EVALUATION OF TRANSIT LINES PLAN

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Abstract: The plan of mass transport or transit lines is a vital part of an organisation of public transportation services. An optimisation of the plan can improve the service either by reducing the operational costs or by improving of service quality. Three subjects interact in the problem namely state or local administration, transport companies and citizens. The plan optimisation should find the best compromise to satisfy interests of all actors. A relevant criterion must be defined to find the best solution. Calculating passenger criterion values must include model of passenger’s decision.

Keywords: Transportation planning, Transit lines, Costs criteria, Service quality, Optimization.

JEL Classification: R41, C61.

Introduction

Mass transportation or transit services carry passengers to a desired destination. An offer of transit services helps to reduce individual use of private cars and thus improve traffic in towns and their environments. The services must be offered for a reasonable price frequently under real operational costs of service providers and thus they must be subsidised to be attractive for passengers.

1 Statement of a problem

Transit lines optimisation helps to achieve the mentioned goals, namely an offer of quality services at acceptable costs. Basic criteria for evaluating quality of public transportation services are defined in European standard EN 13816 or Slovak standard STN EN 13816. Basic 7 criteria-principles mentioned in the standards are: Accessibility, Information, Time, Customer care, Comfort, Security, Environmental impact. Most of them can not be directly respected in the line plan optimisation as they can be hardly quantified to fit into an optimization model. So only time factors seem to be suitable for use in an optimization process as discussed in the following text.

An optimisation of transit services should find the best equilibrium among the interests of all participating parties, namely:

• Governmental or local administration which subsidises transit services,
• service providers who run the services and try to gain some profits,
• passengers who desire to have affordable and quality services.

The administration bodies try to minimise their involvement and to pay as little as possible to subsidise the transit system. The optimisation model of transit services can
suppose that a fixed contribution is agreed on and available from administration bodies.

Under such an assumption only two parties remain and an optimal solution should respect relevant interests of passengers and service providers. Passengers demand quality service, which can be measured by a total travel time and a service comfort. The travel time (which includes waiting for a vehicle, ride in a vehicle and changing vehicles) should be preferably short and comfort of the offered services should be as good as possible (comfort of the ride in a vehicle, walking to the next bus stop or railway station, changing vehicles or transportation modes etc.)

The better a service quality is the higher costs will arise to providers ensuring the services. So there is a contradiction between interests of passengers and service providers. The passengers are interested especially in:

- Travel costs which is pre-eminently given by pricing policy and by real operation costs of providers;
- travel time;
- comfort.

Service providers try to gain some profits which they can achieve by reducing their operational and fixed costs or in other words by minimisation of a total distance driven by vehicles and their crews (what reduces fixed costs as well - less distance driven equals less busses or trains needed at a time, less staff, etc…).

The discussed interests of service providers and passengers can be formulated in a simple model shown in the next paragraph. A relevant criterion for an optimisation of a transit system is then a total distance covered by vehicle rides or a travelled distance and time spent by passengers.

In principle, we have to determine the overall market size, based on characteristics of the population and the service. We then assume a launch date and model the diffusion of the service to determine the shape of logistic curve by which saturation level is reached. The diffusion model begins by identifying the total potential market for a service - those users that could potentially be interested in the service, if conditions (prices, network size, etc.) were suitable. From there, we implement a two-stage model showing how members of the potential market becoming aware of the new product, and then how those who are aware decide whether or not to subscribe. The decision to become a subscriber is a result of comparing the benefits and costs of the service.

2 Methods

The optimisation of transit lines is an NP hard optimisation problem. There are many known approaches and optimisation models such as models presented in [2], [4], [5], [6]. Many of them are based on a network design model defined as follows.

Transportation infrastructure is represented as a graph $G$ consisting of a set of nodes $V$, a set of edges $H$ and edge costs $c$

$$G = (V, H, c)$$
Nodes describe stops in a transit system (bus stops, railway stations etc.) and edges stand for a transport service between origin and destination nodes.

