SCIENTIFIC PAPERS
OF THE UNIVERSITY OF PARDUBICE
Series B
The Jan Perner Transport Faculty
18 (2012)

LOSS OF STABILITY OF UNREINFORCED CONICAL SHELLS

Doubravka STŘEDOVÁ, Petr TOMEK

Department of Mechanics, Materials and Machine Parts

1. Introduction

Conical shells have practical applications in the field of process and power industry (conical roofs of small vertical storage tanks). Unfortunately, they are considerably sensitive to loss of stability. Available standards and recommendations [1], [2] provide engineers useful procedures of solving stability of the cones with the edge angle higher than 25 degrees. Standard methods are not applicable due to the geometry and boundary conditions of the examined shells. The rules should be used only for cones with boundary conditions BC1r and BC2f (clamped edges or edge with stiff ring). These rules do not cover shells with free edges (BC3) or with edges restrained using a light flexible ring, which represent most practical cases.

The area of designing conical shells with edge angle $\alpha_c < 25°$ is not yet adequately charted. It is not possible to use procedures described in standards because of the nonlinearity of the solving problem. Standard methods are based on linear theory of shells. The main aim of the article is to devise appropriate method of calculating the load carrying capacity of smooth conical shells with small camber loaded by external pressure with different types of boundary conditions – hinged (infinite radial stiffness) and simply supported (zero radial stiffness) lower edge. Solved range of dimension of *lower edge* *angle is* $\alpha_c = 5° \div 15°$. The procedure is based on sets of numerical analyses of load carrying capacity of conical shells under external pressure.

2. Theoretical background

The loss of stability is one of the limit states which can occur in an excessively loaded thin-wall structure. It is proved in the shell theory that the thin-walled structures can collapse in various shapes depending on geometrical parameters, boundary conditions, loading conditions, material characteristics and initial imperfections. The stability collapse is induced by minimum load corresponding to a particular form of deformation. The membrane strain energy is converted to both the membrane and bending strain energy. As the membrane stiffness of the shell structures is several orders higher than the bending stiffness, the loss of stability is attended by large displacements of a wave character often visible to the naked eye. Examples of loss of stability of excessively loaded thin-walled structures are shown in Fig. 1.

Fig. 1 *Practical examples of the limit state of the loss of stability*
Obr. 1 *Příklady mezního stavu boulení v praxi*

Load carrying capacity of conical shells according to ECCS

Method of design of conical shells described in this chapter is taken from ECCS (Buckling of Steel Shells, European Design Recommendations, [1]). This recommendation is very important document to design thin-walled structures. The part dedicated to design of conical shell is written by R. Greiner and C. Poggi.

Geometry of the conical shell is converted to osculating cylinder (Figure 4). Then is solved the load carrying capacity of a cylindrical shell with dimensions l_e and r_e (length and radius of the osculating cylinder).

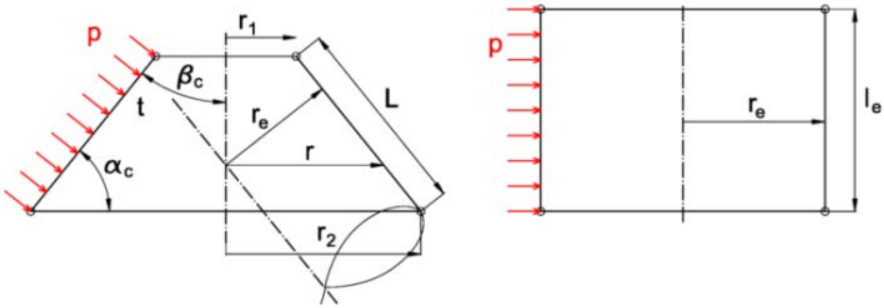

***Fig. 2** Converting of geometry of the conical shell loaded by external pressure*
***Obr. 2** Převod geometrie kuželové skořepiny na náhradní (oskulační) válec*

Length of the osculating cylinder loaded by external pressure is expressed by the relation

$$l_e = \min\left[L; \left(\frac{r_2}{\sin\beta_c}\right)(0{,}53 + 0{,}125\beta_c)\right] \quad (1)$$

