# Modelling Insured Catastrophe Losses

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**Abstract**

Catastrophic events affect various regions of the world with increasing frequency and intensity. Large catastrophic events can be caused by natural phenomena or are caused by man. Serious events in recent years are often the result of terrorist acts. Catastrophe modelling is a risk management tool that uses specific methods and computer technology to help insurers, reinsurers and risk managers better assess the potential losses caused by natural and man-made catastrophes.

This article describes and applies the parametric curve-fitting methods for modelling historical insured catastrophe losses. Article provides theoretical description of the Excess over Threshold Method (EOT) and presents its application to the data about insured catastrophe losses in the world in period 1970-2014, published in No 2/2015 Swiss Re study Sigma.

The modelling using the EOT method follows the assumptions and conclusions in a generalized Pareto family with unknown parameters. Consequently application part of the article comprises the results of fitted insured catastrophe losses by generalized Pareto distribution using the maximum likelihood method for parameters estimation to the data above a high threshold.

***Keywords:*** *Insured catastrophe losses, Excess over Threshold Method, Generalized Pareto Distribution*.

***JEL Classification:*** C4, C6, C8

## 1. Introduction

The enormous impact of catastrophic events on our society is deep and long. Not only we need to investigate the causes of such events and develop plans to protect against them, but also we will have to resolve the resulting huge financial loss. Obviously, the insurance and reinsurance industry needs to reevaluate the risk in insuring future damages. Extreme Value Theory (EVT) (McNeil, 1997) emerges as a basic tool in modeling such risk.

Catastrophe modeling is one of many tools in the risk management available to insurers and reinsurers to predict future losses and better manage and prepare for disasters in the years to come.

Based on *Swiss Re Sigma* criteria (2015), an event is classified as a catastrophe and included in the sigma database when insured claims, total losses or the number of casualties exceed certain thresholds (see Table 1).

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| Insured loss thresholds | | | Total economic loss threshold | Casualties | | |
|---|---|---|---|---|---|---|
| Maritime disasters | Aviation | Other losses | | Dead or missing | Injured | Homeless |
| 19.6 million | 39.3 million | 48.8 million | 97.6 million | 20 | 50 | 2000 |

**Table 1** The sigma event selection criteria, 2014 (Sigma No 2/2015).

The occurrences of the catastrophic events are becoming more frequent and also grow indemnity of insurance and reinsurance companies at these events although the difference between the insured and uninsured losses is considerable (Fig. 1).

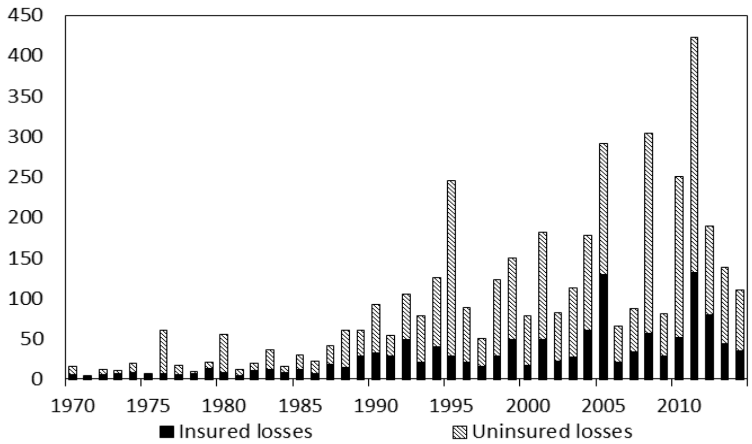


**Fig. 1.** Insured vs uninsured losses, 1970-2014, in USD billion in 2014 prices.

According to the latest sigma study, global insured losses from natural catastrophes and man-made disasters were USD 35 billion in 2014, down from USD 44 billion in 2013 and well below the USD 64 billion-average of the previous 10 years. There were 189 natural catastrophe events in 2014, the highest ever on sigma records, causing global economic losses of USD 110 billion. Around 12 700 people lost their lives in all disaster events, down from as many as 27 000 in 2013, making it one of the lowest numbers ever recorded in a single year. Total economic losses from all disaster events in 2014 were USD 110 billion, down from USD 138 billion in 2013, and well below the previous 10-year annual average of USD 200 billion (Swiss Re Sigma No 2/2015).

