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OPTIMAL IMPLANTATION OF JUNK CAR RECYCLING PLANTS

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1. Introduction

One of the environment problems for all economic well developed countries is the way of liquidation of cars the service life of whose has already finished (we use the term „junk car" which is also currently used in the U.S.A.). Nowadays also for Czech Republic economy it begins to be a serious problem. In the next future in the connection with the hypothetical growth of economic power of the population enormous arising of old cars is expected that should be liquidated in a way that does not harm the environment. There are several possibilities that can be taken into consideration. One of the most frequently mentioned ways in Czech Republic is a construction of recycling plants network. Because the problem is relatively complex and there are several possibilities to solve it, we imagine to design a mathematical model which would enable to choice the solution by information means. Following paper is a small contribution to the discussion in the field of problem formulation and model designing by operational research tools.

2. Description of the problem

Optimal implantation of junk car recycling plants in geographical area must take into consideration:

- supply expenses of junk cars to recycling centres,

- delivery expenses of recycling plants final products to processing industry,
- production expenses including:
  - technological expenses,
  - financial expenses.

Let

$A=(a_i)$, $i= 1,..., n$ is a set of supply points of junk cars,

t_i, $i=1,..., n$ are transportation expenses of junk cars per kilometre,

$k = 1,\dots,m$ are recycling plants,

$\vec{X}_k, k = 1,\dots,m$ is a location of recycling plants in geographic area,

$c_k, k = 1,\dots,m$ is a capacity of recycling plants,

$\mu_{ik}, i = 1,\dots,n; k = 1,\dots,n$ is quantity of junk cars delivered from supply point to the recycling centre *k* located in $\vec{X}_k$.

We will suppose the most simplified case, that the production capacity of each recycling plant equals the total quantity of junk cars delivered from all supply points to the recycling centre:

$$\sum_{i=1}^{n} \mu_{ik} = c_k, k = 1,\dots,m.$$

Consumption of final products $B = (b_j), j = 1,\dots, M$ in the considered area is located in N points of appropriate processing industry plants with co-ordinates $\vec{Y}_l, l = 1,\dots,N.$

Let us denote:

$p_l, l = 1,\dots,N$ is a demand of final product $b_j, j = 1,\dots, M$ in $\vec{Y}_l$,

$t_{b_j}, j = 1,\dots, M$ are transportation expenses of final product b_j per kilometre,

$\lambda_{kl}^{b_j}, k = 1,\dots,m; l = 1,\dots,N; j = 1,\dots, M$ is a quantity of final product b_j delivered by recycling centre *k* to processing industry plant located in $\vec{Y}_l$.

Hypotheses:

1. Every recycling centre can deliver its production to all processing industry points (plants), therefore we can write:

$$\sum_{l=1}^{N} \lambda_{kl}^{b_j} = c_k^{b_j}, k = 1,\dots, m; j = 1,\dots, M$$

2. Demand of every processing plant will be covered (there is not a shortage of final production), therefore we can write:

$$\sum_{k=1}^{m} \lambda_{kl}^{b_j} = p_{lk}^{b_j}, l = 1,\dots, N; j = 1,\dots, M$$

3. Criterion of optimisation

The main components of global expenses are as mentioned above:

a) supply expenses,
b) production expenses,
c) delivery expenses.

Ad a) Expenses of junk cars supply

Let $d(a_i, \vec{X}_k)$ is a distance between the source of junk cars in a_i and recycling plant located in $\vec{X}_k$. Than the supply expenses for this couple will be:

$$q(a_i, \vec{X}_k) = d(a_i, \vec{X}).t_i.\mu_{ik}$$

Total expenses for sphere of attraction of recycling plant in $\vec{X}_k$ will than be:

$$q_k = \sum_{i=1}^{n} d(a_i, \vec{X}).t_i.\mu_{ik}$$

Total supply expenses for the whole area will be:

$$Q_s = \sum_{k=1}^{m} q_k = \sum_{k=1}^{m} \sum_{i=1}^{n} d(a_i, \vec{X}).t_i.\mu_{ik}$$

Ad b) Expenses of final product - b_j delivery to processing industry plants

Let $d(\vec{X}_k, \vec{Y}_l)$ is a distance between recycling plant located in $\vec{X}_k$ and processing plant of final product - b_j located in $\vec{Y}_l$.