If the length of the osculating cylinder is $l_e = \left(\frac{r_2}{\sin\beta_c}\right)(0{,}53 + 0{,}125\beta_c)$ than radius is

$$r_e = 0{,}71 r_2 \frac{1-0{,}1\beta_c}{\cos\beta_c} \quad (2)$$

Critical stress of the cylinder loaded by external pressure is expressed by the relation

$$\sigma_{\theta kr} = 0{,}92E\frac{t}{l_e}\sqrt{\frac{t}{r_e}} \quad (3)$$

Critical stress (critical elastic buckling stress in the circumferential direction) by the European recommendation express the relationship

$$\sigma_{\theta Rcr} = 0{,}92E\,\frac{C_\theta}{\omega}\frac{t}{r_e} \quad (4)$$

where through coefficient C_θ is taken into account the influence of geometry and boundary conditions on the critical stress. Dimensionless parameter ω depends on the specific geometry of the cylindrical shell. Value $\sigma_{\theta Rcr}$ is further modified by other coefficients taking into account the influence of the nonlinear behavior of the material, the influence of initial imperfections, etc. The resulting design load in the circumferential direction is dependent on the external pressure according to the formula

$$\sigma_{\theta Ed} = p\left(\frac{r_e}{t}\right) \quad (5)$$

Conical shells with small camber

Behavior of the excessively loaded conical shell with small angle α_c is considerably nonlinear. A possible explanation of this nonlinearity follows. In the process of the loading, the membrane meridian force F_x occurs in the wall of shell. This force grows in dependence on the size of the edge angle α_c according to relation $1/\sin\alpha_c$ (see Fig. 2). For $\alpha_c \to 0$ is theoretically $F_x \to \infty$. Meridian force causes both the vertical and horizontal (in radial direction) displacement. Horizontal displacement generates further increase of meridian force F_x. This process can lead to breaking the shell into the inverse position.

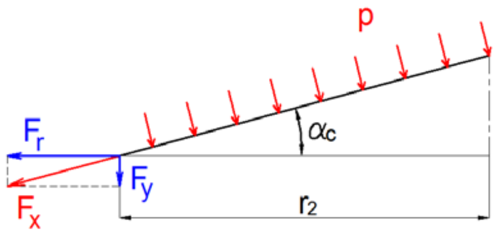

***Fig. 3** Forces acting in the location of the support of the conical shell*
***Obr. 3** Síly působící v místě uložení kuželové skořepiny*

Example of the collapse of the construction into its inverse position is shown in Fig. 4. The roof of the stadium in Minneapolis collapsed because of the weight of snow falling during the night storm.

***Fig. 4** Roof of the stadium in Minneapolis*
***Obr. 4** Střecha stadionu v Minneapolis*

The problem should be solved using numerical analysis taking into account geometrical nonlinearity. Furthermore, it is necessary to experimentally verify the results of the analyses. The main aim of this paper is to find a simple pseudo-analytical formula based on the critical stress equations (4) and (5)

$$p = K \cdot E \frac{1}{\omega} \left(\frac{t}{r_e} \right)^m \qquad (6)$$

where the coefficients K and m take into account the effect of the geometry and boundary conditions on the stability of the cone. Due to the significant complexity and breadth of the described problem would be very difficult and costly to investigate the stability of conical shells using only experiments. Using numerical analyzes it is possible to quickly and cheaply simulate a wide range of experiments. Nonlinear analyses GNA (geometrically nonlinear analysis) of the load carrying capacity of conical shells are performed in a computer program COSMOS/M based on the Finite Element method (FEM).

3. Unreinforced conical shells with small camber

The aim of this chapter is to suggest an approximate analytical method that could allow inexpensive and fast computational control of *stability of conical shells with lower edge angle* $\alpha_c = 5° \div 15°$ *with hinged lower edge*.

Numerical model

Scheme of the simply supported and hinged conical shell is shown in the Fig. 5. The numerical model (see Fig. 6) consists of four-node shell elements SHELL4.

