Multiple state models for critical illness policy.

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Abstract: The main goal of this paper is to apply multiple state models for an insurance policy combining disability income insurance benefits and critical illness benefits. We consider a policy with term 40 years to a life aged 25 which provides a death benefit, a disability benefit and a critical illness benefit. Using the data supplied by the Continuous Mortality Investigation (CMI) we calculate the premium payable continuously for this policy.

Key words: multiple state model, stochastic process, Markov process, critical illness,

JEL Classifications: C51, C52, G22, J11,

1 Introduction

The main goal of this paper is to apply multiple state models for an insurance policy combining disability income insurance benefits and critical illness benefits. The data we used in our contribution were supplied by the Continuous Mortality Investigation (CMI). The CMI is a research organisation established by UK actuarial profession.

Disability insurance, long-term care insurance and critical illness cover are becoming increasingly important in developed countries as the problems of demographic aging (Pacáková, V., Jindrová, P. (2014)) come to the fore. The private sector insurance industry is providing solutions to problems resulting from these pressures and other demands of better educated and more prosperous populations.

Critical illness insurance (CII) is a type of long term insurance that provides a lump sum on the diagnosis of one of a specified list of critical illnesses within the policy conditions. CII coverage includes (but is not limited) cancer, heart attack, stroke, coronary artery by-pass graft, kidney failure, major organ transplant, multiple sclerosis and other causes. CII has been very popular in the UK. UK sales peaked in 2002 when around 1 million new policies were issued by CMI Working paper 50 (2011). There is no restriction on how to spend the CII benefit. Most of the CII policies in the UK are linked to mortgages as this is a considerable financial commitment and diagnosis with a critical illness could affect the individual's ability to repay the mortgage. There are two types of critical illness policy: Full Accelerated, which covers both critical illness and death, and Stand Alone, which covers only critical illness. Most of the policies in UK are accelerated policies (88%) and they are attached to life insurance, term insurance or endowments. Typically, regular premiums are payable throughout the term while the policy is in force.

We describe the actuarial structure of disability insurance, long-term care insurance, and critical illness cover. Actuarial problems such as pricing and reserving are considered within the context of multiple state modelling, providing a strong and sound framework for analysing personal insurances.

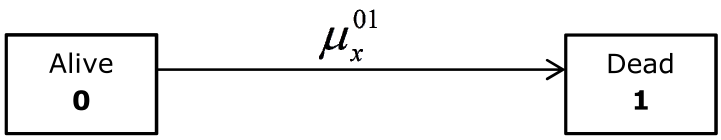
Our contribution is based on Markov process that can be used to develop a general, unified and rigorous approach for describing and analysing disability and related insurance benefits. The use of Markov process or Markov chain in life contingencies and their extensions has been proposed by several authors; for example Dickson, D. C., Hardy, M. R., & Waters, H. R. (2013), Haberman, S., & Pitacco, E. (1998).

2 Methodology and Data

Multiple state models are one of the most exciting developments in actuarial science nowadays. They are a natural tool for many important areas of practical interest to actuaries. They provide solid foundation for pricing and valuing complex insurance contracts.

We can represent life insurance survival model diagrammatically as shows Figure 1. An individual is, at any time, in one of two states, "Alive" or "Dead".

Figure 1 The alive-dead model



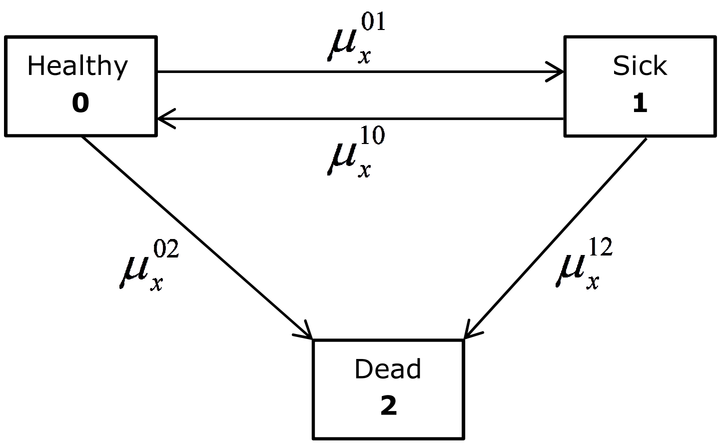
Source: Own processing

Transition from state '0' to '1' is allowed, as indicated by the direction of the arrow, but transition in the opposite direction is not possible.

We can use this simple two state model to reformulate the survival model such as we define a random variable $Y(t)$ which takes one of the two values '0' and '1'. Suppose we have an individual aged x years at time t=0. The event $Y(t)=0$ means that an individual is alive at age x+t, and $Y(t)=1$ means that an individual died before age x+t. The set of random variables $\{Y(t)\}_{t\geq 0}$ is an example of a continuous time stochastic process. We will assume that $\{Y(t)\}_{t\geq 0}$ is a Markov process. The alive-dead model represented by Figure 1 captures all the life contingent information that is necessary for calculating insurance premiums and policy values. The force of mortality μ_x^{01} fully describes the lifetime distribution.