The delivery expenses of b_j between $\vec{X}_k$ and $\vec{Y}_l$ are:

$$q_{kl}^{b_j} = \lambda_{kl}^{b_j}.t_{b_j}.d(\vec{X}_k, \vec{Y}_l)$$

Total delivery expenses for all recycling plants and all final products will than be:

$$Q_d = \sum_{l=1}^{N} \sum_{k=1}^{m} \sum_{j=1}^{M} \lambda_{kl}^{b_j} . t_{b_j} . d\left(\vec{X}_k, \vec{Y}_l\right)$$

Ad c) Production expenses

The most important of all expenses that we include into production expenses are technology a financial expenses.

- technology expenses - $t(c_k)$ depend on installed capacity - c_k and used technology of production. They can be characterised by function of dependence between average expenses and realised production. This function has a minimum corresponding to maximum average profit. Providing that marginal expenses increase, the curve of expenses for determined technology has a shape of convex function. In case of unique technology of the processing the technological expenses can be described by quadratic function.

- financial expenses - $f(c_k)$ depend on set-up costs of installed capacity c_k and amortisation policy. In simplified case we can presume amortisation of set-up costs during the whole service life of installed technology. Than we can consider financial expenses related to the amortisation of installed capacity in the following form:

$$f\left(c_k\right) = \frac{1}{v} . h\left(c_{k,}\right)$$

where $h\left(c_{k,}\right)$ are set-up costs and v is service life of installed devices.

Production expenses of recycling plant k located in $\vec{X}_k$ will be:

$$q_k = t\left(c_k\right) + f\left(c_k\right).$$

Total production expenses than will be:

$$Q_p = \sum_{k=1}^{m} q_k .$$

4. Formulation of the problem

The problem that is to be solved is:

- determine the number of recycling plants (allocation task),
- determine location of recycling plants (location problem),
- determine the spheres of attraction of recycling plants,
- determine the capacities to be installed in recycling plants,
- determine plan of supply and delivery (transportation task),

so that common total expenses of production, supply of junk cars and distribution of final recycling plants production

$$Q = Q_S + Q_D + Q_P$$

would be minimised.

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Literatura

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Resumé

OPTIMÁLNÍ IMPLANTACE RECYKLAČNÍCH CENTER OJETÝCH VOZIDEL

Josef VOLEK

V článku je popsán a formulován problém optimalizace počtu a rozmístění recyklačních center pro ojetá osobní vozidla. Popisovaný problém patří do skupiny úloh diskrétní optimalizace, které jsou též označovány jako alokačně-lokační úlohy. Pro formulaci úlohy a sestavení kritéria optimalizace je použit matematický aparát lineárního programování a teorie grafů.

Summary

OPTIMUM IMPLANTATION OF JUNK CAR RECYCLING PLANTS

Josef VOLEK

The Described problem is one of the tasks of discrete combinatorial optimisation and can be solved by means of operational research methods. The most appropriate are methods of theory of graphs and methods of linear programming. The papers involves the description of the problem, formulation of the optimisation criterion and the simplified verbal formulation of the problem.

Zusammenfassung

OPTIMALE IMPLANTIERUNG DER REZYKLATIONSZENTEREN

Josef VOLEK

Im Aufsatz wird das Problem der Optimierung der Anzahl und der Unterbringung der Razyklationszenteren beschreiben und formuliert. Das beschreibene Problem gehört zur Gruppe der Aufgaben der diskreten Optimierung, die auch als Alokalisierungs-Lokalisierungsaufgaben gezeichnet werden. Für die Problemformulierung und für die Zusammensetzung des Kriteriums der Optimilität wird der mathematische Apparat der linearen Programmierung und der Grapfentheorie verwendet.