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ON THE DRIVE DYNAMICS OF A 4WS-CAR

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1. Introduction

Nowadays the "by-Wire" systems assert in vehicle construction. These systems are free from mechanical connection between operating components (steering wheel) and elements under control (front wheels).

In comparison with common mechanical steering assembly it is not so difficult to build the Steer-by-Wire (SbW) into a car. (See figures below.) This is one of the qualities making possible to revive the idea to drive not only through front wheels but also through back wheels (Four-Wheel Steering, 4WS). In this way, stability of vehicle can be markedly improved in critical situations.

Systems for active turning of back wheels are in first-generation 4WS-cars complicated extensions of common steering assemblies. Via synthesis of SbW and 4WS qualitatively new system will be created. However, to exploit its possibilities further intensive research will be needed.

2. Mathematical model

To obtain a fundamental idea about behaviour of a 4WS-car the abstract plane one-track model (sometimes called "bicycle", Fig. 2-1) can be used.

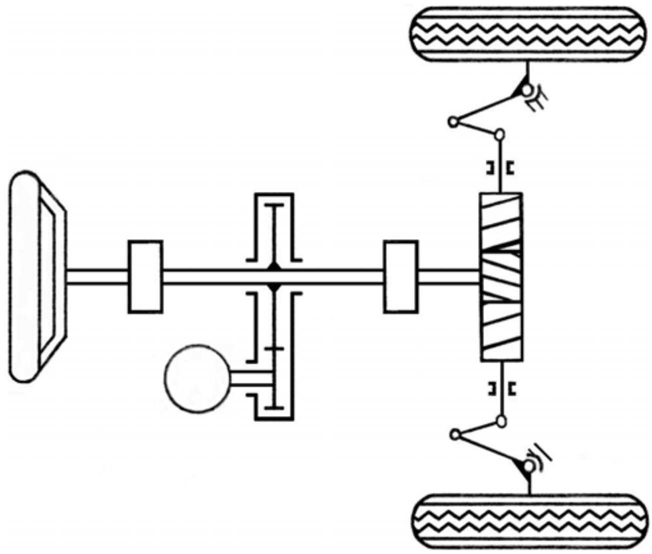


Fig. 1-1 *Common mechanical steering assembly with electrical assistance*

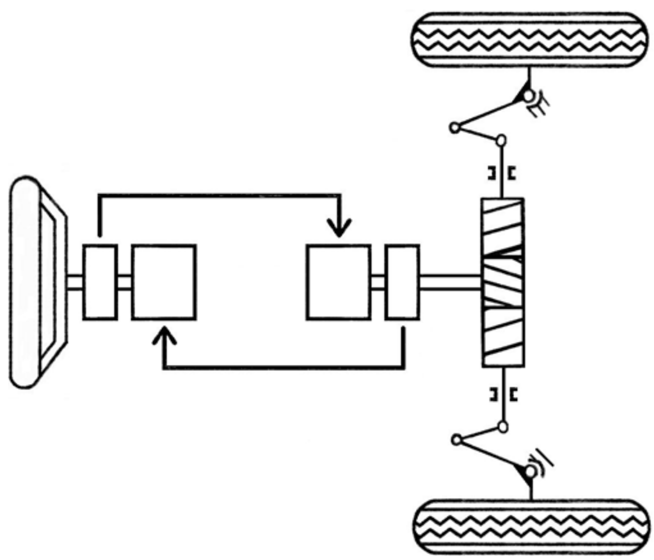


Fig. 1-2 *Information flow in Steer-by-Wire*

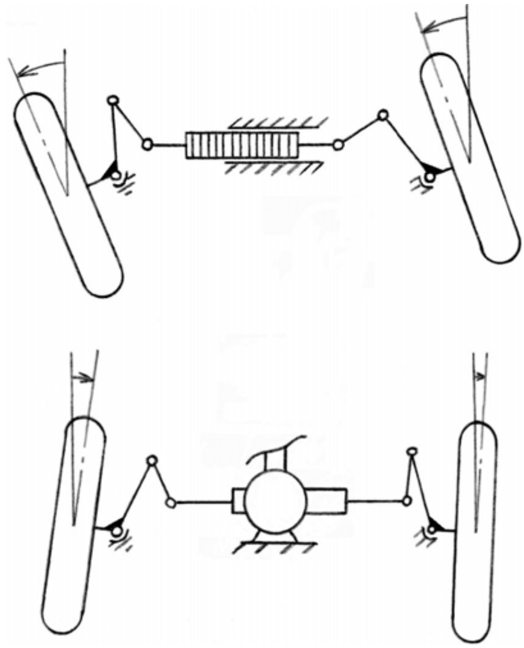


Fig. 1-3 *A first-generation 4WS-car*
(back Wheels controlled by an electric motor)

