horizontal cylindrical column (10 mm in diameter and 62 mm in length) filled with the Krasten particles and using a filter screen pressure drop measuring cell of 50 mm in inner diameter, respectively. All experiments are described in detail in the report by Surý [6].

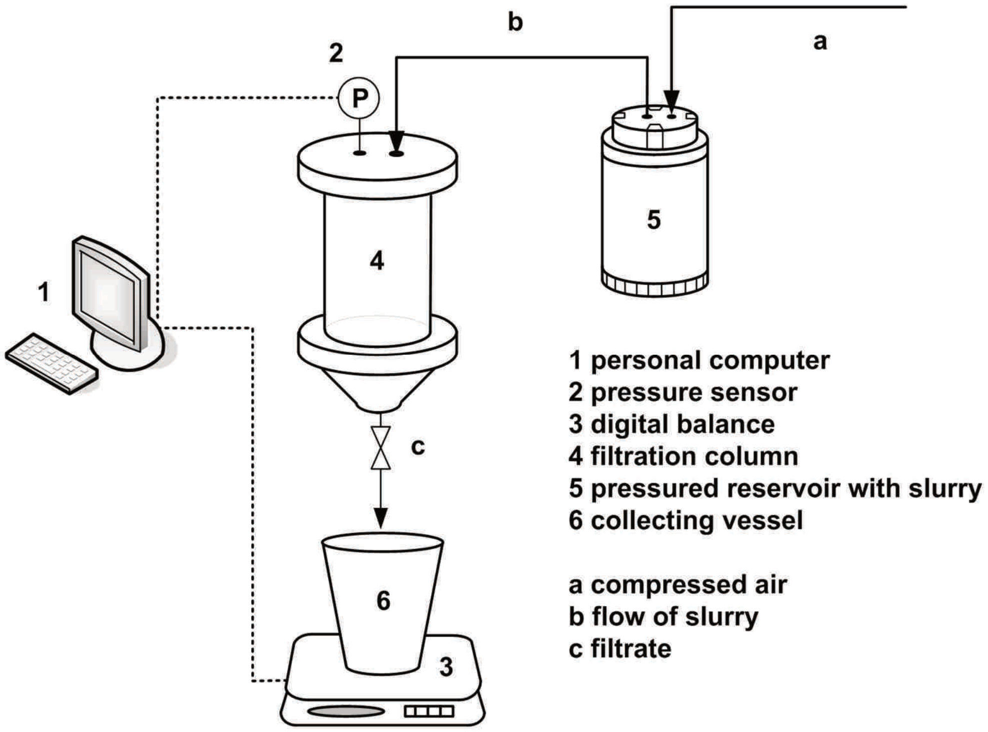


Fig. 2 *Filtration equipment – a scheme*

Results and discussion

Rheological measurements

Parameters K and n of the power law model

$$\tau = K\dot{\gamma}^{\mathrm{n}} \tag{12}$$

that were evaluated from the shear rate-shear stress rheometric data are summarized in Tab. 2; the corresponding shape of viscosity functions $\eta(\dot{\gamma})$ being shown in Fig. 3. From the capillary thinning measurements, the values of relaxation times λ_{E} for the viscoelastic stress growth were evaluated. The liquid times λ_{E} given in Tab. 2 show that these liquids are evidently viscoelastic.

Table 2 *Characteristics of dispersing liquids*

Liquid	ρ_{l} [kg m^{-3}]	$\Delta\dot{\gamma}$ [s^{-1}]	K [Pa s]	n [–]	δ [%]	λ_{E} [s]
Pr08	1001	20–2000	1.01	0.436	3.52	0.44
Ke06	1002	10–2000	1.51	0.382	3.58	0.39