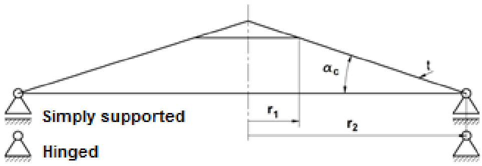

Fig. 5 *Scheme of the conical shell*
Obr. 5 *Schema kuželové skořepiny*

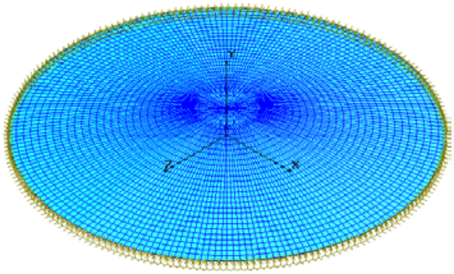

Fig. 6 *Numerical model*
Obr. 6 *Numerický model*

Conical shells with hinged lower edge

Dependence of limit pressure (results of analyzes GNA – point of the loss of stability) of hinged conical shells with edge angle $\alpha_c = 15°$ on the thickness parameter r_e/t is shown in Figure 7 (further graphs are listed in [4]).

It is possible to assemble the regression curves in the form of power functions due to the course of those dependencies

$$\boldsymbol{p} = \boldsymbol{K}' \cdot \left(\frac{\boldsymbol{t}}{\boldsymbol{r_e}}\right)^{\boldsymbol{m}} = \boldsymbol{K}' \cdot \left(\frac{\boldsymbol{r_e}}{\boldsymbol{t}}\right)^{-\boldsymbol{m}} \tag{7}$$

Where K' is the coefficient of power curves which already includes the effect of the material and geometry of the shell by means of the modulus of elasticity E and the dimensionless parameter ω

$$K' = K \cdot E \frac{1}{\omega} \tag{8}$$

The values of coefficients K' and m of regression curves of hinged conical shells are listed in Table 1.

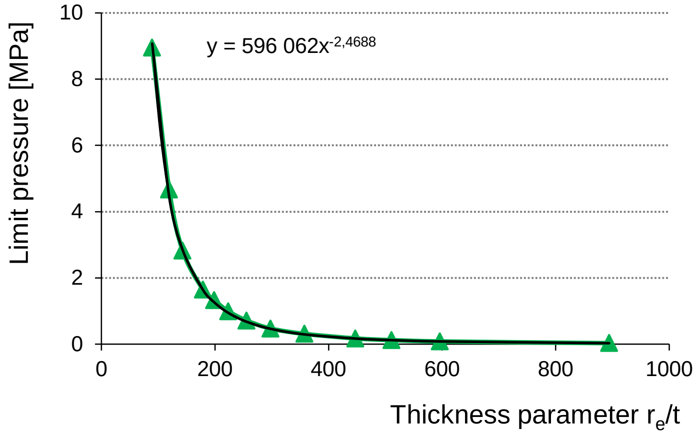

Fig. 7 *Dependence of the limit pressure on the thickness parameter - hinged conical shell with edge angle $\alpha_c = 15°$; regression equation displayed*

Obr. 7 *Závislost limitního přetlaku na parametru tenkostěnnosti – kloubově uložená kuželová skořepina s okrajovým úhlem $\alpha_c = 15°$; regresní rovnice zobrazena*

Table 1 *Values of coefficients of regression curves*

Tab. 1 *Koeficienty regresních křivek*

Boundary condition	Edge angle $\boldsymbol{\alpha_c}$ [°]	Range $\boldsymbol{r_e/t}$	Coefficients	
			$\boldsymbol{K'}$	$\boldsymbol{m}$
HINGED	5	260÷2080	204019	2,1796
	10	130÷1050	616499	2,4144
	15	90÷890	596062	2,4688

Coefficients of regression curves are valid only for specific values of examined edge angles. The value of limit pressure the size of the edge angle between values 5°, 10° and 15° could be linearly interpolated. In the following text are performed calculations of the limit pressure of general conical shells with edge angle $\alpha_c = 7.5°$, lower edge radius $r_2 = 2100\ \mathrm{mm}$ and wall thickness $t = 12\ \mathrm{mm}$. Limit pressure computed numerically is compared with the limit pressure obtained by linear interpolation between the values of limit pressures of conical shell with edge angles of 5° and 10° (see Figure 8) with the same thickness parameter.