Let us name

\[(i, j) \in H\] – an edge from node \(i\) to node \(j\), \(H\) is a set of all feasible edges,

\[(r, s) \in Q\] – passenger flow from node \(r\) to node \(s\) and \(Q\) is a set of all passenger flows,

\(q_{rs}\) – intensity of flow \((r, s)\) characterised by a number of passengers travelling during a time period,

\(f_{ij}\) – fixed costs of creating an edge \((i, j)\) which means costs of vehicle operations on line \((i, j)\),

\(c_{ij}\) – passenger costs for travelling along an edge \((i, j)\), frequently can be substituted by cost \(c_{ij}\) independent origin and destination of passenger’s travel,

\(y_{ij}\) – binary decision variable for creating an edge, which means edge \((i, j)\) is created \((y_{ij}=1)\) or is not \((y_{ij}=0)\),

\(x_{ij}^{rs} \in \{0,1\}\) for all edges \((i, j) \in H\) and flows \((r, s) \in Q\) are binary decision variables signifying that the edge \((i, j)\) (in other words transport service between nodes \(i\) and \(j\)) is \((for\ x_{ij}^{rs}=1)\) or is not \((for\ x_{ij}^{rs}=0)\) used for transportation of a passenger flow \((r, s)\).

The optimisation model for the network design problem can be formulated as follows:

Costs function

\[
\min \sum_{(i,j) \in H} f_{ij} \cdot y_{ij} + \sum_{(r,s) \in Q} q_{rs} \cdot \sum_{(i,j) \in H} x_{ij}^{rs} \cdot c_{ij}^{rs} \tag{1}
\]

subject to constraints:

\[
x_{ij}^{rs} \leq y_{ij} \quad \text{for} \quad (i, j) \in H, \quad (r, s) \in Q \tag{2}
\]

\[
\sum_{(i,k) \in H} x_{ik}^{rs} - \sum_{(k,j) \in H} x_{kj}^{rs} = \begin{cases} 
-1 & \text{for} \ k = r \\
1 & \text{for} \ k = s \\
0 & \text{for} \ k \neq r \ a \ k \neq s 
\end{cases} \quad \text{for} \ (r, s) \in Q \ \text{and} \ k \in V \tag{3}
\]

\[
x_{ij}^{rs} \in \{0,1\} \quad \text{for} \ (i, j) \in H, \ (r, s) \in Q \tag{4}
\]

\[
y_{ij} \in \{0,1\} \quad \text{for} \ (i, j) \in H \tag{5}
\]

The costs function consists of two parts where the first term stands for operating costs of service providers and the second one for passenger costs. If passenger interests are neglected the first term only is significant and corresponding optimisation model is known as travelling salesman problem. If only passenger costs are respected the optimisation will find shortest paths in a complete set of feasible edges (in the graph \(G\)) for every passenger. A suitable compromise between interests of providers and passengers must be found in real life.
2.1 Provider costs

The function of provider costs can be discussed in detail now. The first term of the costs function (1) stands for operational costs, which must be paid by a service provider. A more detailed expression can define the operational costs as

\[ f_{ij} = \sum (d_{ij} \cdot p_{ij}) \quad \text{for all } (i, j) \in H \]

\[ d_{ij} \] — costs of one ride along line \((i, j)\),

\[ p_{ij} \] — number of line trips \((i, j)\).

Variable \(y_{ij}\) can be in fact omitted and replaced only by variable \(p_{ij}\), which will be set to zero if the line is not chosen and so no vehicle run will serve it in a transit service plan.

Distances or costs \(d_{ij}\) depend mostly on the length of the line and are well known at the moment of a design of a set of feasible lines. The problem arises with number of rides \(p_{ij}\). The service frequency on a line is determined by several factors as:

- expected (predicted) number of passengers known from an O-D matrix,
- minimum frequency of a service estimated by enforced standards,
- expected (desired) occupancy of vehicles.

A significant criterion is needed to evaluate a quality of a transit lines plan. For a comparison of a new plan quality against the old one currently in use the best way would be to suppose that the number of runs on a line ensures the same quality of service at all bus stops or railway stations as is currently offered. Further optimisation can estimate a proper number of line runs and their departure times so that a better efficiency of the system is attained and operational costs are lowered or service quality is improved.