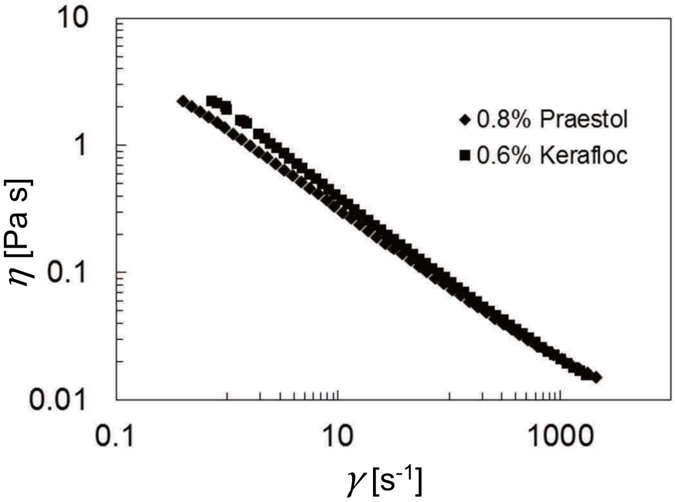


Fig. 3 *Viscosity functions of 0.8% Praestol and 0.6% Kerafloc*

Filtration tests

The characteristics of the filtration experiments performed are given in Tabs. 3–6. For evaluation of filtration tests, the Eq. (9) was used. Our previous filtration experiments with suspensions of Krasten [5,6] have shown that the filter cake formed from Krasten particles can be treated as incompressible.

Therefore, the effective specific filter cake resistance γ_{ef} could be identified with the value of the local specific filter cake resistance calculated from the theory of filtration from purely viscous power-law fluids [6]. Furthermore, the aforementioned pressure drop measurements in the flow through fixed beds of Krasten and through the filter screens have shown that the value of parameter $B = 1$ can be considered for both Praestol and Kerafloc solutions.

The experiments encompassing the filtrations on the filter screen I with a relatively small resistance were used for determination of the parameter A_c. For such a purpose, the second term on the right side of Eq. (9) was neglected and parameter A_c, denoted as $A_{c,exp}$, evaluated from the simplified Eq. (9) by optimizing the experimental data for each filtration test. The obtained values of $A_{c,exp}$ are summarized in Tabs. 3 and 4. The values $A_{c,exp}$ were compared with the values $A_{c,th}$ evaluated from the pressure drop measurements in the flow through the Krasten fixed bed [6]. The example of the pressure drop measurement in the flow of 0.6% Kerafloc solution is then shown in Fig. 4. Herein, the theoretical values of the pressure drop Δp_{ve} calculated from Eq. (7) using the parameters $A_{c,th} = 0.0159$ and $B = 1$ are displayed by a full line. The values of $A_{c,th}$ along with the relative deviations δ_{A_c} between $A_{c,th}$ and $A_{c,exp}$ are given in Tabs. 3 and 4 as well. It is evident that the agreement between $A_{c,th}$ and $A_{c,exp}$ is from an engineering point of view satisfying as it verifies the possibility to express the elongation coefficient E_c according to Eq. (10).

Table 3 *Characteristics of filtration from of 0.6% Kerafloc solution at filter screen I*

Conc./Δp	ε [–]	$\overline{\Delta p}$ [kPa]	$\gamma_{\mathrm{ef}} \times 10^{-4}$ [$\mathrm{m}^{2-n}\,\mathrm{kg}^{-1}$]	$A_{\mathrm{c,exp}} \times 10^{2}$ [s^{B}]	$A_{\mathrm{c,th}} \times 10^{2}$ [s^{B}]	δ_{A_c} [%]
5 % 15 kPa	0.417	14.97	0.834	1.724	1.592	8.30
7 % 15 kPa	0.410	15.12	0.867	1.950		22.50
9 % 15 kPa	0.403	14.99	0.897	1.807		13.51
5 % 30 kPa	0.411	29.91	0.862	1.843		15.74
7 % 30 kPa	0.406	29.97	0.880	1.604		0.74
9 % 30 kPa	0.410	29.98	0.863	1.770		11.16
5 % 45 kPa	0.411	44.94	0.862	1.754		10.21
7 % 45 kPa	0.411	44.85	0.860	1.730		8.68
9 % 45 kPa	0.411	45.07	0.860	1.714		7.65

Table 4 *Characteristics of filtration from of 0.8% Praestol solution at filter screen I*