In the modelling of catastrophe events statistical methods are commonly used for inference from historical data. Extreme Value Theory (EVT) (Embrechs, Kluppelberg, Mikosch, 1997) emerges as a basic tool in modelling such risk. It began with the paper by Dodd in 1923, followed by the paper Fisher and Tippett in 1928, after by the papers by de Finetti in 1932, by Gumbel in 1935 and by von Mises in 1936, to cite the more relevant; the first complete frame in what regards probabilistic problems is due to Gnedenko in 1943. Following the theoretical developments of the extreme value theory many scholarly papers, as (Han, 2003; Skřivánková and Tartaľová, 2008; Jindrová and Sipková, 2014; Jindrová and Jakubínský, 2015) dealing with the variety of practical applications of the theory were published. The Generalized Extreme Value (GEV), Gumbel, Frechet, Weibull, and the Generalized Pareto (GPD) distributions are just the tip of the iceberg of an entirely new and quickly growing branch of statistics. Various authors have noted that this theory is relevant to the modelling of extreme insurance losses.

## 2. Methodology and data

Catastrophic events are undoubtedly extreme events, as seen from the Table 1. They are also extremal events, also called "rare" events. Extremal events share three characteristics: relatively rareness, huge impact and statistical unexpectedness. Although catastrophic events are rare events and their occurrence is very small, over longer period we have observed several.

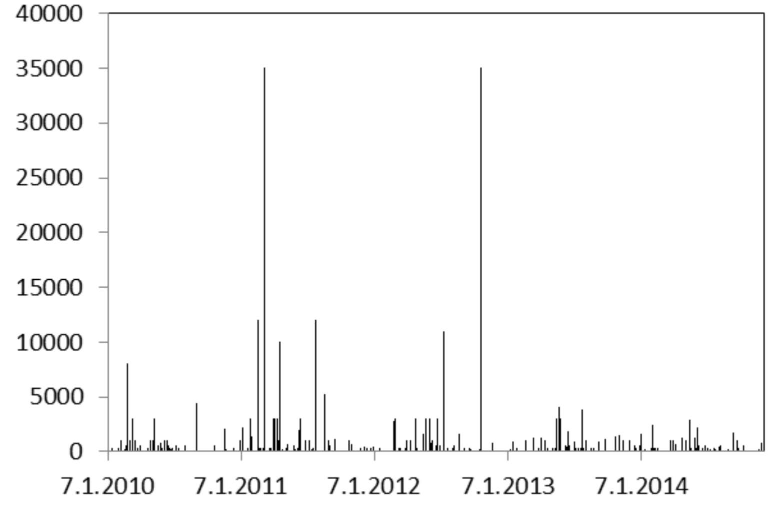


**Fig. 2.** Chronologically arranged the insured losses of natural catastrophes, 2010-2014.

Our modelling focus on chronological list of 323 insured losses (in USD millions) of natural catastrophes in time period from January 1st 2010 to December 31th 2014, published in Swiss Re Sigma 2011-2015 (Fig. 2).

The time series plot (Fig. 2) allows us to identify the most extreme losses and their approximate times of occurrence.

In the modelling of extremal events different approaches had been proposed for certain circumstances. In this paper we are concerned with fitting the generalized Pareto distribution (GPD) to losses which exceed high enough thresholds using the Excess over Threshold Method (EOT) (Embrechs and Kluppelberg and Mikosch, 1997). The Generalized Pareto Distribution (GPD) is the limit distribution of values excess of high thresholds. The main connection is in the following GPD theorem (Fisher and Tippett, 1928).

Suppose $X_1, X_2, \ldots$ are independent, identically distributed with distribution *F*. Then for a large enough threshold *u*, the conditional distribution function of *Y* = (*X* – *u* / *X* > *u*) is approximately

$$P[X-u<x/X>u] \sim H(x)=1-\left(1+\frac{\xi x}{\tilde{\sigma}}\right)^{-1/\xi} \tag{1}$$

defined on $\{x: x>0 \text{ and } (1+\xi x/\tilde{\sigma})>0\}$.

The family of distributions defined by equation (1) is called the General Pareto Distribution (GPD) family. For a fixed high threshold *u*, the two parameters are the shape parameter $\xi$ and the scale parameter $\tilde{\sigma}$.