2.2 Passenger costs

Passenger’s costs are formulated in the second term of the costs function (1). A more detailed expression can define the passenger’s costs as

\[ c_{ij} = X(t_{ij} + t_p) + Yl + cp_{ij} \quad \text{for all } (i, j) \in H \]

\[ t_{ij} \] — in vehicle time \((i, j)\),

\[ t_p \] — access time \((i, j)\),

\[ cp_{ij} \] — price of a ride along line \((i, j)\),

\(l\) — quantification of comfort

\(X\) — weight of time parameters in the formula

\(Y\) — weight of comfort parameter in formula

Passenger costs are rather difficult to evaluate relevantly, because of many factors included. This part of costs function is actually determining, whether a passenger will choose to ride public transportation or not. All of above mentioned parameters are relevant in deciding whether to ride public transportation or not, but there are many parameters that can be hardly included in a costs function formula such as:

- Current weather condition
• outside temperature
• what time of year it is (summer holiday, winter, spring, autumn….)
• quality of actual ride (comfort of seats, comfort of vehicle and driving)
• distance from home to bus stop or station
• distance from final bus stop or station to destination
• total time of the whole trip
• social and economical status of a passenger
• local customs of passengers etc.

Relevancy of mentioned factors is indisputable, however determining weights of time and comfort parameters along with weights of all other mentioned factors not included in formula would demand a serious analysis by itself. Despite of all the efforts the analysis may bring no valuable results while the final decision on choice of a transportation mode is still taken by a human being. Passenger decisions which depend on actual plan of transit line and other factors should also affect provider costs in costs formula, because it determines number of passengers actually riding a vehicle.

3 Problem solving

Relevant decision criteria and description of passenger’s behaviour should result from the discussed costs analysis.

3.1 Relevant criteria

Evaluating quality of line plan is the same problem as a value estimation of costs function (1), however passenger flows as input data to the above mentioned optimisation model depend on many parameters mentioned in chapter 4. In other words passenger flows depend on a designed plan of transit lines and transit line design is done using data on passenger flows what is a circular dependency which leads to a completely different view on a problem. Costs function value of a solution (designed plan) represents costs assuming that all passengers actually use the public transportation for their whole trip from origin to destination. In reality a lot of potential passengers may switch to another type of transport if the offered services do not comply with their needs. In this point provider income and costs may differ from model results. The correct way to design a line plan would be to design a line plan based on input data from current system, then to implement the designed plan in real traffic, and after stabilisation of passenger flows to collect data on passenger flows and make a new design based on these new data. Designed plan must be evaluated using criteria from both passenger and provider’s point of view:

From passenger’s point of view:
• Total ride time,
• total distance,
• total count of line transfers.

From provider’s point of view:
• Total number of rides,
• total distance driven.

**Total ride time** is a sum of time that passengers spend in a mean of mass transportation or waiting for a mean of mass transportation what reflects a speed of transportation. The only problematic point in evaluation of the criteria is a determination of waiting times for transfers. Number of line trips for every line can be estimated but also departure times of individual trips and intensities of passenger flows during the hours of a day are necessary for a precise calculation of waiting times at transfers. Waiting time can be substituted by a constant value that will not suppress relevance of actual travelling time and also will not become irrelevantly small.

**Total distance** is a sum of all distances that all passengers travelled in means of transport. This value can be calculated by choosing certain decision strategy for transportation route for every passenger and calculating route’s distance. This parameter is equivalent with previous one excluding waiting time.

**Total count of transfers** is sum of all transfers of all passengers as getting to their destination using designed plan and certain decision strategy. This parameter describes comfort factor of the designed line plan.

**Total number of rides** is determined by count of runs per day of every designed line. This number can be set only by statistical research, but it cannot be precisely determined while its count is being influenced by many factors where some are not even quantifiable.

**Total distance driven** is a parameter derived from **total number of rides** and there are same problems determining the exact value.

### 3.2 Model of passenger’s behaviour

Usual input for evaluating transit line plan includes OD matrix, which consists of numbers of passengers for certain period of time that request to be transported among each pair of nodes in a transportation network. Proposed line plan includes line tracks and their count for one day or for any other defined time period. In this point we can shrink the whole problem of determining evaluation criteria values to a problem of modelling passenger’s behaviour.