Conc./Δp	ε [–]	$\overline{\Delta p}$ [kPa]	$\gamma_{\mathrm{ef}} \times 10^{-4}$ [$\mathrm{m}^{2-n}\,\mathrm{kg}^{-1}$]	$A_{\mathrm{c,exp}} \times 10^{2}$ [s^{B}]	$A_{\mathrm{c,th}} \times 10^{2}$ [s^{B}]	δ_{A_c} [%]
5 % 15 kPa	0.402	14.99	1.748	1.651	1.796	–8.09
7 % 15 kPa	0.404	15.01	1.725	1.593		–11.32
9 % 15 kPa	0.397	15.05	1.789	1.794		–0.11
5 % 30 kPa	0.394	30.01	1.822	1.680		–6.44
7 % 30 kPa	0.413	30.06	1.645	1.899		5.75
9 % 30 kPa	0.417	29.94	1.611	2.088		16.27
5 % 45 kPa	0.392	44.87	1.841	1.777		–1.04
7 % 45 kPa	0.393	45.15	1.830	1.548		–13.80
9 % 45 kPa	0.413	45.03	1.643	2.027		12.89

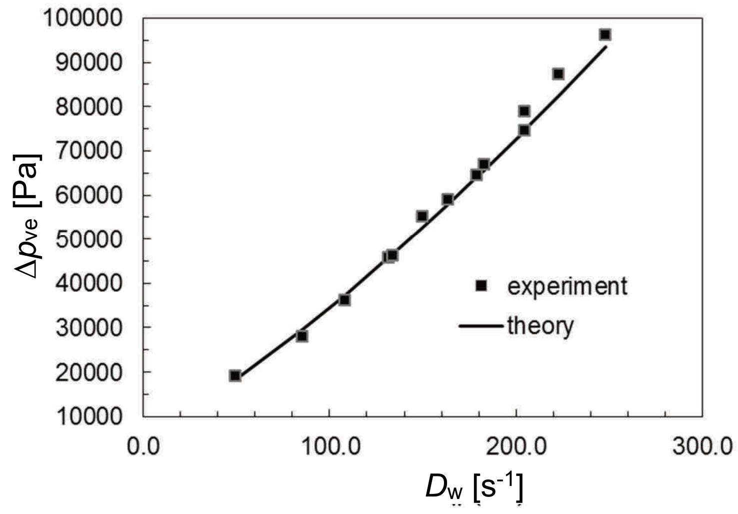


Fig. 4 *Dependence of the pressure drop* Δp_{ve} *on the kinematic variable* D_w *in the flow of 0.6% Kerafloc through the fixed bed of Krasten*

The filtration experiments encompassing the filtrations on the filter screen II were used for determination of the filter screen resistance R_{fs} and parameter A_{fs}. In this case, the values $A_{c,th}$ were substituted for A_c into Eq. (9) and the values of R_{fs} and A_{fs} evaluated by optimizing the experimental filtration data for each filtration test. The obtained values of R_{fs} and A_{fs} (denoted as $R_{fs,exp}$ and $A_{fs,exp}$) are summarized in Tabs. 5 and 6. The values $R_{fs,exp}$ and $A_{fs,exp}$ were compared with the values $R_{fs,th}$ and $A_{fs,th}$ evaluated from the pressure drop measurements in the flow through filter screen II [6].

Table 5 *Characteristics of filtration from of 0.6% Kerafloc solution at filter screen II*

Conc./Δp	$\gamma_{ef,th}\times10^{-4}$ [m^{2-n} kg^{-1}]	$A_{c,th}\times10^{2}$ [s^{B}]	$R_{fs,exp}\times10^{-4}$ [m^{-n}]	$R_{fs.th}\times10^{-4}$ [m^{-n}]	$\delta_{R_{fs}}$ [%]	$A_{fs,exp}\times10^{-4}$ [s^{B} m^{-B}]	$A_{fs,th}\times10^{-4}$ [s^{B} m^{-B}]	$\delta_{A_{fs}}$ [%]	δ_{W-t} [%]
5 % 15 kPa	0.933	1.592	1.806	2.712	–33.4	0.852	1.177	–27.6	–10.23
7 % 15 kPa	0.888		1.823		–32.8	0.953		–19.0	–4.53
9 % 15 kPa	0.924		1.701		–37.3	0.843		–28.4	–3.82
5 % 30 kPa	0.892		2.496		–8.0	1.097		–6.7	–0.97
	0.866		2.805		3.4	1.015		–13.7	0.03
7 % 30 kPa	0.888		2.107		–22.3	1.017		–13.6	–5.58
9 % 30 kPa	0.937		1.875		–30.8	0.866		–26.4	–8.02
5 % 45 kPa	0.922		2.667		–1.7	1.253		6.5	–0.04
7 % 45 kPa	0.888		2.581		–4.8	1.039		–11.7	–2.27
9 % 45 kPa	0.880		2.574		–5.1	1.134		–3.6	–3.75