The modelling using the excess over threshold method follows the assumptions and conclusions in GPD Theorem. Suppose $x_1, x_2, \ldots, x_n$ are raw observations independently from a common distribution *F*(*x*). Given a high threshold *u*, assume $x_{(1)}, x_{(2)}, \ldots, x_{(k)}$ are an observation that exceeds *u*. Here we define the ascendances as $x_i = x_{(i)} - u$ for $i = 1, 2, \ldots, k$.

By GPD Theorem $x_i$ may be regarded as realization of independently random variable which follows a generalized Pareto family with unknown parameters $\xi$ and $\tilde{\sigma}$. In case $\xi \neq 0$, the likelihood function can be obtained directly from (1) (Han, 2003):

$$L(\xi, \tilde{\sigma}/x) = \prod_{i=1}^{k}\left[\frac{1}{\tilde{\sigma}}\left(1+\frac{\xi x_i}{\tilde{\sigma}}\right)^{-1/\xi-1}\right] \tag{2}$$

## 3. Results and Discussion

Procedures for goodness-of-fit tests with GPD are part of a number of statistical software packages. We have used for modelling insured catastrophic losses by GPD the statistical package Statistica 12.

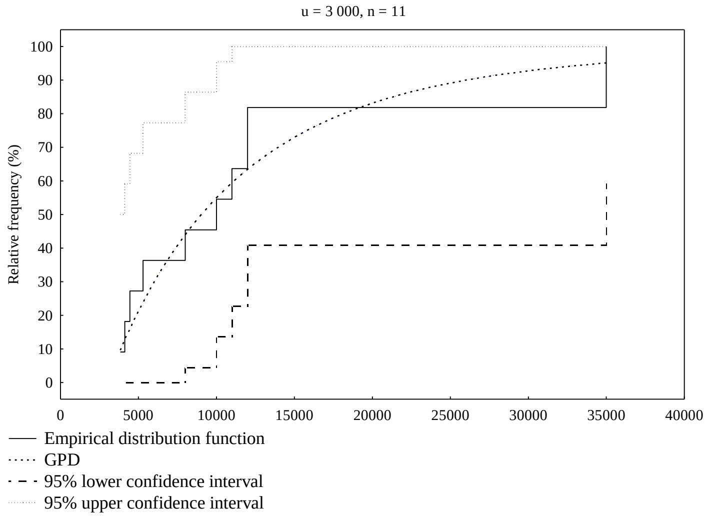


**Fig. 3.** GPD fitted to 11 exceedances of the threshold 3000.

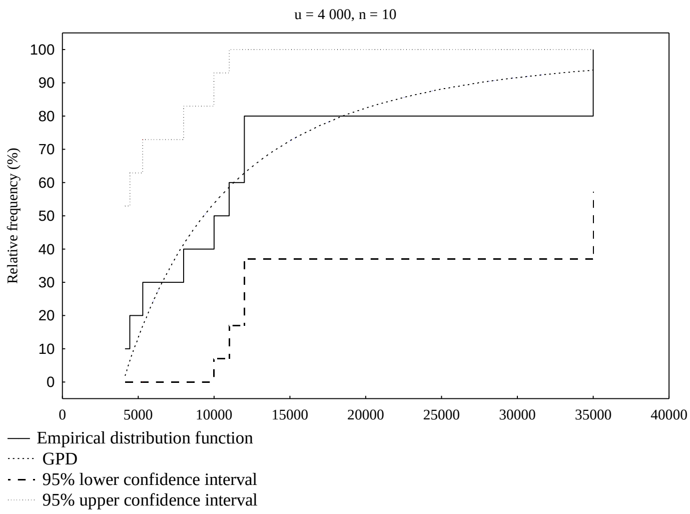


**Fig. 4.** GPD fitted to 10 exceedances of the threshold 4000.

We have fitted a generalized Pareto distribution using the maximum likelihood method for parameters estimation to the data above threshold of 3000 (Fig. 3) and above threshold of 4000 (Fig. 4).

These plots are useful for examining the distribution based on sample data. We have overlaid a theoretical CDF on the same plot with empirical distribution of the sample to compare them.