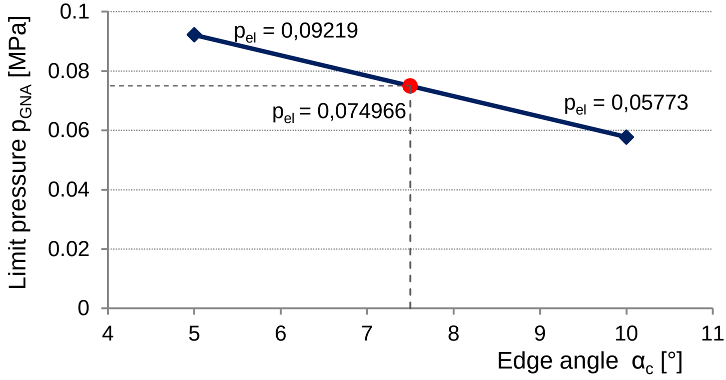

Fig. 8 *Linear interpolation of limit pressure of hinged conical shell*

Obr. 8 *Lineární interpolace limitního přetlaku kloubově uložené kuželové skořepiny*

Results of numerical analysis GNA and the calculated value of limit pressure are compared in table 2. Relative error of results is listed in the last row of the table (specified to the result of numerical analysis).

Table 2 *Comparison of results of calculated limit pressure*

Tab. 2 *Porovnání vypočítaných hodnot limitního přetlaku*

$p_{GNA}[MPa]$	$p_{el}[MPa]$
0,07481	0,074966
0,2%	

Conical shells with simply supported lower edge

Dependence of limit pressure (results of analyzes GNA – point of the loss of stability) of hinged conical shells with edge angle $\alpha_c = 15°$ on the thickness parameter r_e/t is shown in Figure 9 (further graphs are listed in [4]).

The values of coefficients K' and m of regression curves of simply supported conical shells are listed in Table 3.

Table 3 *Values of coefficients of regression curves*

Tab. 3 *Koeficienty regresních křivek*

Boundary condition	Edge angle α_c [°]	Range r_e/t	Coefficients	
			K'	m
SIMPLY SUPPORTED	5	260÷2080	104300	2,2175
	10	130÷1050	91350	2,2499
	15	90÷890	91858	2,28

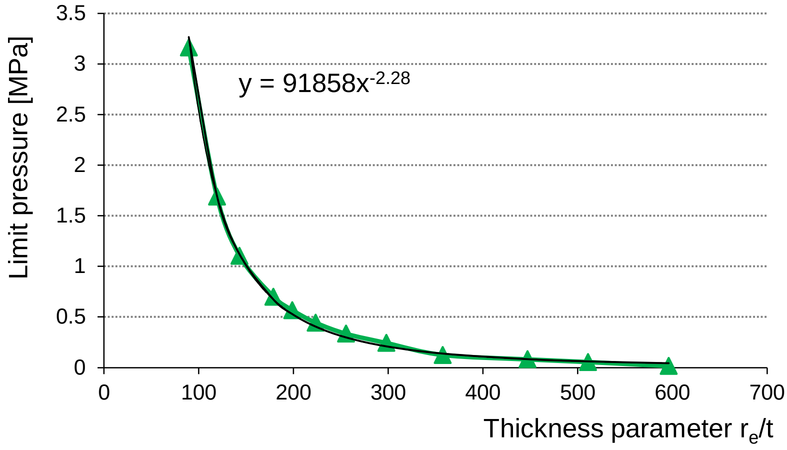

Fig. 9 *Dependence of the limit pressure on the thickness parameter – simply supported conical shell with edge angle* $\alpha_c = 15°$*; regression equation displayed*

Obr. 9 *Závislost limitního přetlaku na parametru tenkostěnnosti – prostě podepřená kuželová skořepina s okrajovým úhlem* $\alpha_c = 15°$*; regresní rovnice zobrazena*

Coefficients of regression curves are valid only for specific values of examined edge angles. The value of limit pressure the size of the edge angle between values 5°, 10° and 15° could be linearly interpolated. In the following text are performed calculations of the limit pressure of general conical shells with edge angle $\alpha_c = 7.5°$, lower edge radius $r_2 = 2100\ \mathrm{mm}$ and wall thickness $t = 12\ \mathrm{mm}$. Limit pressure computed numerically is compared with the limit pressure obtained by linear interpolation between the values of limit pressures of conical shell with edge angles of 5° and 10° (see Figure 8) with the same thickness parameter.