Table 6 *Characteristics of filtration from of 0.8% Praestol solution at filter screen II*

Conc./Δp	$\gamma_{\mathrm{ef,th}}\times 10^{-4}$ [m^{2-n} kg^{-1}]	$A_{\mathrm{c,th}}\times 10^{2}$ [s^{B}]	$R_{\mathrm{fs,exp}}\times 10^{-4}$ [m^{-n}]	$R_{\mathrm{fs.th}}\times 10^{-4}$ [m^{-n}]	$\delta_{R_{\mathrm{fs}}}$ [%]	$A_{\mathrm{fs,exp}}\times 10^{-4}$ [s^{B} m^{-B}]	$A_{\mathrm{fs,th}}\times 10^{-4}$ [s^{B} m^{-B}]	$\delta_{A_{\mathrm{fs}}}$ [%]	$\delta_{\mathrm{W-t}}$ [%]
5 % 15 kPa	1.689		4.643		–10.6	1.294		24.0	–0.54
7 % 15 kPa	1.619		4.315		–16.9	1.182		13.3	–1.24
9 % 15 kPa	1.833		6.319		21.7	1.093		4.8	–3.20
5 % 30 kPa	1.736		5.012		–3.4	1.271		21.8	–0.78
7 % 30 kPa	1.703	1.796	5.419	5.191	4.4	1.001	1.044	–4.0	–4.52
9 % 30 kPa	1.785		6.118		17.9	1.234		18.3	–1.92
5 % 45 kPa	1.824		5.589		7.7	1.130		8.2	–0.24
7 % 45 kPa	1.704		6.434		23.9	1.097		5.1	–2.84
9 % 45 kPa	1.836		6.590		27.0	1.223		17.2	–3.36

An example of the pressure drop measurement in the flow of 0.6% Kerafloc solution through the screen II is shown in Fig. 5. Here, the theoretical values of the pressure drop Δp_{ve} calculated from Eq. (8) using parameters $A_{\mathrm{fs,th}} = 11772$ and $B = 1$ are displayed by a full line. The values of $R_{\mathrm{fs,th}}$ and $A_{\mathrm{fs,th}}$ along with the relative deviations and $\delta_{\mathrm{R,fs}}$ between $R_{\mathrm{fs,th}}$ and $R_{\mathrm{fs,exp}}$, and relative deviations $\delta_{\mathrm{A,fs}}$ between $A_{\mathrm{fs,th}}$ and $A_{\mathrm{fs,exp}}$ are also given in Tabs. 5 and 6.

The achieved agreement is again satisfying, documenting the applicability of Eq. (11) for the expression of the elongation coefficient E_{fs}.

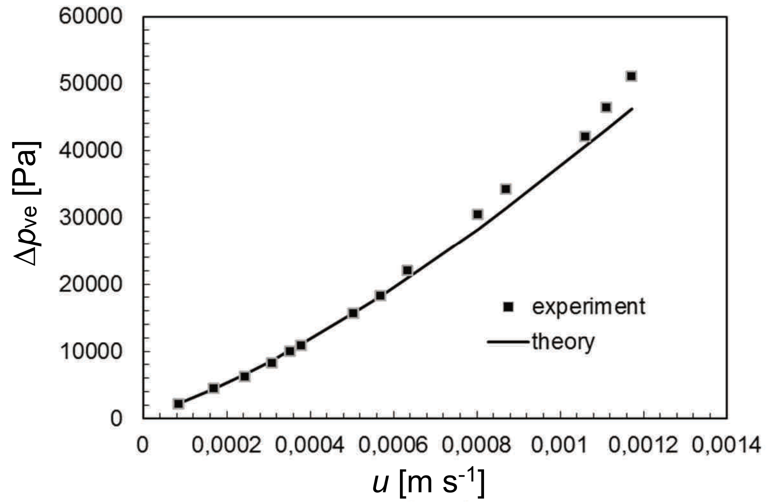


Fig. 5 *Dependence of the pressure drop Δp_{ve} on the superficial filtrate velocity u_1 in the flow of 0.6% Kerafloc through the filter screen II*