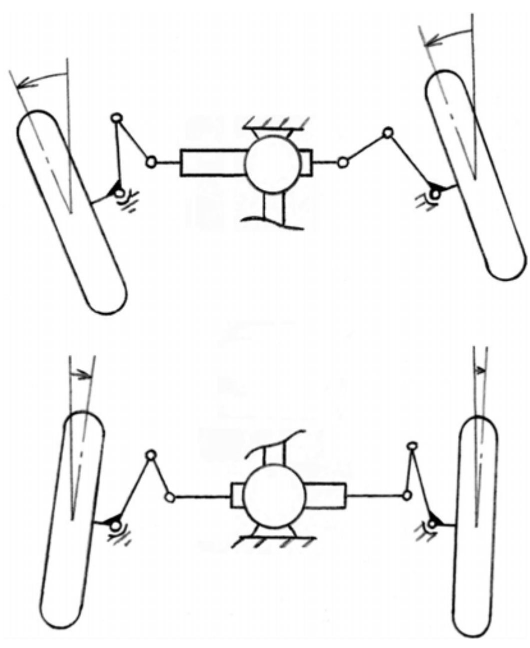


Fig. 1-4 *A 4WS-car with Steer-by-Wire*

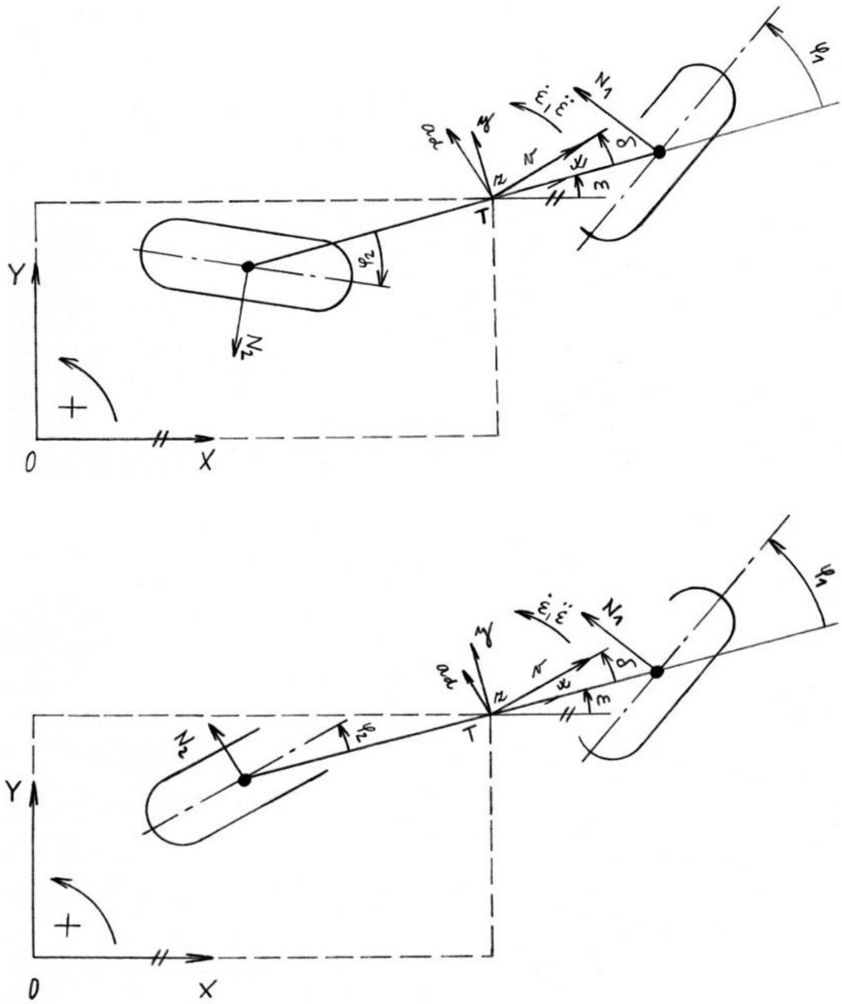


Fig. 2-1 *One-track model, to the equations of motion*

A 4WS-car can steer with its wheels in two ways. Back wheels can be *turned* opposite to front wheels in *converse* (mode OP) or in *same* (mode ST) sense.

Wheels of the model (to differentiate the model from a real car big initial letter is used in text) can be as turned as wheels of a 4WS-car.