The stair lines on Fig. 3 and Fig. 4 show the empirical distribution functions of empirical data and the dashed lines present the theoretical CDF of the estimated generalized Pareto distributions for different thresholds. The dotted lines are the lower and upper bounds of the 95% confidence interval estimates of the CDF. It can be seen that the estimated parametric CDF falls inside the bands.

In Fig. 3 and Fig. 4 we see the good fit of both generalized Pareto distributions of insured losses on natural catastrophes.

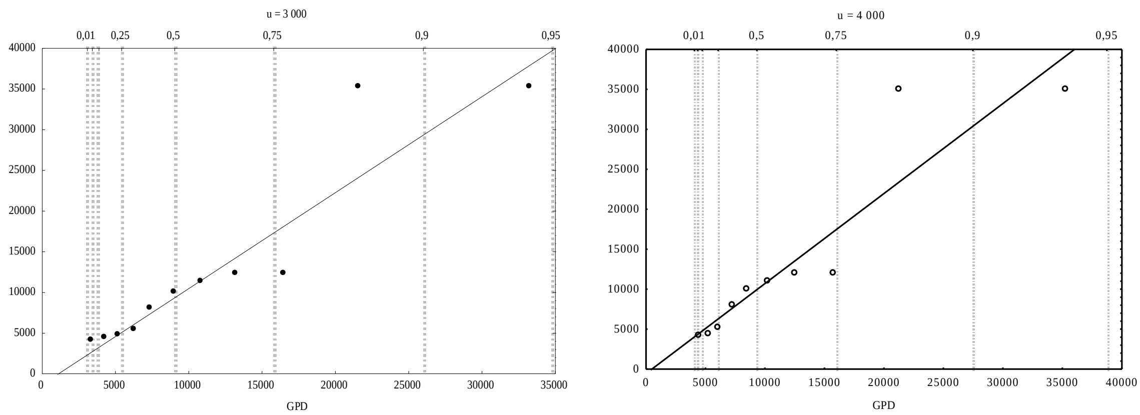


**Fig. 5.** QQ-plots against the GPD fitted to exceedances of the thresholds 3000 and 4000.

The QQ-plots (Fig. 5) against the generalized Pareto distributions there are another way to examine visually the hypothesis that the losses which exceed a very high threshold come from estimated distributions.

| | ***u* = 3 000** | ***u* = 4 000** |
|---|---|---|
| Parameter $\xi$ | 8 222.9790 | 6 849.8860 |
| Parameter $\tilde{\sigma}$ | -0.1636 | -0.3253 |
| *p*-value | 0.7901 | 0.8835 |

**Table 2** Comparison of estimated GPD for different thresholds.

Table 2 presents the parameters of the fitted generalized Pareto distributions on the data above the different thresholds. By *p*-values in this table we can state the best fit in the case of threshold $u$ = 4 000.

The publication Swiss Re Sigma No 2/2015 provides data about the 40 most costly insurance losses in time period 1970-2015. These data are the basis for continuing of our analysis. These values are ranging from 3410 to 78638 million USD in 2014 prices. We want to verify whether the 2-parameter Pareto distribution with cumulative distribution function defined by form

$$F_a(x) = p = 1 - \left(\frac{a}{x}\right)^b, \quad x \geq a \tag{3}$$

fits the data adequately by selecting Goodness-of-Fit Tests, analogously to the (Pacáková and Linda, 2009) or (Pacáková and Zapletal, 2014). The first step is parameters estimation by maximum likelihood method analogously to the (Pacáková and Gogola, 2013). The estimated parameters of the fitted distribution as the output from Statgraphics Centurion XV are shown in Table 3. By (3) estimated parameters are $a$ = 3410 and $b$ = 1.0478.

| **Shape (parameter *a*)** | **Lower thereshold (parameter *b*)** |
|---|---|
| 1.0478 | 3410 |

**Table 3** Parameters of fitted distribution for Pareto (2-Parameter).

| **DPLUS** | **DMINUS** | **DN** | ***p*-value** |
|---|---|---|---|
| 0.0576 | 0.0955 | 0.0955 | 0.8588 |

**Table 4** Results of Kolmogorov-Smirnov Test for Pareto (2-Parameter).

Table 4 shows the results of test run to determine whether the most costly insured losses can be adequately fit by a 2-parameter Pareto distribution (3). Since the smallest p-value = 0.8588 amongst the tests performed is greater than or equal to 0.05 we cannot reject the idea that losses comes from a 2-parameter Pareto distribution with 95% confidence.