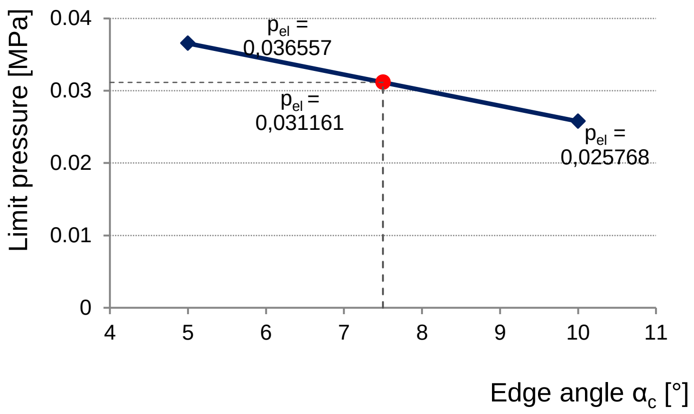

Fig. 10 *Linear interpolation of limit pressure of simply supported conical shell*

Obr. 10 *Lineární interpolace limitního přetlaku prostě podepřené kuželové skořepiny*

Results of numerical analysis GNA and the calculated value of limit pressure are compared in table 4. Relative error of results is listed in the last row of the table (specified to the result of numerical analysis).

***Table 4** Comparison of results of calculated limit pressure*

***Tab. 4** Porovnání vypočítaných hodnot limitního přetlaku*

$p_{GNA}[MPa]$	$p_{el}[MPa]$
0,03215	0,031161
3,07%	

4.Experiment

Validity of the method must be verified experimentally. In this chapter are introduced experiment results of loss of stability of simply supported conical shells. Test specimens (Fig. 11) are made of ordinary structural steel. Test equipment (Fig. 12) is made of tube of external diameter 273 mm and wall thickness of *7.5* mm. The flange (outer diameter *345* mm and thickness *18* mm) is welded to the tube. The rubber gasket with a thickness of *5* mm is glued on the flange. From the other side, the tube is fitted with a lid. Using a vacuum pump air is creating a vacuum inside the tube (external pressure). Gauge pressure is read on the scale of an analog manometer until the limit pressure is reached.

***Fig. 11** Test specimens*

***Obr. 11** Zkušební vzorky*

***Fig. 12** Test specimen after experiment*

***Obr. 12** Vzorek po provedeném experimentu*

The experimental results of stability loss of real samples are compared with the results of numerical analyzes GMNA which considered the influence of elastic-plastic behavior of material in Table 5. Test specimens 1÷8 are freely placed on the rubber seal. Test specimen No. 9 was a little bit smaller due to the manufacturing inaccuracies. This sample was placed directly on the steel flange. This placement represents the boundary condition of simple support.

Table 5 *Comparison of results of experiment and numerical analysis*
Tab. 5 *Porovnání výsledků experimentu a numerické analýzy*

No.	Edge angle $\alpha_c[°]$	Limit pressure $p_{GMNA}[MPa]$	Limit pressure $p_{exp}[MPa]$	Relative error $\delta[\%]$
1	10	0,03195	0,022	31,1
2			0,024	24,9
3			0,024	24,9
4			0,025	21,8
5			0,026	18,6
6	15	0,04485	0,039	19,8
7			0,040	16,8
8			0,041	14,0
9		0,04672	0,046	1,6

The aim of the first series of experiments was to verify the production technology of test specimens and the functionality of the equipment. Quite large relative error is most likely attributable to manufacturing inaccuracies and to the way of placing of conical shell. Experimental results show that the production of test specimens using welding technology is not quite appropriate. Other test samples will be manufactured with the technology of pressing.

5. Conclusion

Method of solving the elastic limit pressure of conical shells with edge angle of the range $\alpha_c = 5 \div 15°$ with two extreme types of boundary conditions is presented in this paper. This method is based on the formula of critical stress of osculating cylinder (4). Formula for the critical pressure (7) is supplemented by coefficients (K, m) which reflected conical shell geometry (edge angle α_c).

$p_{el} = K \cdot E \frac{1}{\omega}\left(\frac{t}{r_e}\right)^m$, resp. $p_{el} = K' \cdot \left(\frac{t}{r_e}\right)^m$

Coefficients K', m are determined from regression curves of limit pressure dependence on the thickness parameter (see Figure 6). After substituting the coefficients into the equation of limit pressure is calculated elastic limit pressure. Using similarity criteria (dimensionless thickness parameter r_e/t) can be calculated limit pressure conical shells with arbitrary dimensions within the studied range. The attained results are presented in the form of clear graphs and tables. Using similarity criteria (dimensionless thickness parameter r_e/t) can be calculated elastic limit pressure of conical shells with arbitrary dimensions (within the studied range).