For comparison of the individual filtration resistances, the resistances $R_{\mathrm{c,pv}} = \gamma_{\mathrm{ef}} x'W$, $R_{\mathrm{c,ve}} = \gamma_{\mathrm{ef}} x'WA_{\mathrm{c}}D_{\mathrm{w}}$, $R_{\mathrm{fs,pv}}$ (given in Tabs. 5 and 6), and $R_{\mathrm{fs,ve}} = R_{\mathrm{fs,pv}} A_{\mathrm{fs}}\, u_1^{\mathrm{B}}$ could be evaluated. The corresponding values of u_1 have been determined by numerical differentiation of experimental "*W vs. t*" dependences. Examples of dependences of resistances $R_{\mathrm{c,pv}}$, $R_{\mathrm{c,ve}}$, $R_{\mathrm{fs,pv}}$, and $R_{\mathrm{fs,ve}}$ *vs. W* are plotted in Fig. 6 for filtration of 9% (wt.) Krasten suspension in 0.6% Kerafloc at 45 kPa. It can be seen that, in the all period of filtration, the liquid elasticity manifests itself and the flows through the filter cake and filter screen are not purely viscous as approaching the end of filtration.

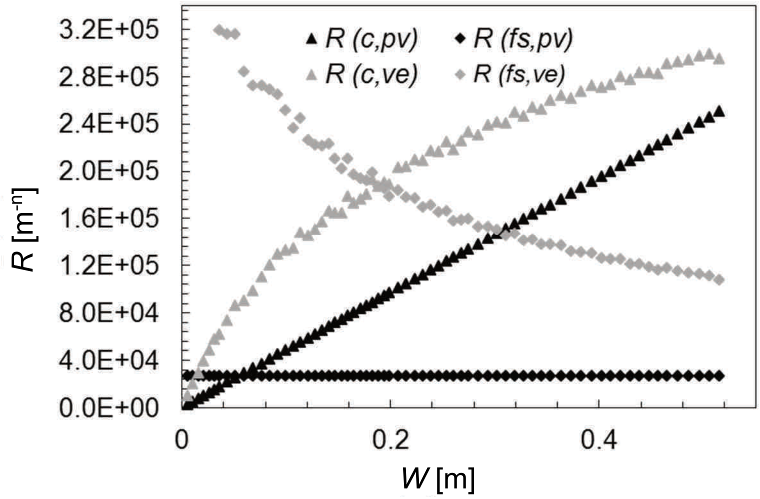


Fig. 6 *Dependences of resistances $R_{c,pv}$, $R_{c,\,ve}$, $R_{fs,pv}$, and $R_{fs,ve}$ on filtrate volume W for filtration of 9% wt. Krasten suspension in 0.6% Kerafloc solution on filter screen II at 45 kPa*

The experimental dependences of the filtrate volume on the filtration time, *W vs. t*, were compared with theoretical ones calculated according to Eq. (9), which was solved numerically by means of the Runge-Kutta 4th order method. In the numerical calculations the values $A_{\mathrm{c,th}}$ and $A_{\mathrm{fs,th}}$ were substituted into Eq. (9). The results obtained for the filtrations from Preastol solutions are portrayed in Fig. 7. The relative mean deviations $\delta_{\mathrm{W-t}}$ between the experimental and calculated values of *W* are given in Tabs. 5 and 6. The agreement between the experimental and calculated data is very good.

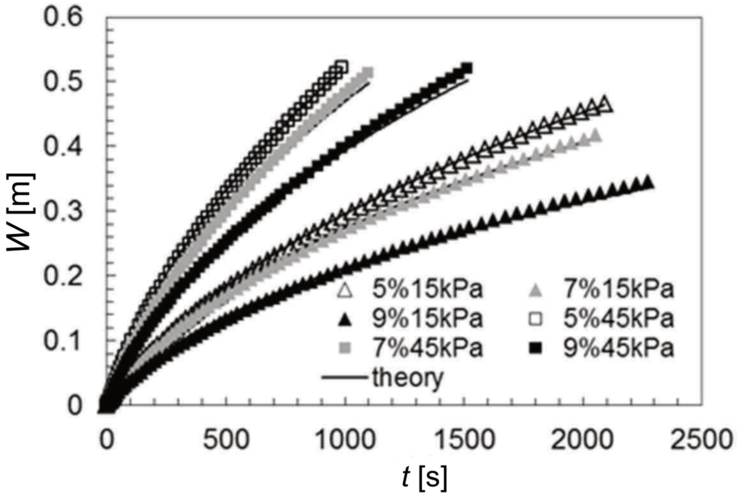


Fig. 7 *Dependences of the filtrate volume on filtration time, W vs. t, for filtration of Krasten from 0.8% Prestol solution*

Conclusions

The constant-pressure cake filtration of Krasten polystyrene particles from viscoelastic aqueous solutions of polyacryl amides Kerafloc and Praestol on the ilter screens with different resistance has been investigated experimentally.