Table 5 contains the selected quantiles of Pareto distribution, which is well fitted model for the most costly insured catastrophe losses. If will not change conditions of the occurrence of these events on the globe, will not change even their distribution. Then 50% of the most costly insurance losses in future will exceed 6 607.8 million USD, 10% will exceed 30 701.5 million USD, 1% will exceed 276 417 million USD.

| Lower Tail Area (<=) | Pareto (2-Parameter) |
|---|---|
| 0.50 | 6607.8 |
| 0.75 | 12804.5 |
| 0.90 | 30701.5 |
| 0.95 | 59492.8 |
| 0.99 | 276417.0 |

**Table 5** Quantiles of fitted Pareto distribution.

**Conclusion**

We have shown that fitting the generalized Pareto distribution to insured natural catastrophic losses which exceed high thresholds is a useful method for estimating the tails of loss severity distributions. This is not altogether surprising. As we have explained in part 2, the method has solid foundations in the mathematical theory of the behavior of extremes; it is not simply a question of ad hoc curve fitting.

The results of the analysis based on data of insured losses in the world natural catastrophes in time period 1970-2014 are alarming. Are justified concerns that the capacity of the world's insurance and reinsurance markets in the future will not be sufficient to cover these risks. It is high time for humanity to start emphatically remove the causes of the occurrence of catastrophes and their consequences.

The knowledge the probability models for prediction of consequences of catastrophe events allow to insurance or reinsurance companies to select the best options to cover these risks and correct setting premiums or reinsurance.

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**References**

Embrechts, P., Klüppelberg, C., & Mikosch, T. (1997). *Modelling extremal events for insurance and finance*. New York: Springer.

Fisher, R. A., & Tippett, L. H. (1928). Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Math. Proc. Camb. Phil. Soc. Mathematical Proceedings of the Cambridge Philosophical Society, 24*(02), 180.

Han, Z. (2003). *Actuarial modelling of extremal events using transformed generalized extreme value distributions and generalized pareto distributions.* (Unpublished doctoral dissertation).

Jindrová, P., & Jakubínský, R. (2015). Significance and possibilities of major accident insurance. *E + M Ekonomie a Management, 18*(4), 121-131.

Jindrová, P., & Sipková, Ľ. (2014). Statistical Tools for Modeling Claim Severity. In *Proceedings of the 11th International Scientific Conference European Financial Systems 2014* (pp. 288-294). Brno, Czech Republic: Masaryk University. Retrieved from http://is.muni.cz/do/econ/sborniky/2014/proceedings-EFS-2014.pdf

McNeil, A. (1997). Estimating the Tails of Loss Severity Distributions Using Extreme Value Theory. *ASTIN Bulletin, 27*(1), 117-138.

Natural catastrophes and man-made disasters in 2014, SIGMA No 2/2015, Swiss Re. (2015). Retrieved from http://media.swissre.com/documents/sigma2_2015_en_final.pdf

Pacáková, V., & Gogola, J. (2013). Pareto Distribution in Insurance and Reinsurance. In *Proceedings of the 9th International Scientific Conference Financial Management of Firms and Financial Institutions* (pp. 648-657). Ostrava, Czech Republic: VŠB - Technical university of Ostrava. Retrieved from http://www.ekf.vsb.cz/export/sites/ekf/frpfi/.content/galerie-dokumentu/final-2.pdf

Pacáková, V., & Linda, B. (2009). Simulations of Extreme Losses in Non-Life Insurance. *E + M Ekonomie a Management, 12*(4), 97-103.

Pacáková, V., & Zapletal, D. (2014). Effect of Reinsurance on the Collective Risk Model. *Managing and Modelling of Financial Risks* (pp. 590-597). Retrieved from https://www.ekf.vsb.cz/export/sites/ekf/rmfr/cs/sbornik/Sbornik_parts/Sbornik_II.dil.pdf

Sigma: Insurance research. (2015). Retrieved from http://www.swissre.com/sigma/

Skřivánková, V., & Tartaľová, A. (2008). Catastrophic Risk Management in Non-life Insurance. *E + M Ekonomie a Management, 11*(2), 65-72.