But there are more complicated insurance policies which require more sophisticated models. These policies consist of a finite set of states with arrows indicating possible movements between them. Each model appropriate for a given insurance policy is constructed in a similar manner.

Figure 2 The disability income model



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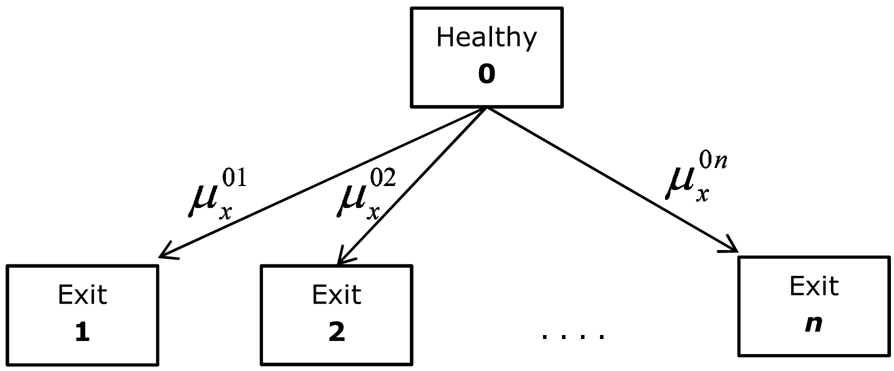
The condition for a payment relating to the policy, for example a premium, sum insured, is either that the individual is in a specified state at that time or that the individual makes an instantaneous transfer between a specified pair of states at that time.

The disability income insurance pays a benefit during periods of sickness, the benefit ceases on recovery. Figure 2 shows a model suitable for policy which provides an annuity while person is sick, with premiums payable while the person is healthy. The model represented by Figure 2 differs from that in Figure 1 in one important aspect: it is possible to transfer from state '1' to state '0', that is, to recover from an illness. This model illustrates an important general feature of multiple state models that is the possibility of entering one or more states many times. This means that several periods of sickness could occur before death, with healthy (premium paying) periods in between. This model has three states, and we can define a continuous time Markov process, $\{Y(t)\}_{t\geq 0}$, where random variable $Y(t)$ takes one of the value '0', '1' and '2'.

Other extension of the model illustrated in Figure 1 is a multiple decrement model. A multiple decrement model is characterized by having a single starting state and several exit states (absorbing states), but no further transitions. Figure 3 illustrates a multiple decrement model with n+1 states. A policyholder is supposed to be healthy at the time of the commencement of the policy and he/she stays in this state until at some time he/she transits to one of the n possible exit states that means a death or a critical illness occurred.

Figure 3 A multiple model with several exits



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In general case, with states 0, 1, 2, …, n, we refer to μ_x^{ij} as the force of transition or transition intensity between states i and j at age x. The transition intensities are fundamental quantities which determine everything we need to know about a multiple state model.

Consider an insurance policy issued at age x and with term m years described by a multiple state model with n+1 states, labelled 0, 1, 2, …, n. Let

μ_y^{ij} denote the transition intensity between states i and j at age y,

δ_t denote the force of interest per year at time t,

$B_t^{(i)}$ denote the rate of payment of benefit at time t while the policyholder is in state i,

$S_t^{(ij)}$ denote the lump sum benefit payable instantaneously at time t on transition from state i to state j.

Then the policy value ${}_tV^{(i)}$ for a life in state i at time t is given by the **Thiele's differential equation**

$$\frac{\mathrm{d}}{\mathrm{d}t}\,{}_tV^{(i)} = \delta_t \cdot {}_tV^{(i)} - B_t^{(i)} - \sum_{j=0,\, j\neq i}^{n} \mu_{x+t}^{ij} \cdot \left(S_t^{(ij)} + {}_tV^{(j)} - {}_tV^{(i)}\right), \qquad (1)$$

for i = 0, 1, …, n and 0 < t < m.

Thiele's differential equation is a continuous time equivalent of the recursions for policies with annual cash flows.

We assume that δ_t, $B_t^{(i)}$ and $S_t^{(ij)}$ are continuous function of t. The premium is included within this model as negative benefit and expenses can be included as addition to the benefits.
We can use formula (1) to calculate policy values numerically. We choose a small step size h and replace the left-hand side of (1) by $\dfrac{{}_tV^{(i)} - {}_{t-h}V^{(i)}}{h}$.

We will then use Euler's method (starting with ${}_mV^{(i)} = 0$) to calculate the policy values at durations m-h, m-2h, …, h, 0.