It is influenced by many factors as:

• Weather (for example we can assume, that during cold or rainy days passenger chooses to wait less on the stop, so the earlier connection is chosen even if it is longer);

• season of the year (similar to weather factor);

• time of day (there are less available connections during night hours, so we can expect strategy „pick first admissible“);

• social and economical status (wealthy passenger chooses higher standard connection, poor (underprivileged) passengers choose cheapest possibility);
• mass transportation comfort level (comfort of vehicle or comfort of certain track can determine passengers decision – vehicle has a restroom, chosen connection’s track is less curvy, etc...);
• speed (chosen connection is faster then other possibility);
• transportation type;
• safety;
• origin and destination stop location;
• track (some track alternatives give nicer views than others);
• day of week;
• waiting time and others.

Combination of all above mentioned factors yields a final decision determining which connection alternative will be chosen to serve a transportation request. Influence factors have an impact on decision whether transportation request will be served by means of mass transportation or not.

Most of above mentioned factors are hardly quantifiable, and can be estimated by statistic methods using observation results of passenger’s behavior in real life situations.

Amount of impact on making a decision by particular factor can be generalized for a group of passengers which have social status, living area, work area, economical status, etc. in common.

Also a set of admissible connections must be determined, what can be done by several methods:
• First n possibilities (fixed number)
• first n% possibilities (percentage of all admissible)
• first n or n% quickest
• connections within time interval (all connections during first n minutes).

Passenger’s decision fulfills following conditions of a discrete choice, namely
• Alternatives are mutually exclusive,
• alternatives are exhaustive
• the number of alternatives is finite.

A very simple decision model can be formulated as choosing a connection with the maximum probability. Connection i will be chosen if \( p_i \) satisfies condition

\[
p_i = \max_{k \in K} (p_k)
\]  

(6)

where

\( K \) – set of available connection alternatives and

\( p_k \) - probability of choosing particular connection \( k \) (connections set must include individual transport, too, not only mass transportation alternatives).
Another simple decision rule distributes passengers to connections proportionally to probabilities \( p_i \). It means from a total number of \( n \) passengers just \( p_i \cdot n \) passengers will use connection \( i \).

A vital step in the decision process is a correct estimation of probabilities \( p_i \). Probability of choosing connection \( i \) can be further estimated as

\[
p_i = \sum_{j \in F} w_j f_{ij}
\]

(7)

where

- \( F \) – set of all impact factors influencing final decision
- \( f_{ij} \) - probability, that connection \( i \) will be chosen based on impact factor \( j \).
- \( w_j \) - weight of impact factor \( j \)

Presuming a passenger has to choose one of available alternatives there must hold

\[
\sum_{j \in F} \sum_{i \in K} f_{ij} = 1
\]

\[
f_{ij} \in < 0,1 >
\]

\[
\sum_{j \in F} w_j \leq 1
\]

\[
w_j \in (0,1)
\]

Above mentioned model is a type of discrete choice model and values \( f_{ij} \) will be modelled using a suitable logit or probit model (see [1]). Even if the model is known, there is a major problem how to acquire necessary input data, and that is why simplified approaches are chosen frequently.

**Conclusion**

As discussed in previous text, decision of potential passengers whether or not to ride public transportation depends on many factors, where some of them are hard to be included into costs function. Passenger’s decision making model must be solved to get estimated decision for calculating criterion value from relevant flow intensities on individual transit lines. An optimisation problem stated in (1) – (5) can be solved using a commercial integer programming solvers (e.g. XPRESS). The problem is NP hard and difficult to be solved so a heuristic approach (as described for example in [4]) can be used to get an acceptable solution in a reasonable time. Anyhow, the model is based on a rough and uncertain estimations of passenger’s behaviour discussed in paragraph 3.2 and so an aproximative heuristic solution seams to deliver appropriate results.

Designing a transit line plan should be periodically repeated and the computed results compared with real passenger’s behaviour. The continuous design process will then consist of analysing currently operated mass transport system and taking steps for its improvement. Every change of the system invokes response of passengers what
needs some time to stabilize whole system to a new status to be analyzed. Passenger decisions must be evaluated after every iteration, while total ride time comes out as the only relevant criteria correctly representing quality of the designed route plan.

Acknowledgement

This contribution was supported by the Scientific Grant Agency of the Ministry of Education of the Slovak Republic and the Academy of Sciences under project VEGA 1/0296/12 “Public service systems with fair access to service”

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Received: 01. 05. 2012
Reviewed: 09. 08. 2012
Approved for publication: 01. 11. 2012