The results achieved in the paper (after the proper opponency and supplements) could be a useful tool in the hands of ordinary designer. At present, must be conical shell construction with a small camber solved numerically. The possibility of using normative relations for the design comes much cheaper. The proposed method is quite simple and has the same physical basis as the calculation of load capacity specified in the standards and recommendations. Calculation of load capacity is based on the well-known formula for the critical stress of cylinder.

References

1. ECCS TC8 TWG 8.4 Buckling of Steel Shells. European Design Recommendations. 5th Edition. Published by ECCS. (2008). ISBN: 92-9147-000-92.
2. ČSN EN 1993-1-6. Navrhování ocelových konstrukcí. *Pevnost a stabilita skořepinových* konstrukcí. Český normalizační institute. (2008).
3. FEM Computer program COSMOS/M, Version 2.95 by SRAC (Structural Research and Corporation), Los Angeles, California.
4. Paščenko, P.: „Non-linear Behavior of Conical Roofs Stiffened by Meridional Stringers", International Conference on Carrying Capacity of Steel Shell Structures, Under the auspices of ECCS, Brno, 1997, 106-111.
5. Paščenko P.: „Stabilita tenkostěnných skořepinových konstrukcí v dopravní technice", Habilitační práce, Univerzita Pardubice, Dopravní fakulta Jana Pernera, 2009.
6. STŘEDOVÁ D.: *Stabilitní prolomení kuželových skořepin s malým vzepětím*. Dizertační práce. Univerzita Pardubice. Pardubice (2012).

Předloženo: 31. 5. 2013

Resumé

ZTRÁTA STABILITY NEVYZTUŽENÝCH KUŽELOVÝCH SKOŘEPIN

Doubravka STŘEDOVÁ, Petr TOMEK

Cílem článku je představení analytické metody výpočtu elastického limitního přetlaku kuželových skořepin s malým vzepětím. Zkoumaný rozsah okrajového úhlu je $\alpha_c = 5° \div 15°$. Tato jednoduchá metoda výpočtu umožňuje levnou a rychlou kontrolu stabilitní únosnosti nevyztužených kuželových skořepin. Vzhledem ke geometrii a podmínkám uložení zkoumaných skořepin není možné použít standardizované metody výpočtu uvedené v normách a doporučeních (Evropské doporučení ECCS [1]; ČSN EN 1993-1-6 [2]) není možné použít. Navržená metoda je založena na výsledcích řady numerických analýz stabilitní únosnosti zkoumaných skořepin. Numerické analýzy jsou provedeny v počítačovém programu založeném na metodě konečných prvků COSMOS/M [3].

Summary

LOSS OF STABILITY OF UNREINFORCED CONICAL SHELLS

Doubravka STŘEDOVÁ, Petr TOMEK

The aim of this paper is to suggest an approximate analytical method that could allow inexpensive and fast computational control of stability of unreinforced conical shells with lower edge angle α_c=5°÷15°. Standard methods of stability calculation according to European Recommendation ECCS [1] or ČSN EN 1993-1-6 [2] are not applicable due to the geometry and boundary condition of the examined shells. Approximate method is based on the results of a set of numerical analyzes of load carrying capacity of examined structures. Numerical analyses are performed by FEM computer program COSMOS/M [3].

Zusammenfassung

STABILITÄTSVERLUST UNVERSTÄRKTEN KEGELSCHALEN

Doubravka STŘEDOVÁ, Petr TOMEK

Das Ziel dieser Arbeit ist, eine ungefähre analytische Methode, die kostengünstig und schnelle rechnerische Kontrolle der Stabilität der unverstärkten Kegelschalen mit Unterkante Winkel α_c=5°÷15° ermöglichen könnte vorschlagen. Standard Methoden der Berechnung der Stabilität nach europäischen Empfehlung ECCS [1] oder ČSN EN 1993-1-6 [2] nicht anwendbar sind aufgrund der Geometrie und Randbedingungen der untersuchten Schalen. Ungefähre Verfahren ist basiert auf den Ergebnissen von einer Reihe von numerischen Analysen der Tragfähigkeit der untersuchten Strukturen. Numerische Analysen werden durch FEM Computerprogramm COSMOS/M [3] durchgeführt.