3 Results

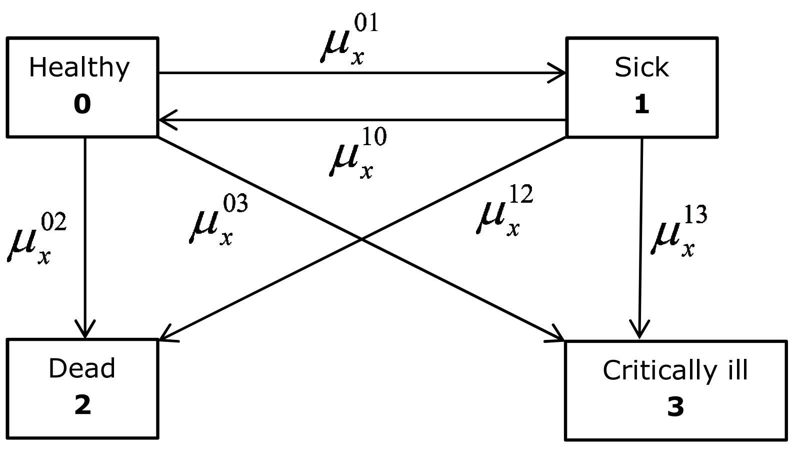
Consider the model (Figure 4) for an insurance policy combining disability income insurance benefits and critical illness benefits. An insurance company issues a policy with term 40 years to a life aged 25 which provides a death benefit, a disability benefit and a critical illness benefit as follows:

- a lump sum payment of 200 000 USD is payable immediately on the life becoming critically ill,
- a lump sum payment of 100 000 USD is payable immediately on death, provided that the life has not already been paid a critical illness benefit,
- a disability income annuity of 25 000 USD per year payable whilst the life is disabled payable continuously,

with no lapses and no expenses. (Expenses can be included in as additions to the benefits)

Premium is payable continuously provided that the policyholder is healthy. We assume an interest rate of 2 % p.a.

Figure 4 Schema of our model



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For calculation we use transition intensities from the CMI Working paper 50. From these data we apply our model for particular critical illnesses: cancer and stroke for female population.

The transition intensities are as follows:

$$\mu_x^{01} = 4 \cdot 10^{-4} + 3{,}5 \cdot 10^{-6} \cdot \exp\{0{,}14 \cdot x\},$$

$$\mu_x^{02} = 5 \cdot 10^{-4} + 7{,}6 \cdot 10^{-5} \cdot 1{,}094174^x, \text{ (Gompertz-Makeham's law of mortality)}$$

$$\mu_x^{03} = \exp\{-13{,}425 + 0{,}09313 \cdot x\}, \text{ for Stroke and}$$

$$\mu_x^{03} = \exp\{-10{,}135 + 0{,}08347 \cdot x\}, \text{ for Cancer,}$$

$$\mu_x^{10} = 0{,}1 \cdot \mu_x^{01},$$

$$\mu_x^{12} = \mu_x^{02},$$

$$\mu_x^{13} = \mu_x^{03}.$$

Thiele's differential equations for ${}_tV^{(0)}$ and ${}_tV^{(1)}$ are

$$\frac{\mathrm{d}}{\mathrm{d}t}\,{}_tV^{(0)} = \delta_t \cdot {}_tV^{(0)} + P - \mu_{25+t}^{01} \cdot \left({}_tV^{(1)} - {}_tV^{(0)}\right) - \mu_{25+t}^{02} \cdot \left(100\,000 - {}_tV^{(0)}\right) - \mu_{25+t}^{03} \cdot \left(200\,000 - {}_tV^{(0)}\right), \quad (2)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,{}_tV^{(1)} = \delta_t \cdot {}_tV^{(1)} - 25\,000 - \mu_{25+t}^{10} \cdot \left({}_tV^{(0)} - {}_tV^{(1)}\right) - \mu_{25+t}^{12} \cdot \left(100\,000 - {}_tV^{(1)}\right) - \mu_{25+t}^{13} \cdot \left(200\,000 - {}_tV^{(1)}\right). \quad (3)$$

Using Euler's method with a step size $h = \frac{1}{12}$ and with the boundary conditions ${}_{40}V^{(0)} = {}_{40}V^{(1)} = 0$ we calculate the policy values.

These equations we solve by using Excel build-in tool "Solver". Requiring ${}_0V^{(0)}$ to be equal to 0 (using equivalence principle) gives P_S = 1 435.59 USD for stroke and P_C = 1 773.26 USD for cancer.

4 Conclusions

We have presented an application of multiple state models to problems in actuarial science. There are various extensions of multiple state models. One way is to allow the transition intensities out of a state to depend not only on individual's current age but also on how long they have been in current state. This breaks the Markov property assumption and leads to the new process known as a semi-Markov process. This could be appropriate for the disability income insurance process where the intensities of recovery and death from the sick state could be assumed to depend on how long the individual had been sick, as well as on current age.

For the numerical solution of differential equations we used Euler's method. Its advantage is that it is relatively simple to implement. There are more sophisticated ways of solving such equations, for example the Runge-Kutta method.

The transition intensities are fundamental quantities which determine everything we need to know about a multiple state models. Therefore it would be useful to have data from domestic insurance industry. Our further research will focus on estimation of transition intensities for the Czech Republic (or other central European countries) in similar manner as in Pacáková, V., Jindrová, P., Seinerová, K. (2013).

There is a need for awareness of model risk when assessing an insurance policy combining disability income insurance benefits and critical illness benefits, especially with long term. The fact that transition intensities can be estimated does not imply that they can sensibly describe future medical development.

Acknowledgments

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