The filtration equation, involving the modified Deborah number correction functions for expressing the elastic pressure drop excess, has been presented to describe this type of filtration. It was found that the parameters A and B of the correction function evaluated from the filtration experiments agreed satisfactorily with those obtained from the pressure drop measurements in the flow of dispersing liquids through separate fixed bed of Krasten particles and filter screen, respectively.

For each filtration test, the individual resistances $R_{\mathrm{c,pv}}$, $R_{\mathrm{c,ve}}$, $R_{\mathrm{fs,pv}}$, and $R_{\mathrm{fs,ve}}$ were evaluated from the filtration equation. The results showed that, in all the period of filtration, the elongation flow mode manifests itself and the flows through the filter cake and filter screen were not purely viscous, including a stage at the end of filtration.

The agreement of experimental dependences achieved for the cumulative filtration volume in dependence on the time of filtration, "*W vs. t*", and confronted with the relations obtained by the numerical solution of the proposed differential equation for filtration has confirmed that the form of the proposed correction functions is suitable to evaluate the elastic effects in the cake filtration from viscoelastic liquids.

Symbols

A	parameter in Deborah number correction function (6)	–
A_{c}	parameter in Deborah number correction function (7)	s^{B}
A_{fs}	parameter in Deborah number correction function (8)	$\mathrm{s}^{\mathrm{B}}\ \mathrm{m}^{-\mathrm{B}}$
a_{ps}	specific surface of particles	m^{-1}
B	parameter in Deborah number correction functions	–
D_{w}	kinematic variable ($=\dfrac{2u_1}{l_{\mathrm{ch}}\varepsilon}$)	s^{-1}
De	Deborah number	–
E_{c}	coefficient characterizing the filter cake resistance associated with the elongation flow mode	$\mathrm{Pa\ s^{n}\ m^{-n}}$
E_{fs}	coefficient characterizing the filter screen resistance associated with the elongation flow mode	$\mathrm{Pa\ s^{n}\ m^{-n}}$
K	power law parameter (consistency)	$\mathrm{Pa\ s^{n}}$
l_{ch}	characteristic linear dimension of the filter cake ($=\dfrac{\varepsilon}{a_{\mathrm{ps}}\phi(1-\varepsilon)}$)	m
m_{c}	mass of filter cake per filter screen unit area	$\mathrm{kg\ m^{-2}}$
n	power law parameter (flow index)	–
Δp_{pv}	pressure drop of purely viscous fluid	Pa
Δp_{ve}	pressure drop of viscoelastic fluid	Pa
R_{fs}	filter screen resistance	$\mathrm{m^{-n}}$
t	time of filtration	s
t_{p}	process characteristic time	s
u_1	superficial filtrate velocity	$\mathrm{m\ s^{-1}}$
W	cumulative filtrate volume per unit area	$\mathrm{m^3\ m^{-2}}$
x'	suspension concentration (kg of solid per m^3 of filtrate)	$\mathrm{kg\ m^{-3}}$

Greek Letters

γ_{ef}	specific filter cake resistance	$\mathrm{m^{2-n}\ kg^{-1}}$
$\dot{\gamma}$	shear rate	s^{-1}
δ	relative deviation	%
ε	filter cake porosity	–
η	non-Newtonian shear viscosity	Pa s
λ_{E}	characteristic time for viscoelastic stress growth	s
λ_{f}	characteristic liquid time	s
τ	shear stress	Pa
φ	filter cake factor defined as the ratio of the total filter cake drag and the viscous drag	–

Subscripts

c	related to the filter cake
exp	experimental value
fs	related to the filter screen
m	modified
pv	purely viscous
th	theoretical value
ve	viscoelastic

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