Hereafter equations describing lateral motion and yaw rotation of the model at turning at drive with constant nonzero running velocity v are written.

$$\left[mv + \left(k_1 l_1 - k_2 l_2\right)/v\right]\dot{\varepsilon} + mv\dot{\delta} + \left(k_1 + k_2\right)\delta = k_1\varphi_1 \mp k_2\varphi_2 \, , \tag{2-1}$$

$$J_z\ddot{\varepsilon} + \left[\left(k_1 l_1^2 + k_2 l_2^2\right)/v\right]\dot{\varepsilon} + \left(k_1 l_1 - k_2 l_2\right)\delta = k_1 l_1 \varphi_1 \pm k_2 l_2 \varphi_2 \, . \tag{2-2}$$

Values of the model parameters used at computations are written in Tab. 2-1.

Model parameter (*symbol*, physical unit)	Value
mass (m, kg)	$2{,}4.10^3$
distance between the centre of gravity T and rotation axis of the front Wheel (l_1, m)	1,2
distance between the centre of gravity T and rotation axis of the back Wheel (l_2, m)	1,8
moment of inertia about the z-axis (J_z, kg.m^2)	$3{,}8.10^3$
directional stiffness of the front Wheel (k_1, N/rad)	$4{,}7.10^4$
directional stiffness of the back Wheel (k_2, N/rad)	$5{,}2.10^4$
maximum front Wheel angle (φ_{1m}, ˚)	30

Tab. 2-1 *Model parameters*

3. Studied problems

Given dependences of the back Wheel angle on the front Wheel angle (Problem I)

The first problem was behaviour of the model at steering with its Wheels in modes OP, ST under *given* dependences of the back Wheel angle on the front Wheel angle ($\varphi_2 = \varphi_2(\varphi_1)$). For both modes the same value 4˚ of the maximum back Wheel angle φ_{2m} was used.

These four dependences were scrutinized

LIN:

$$\varphi_2 = (\varphi_{2m}/\varphi_{1m}).\varphi_1 \,, \tag{3-1}$$

NELIN_NAD: the cubical polynomial

$$\varphi_2 = p_3\varphi_1^3 + p_2\varphi_1^2 + p_1\varphi_1 + p_0 \tag{3-2}$$

ran through [0; 0], [φ_{1m}/3; 0,65.φ_{2m}], [(2/3).φ_{1m}; 0,9.φ_{2m}], [φ_{1m}; φ_{2m}],

NELIN_POD:

$$\varphi_2 = (\varphi_{2m}/\varphi_{1m}^5).\varphi_1^5 \tag{3-3}$$

and SIN:

$$\varphi_2 = \varphi_{2m}.\sin((\pi/\varphi_{1m}).\varphi_1). \tag{3-4}$$

Behaviour of the model at steering with its Wheels in modes OP, ST at the dependence LIN was also compared with behaviour of the model in the case of back Wheel being not turned during a drive (mode ZN, $\varphi_2(\varphi_1) = 0$).

Computations were made for running velocities 5, 30, 80 and 130 km/h.

Back Wheel turn as a function of yaw velocity (Problem II)

The second problem was, with motivation to *restrict* yawing by the back Wheel, behaviour of the model in the case of back Wheel turn being dependent on yaw velocity.

The *linear* relation

$$\varphi_2 = \kappa.\dot{\varepsilon} \tag{3-5}$$

between the back Wheel angle φ_2 and the yaw velocity dε/dt was considered. Two κ values of different order 0,028 s (κ_1) and 0,2 s (κ_2) were taken. Maximum back Wheel turn in this task was not limited.

Computations were made for running velocities 30, 50, 80 and 130 km/h.

Elimination of sideslip angle in the centre of gravity (Problem III)

The third problem was behaviour of the model at *zero* sideslip angle in the centre of gravity ($\delta(t) = 0$).

Maximum back Wheel turn in this task was also not limited. Computations were made for running velocities 30, 50, 80 and 130 km/h again.

As input functions ((drive) maneuvers) at studying of all three problems these two dependences of the front Wheel angle on time ($\varphi_1 = \varphi_1(t)$) were used

RAM:

$$\varphi_1 = \left(\varphi_{1m}/t_m\right).t \, , \tag{3-6}$$

PUL:

$$\varphi_1 = \left(\varphi_{1m}/2\right).sin\left(\left(\pi/t_m\right).t\right) . \tag{3-7}$$

The time interval <0, t_m = 4> s was taken.

4. Methods of solution

Problems I and II

From the systems

$$F_1\left(t,\dot{\varepsilon},\delta,\dot{\delta}\right) = 0 \, , \tag{4-1}$$

$$F_2\left(t,\dot{\varepsilon},\ddot{\varepsilon},\delta\right) = 0 \tag{4-2}$$

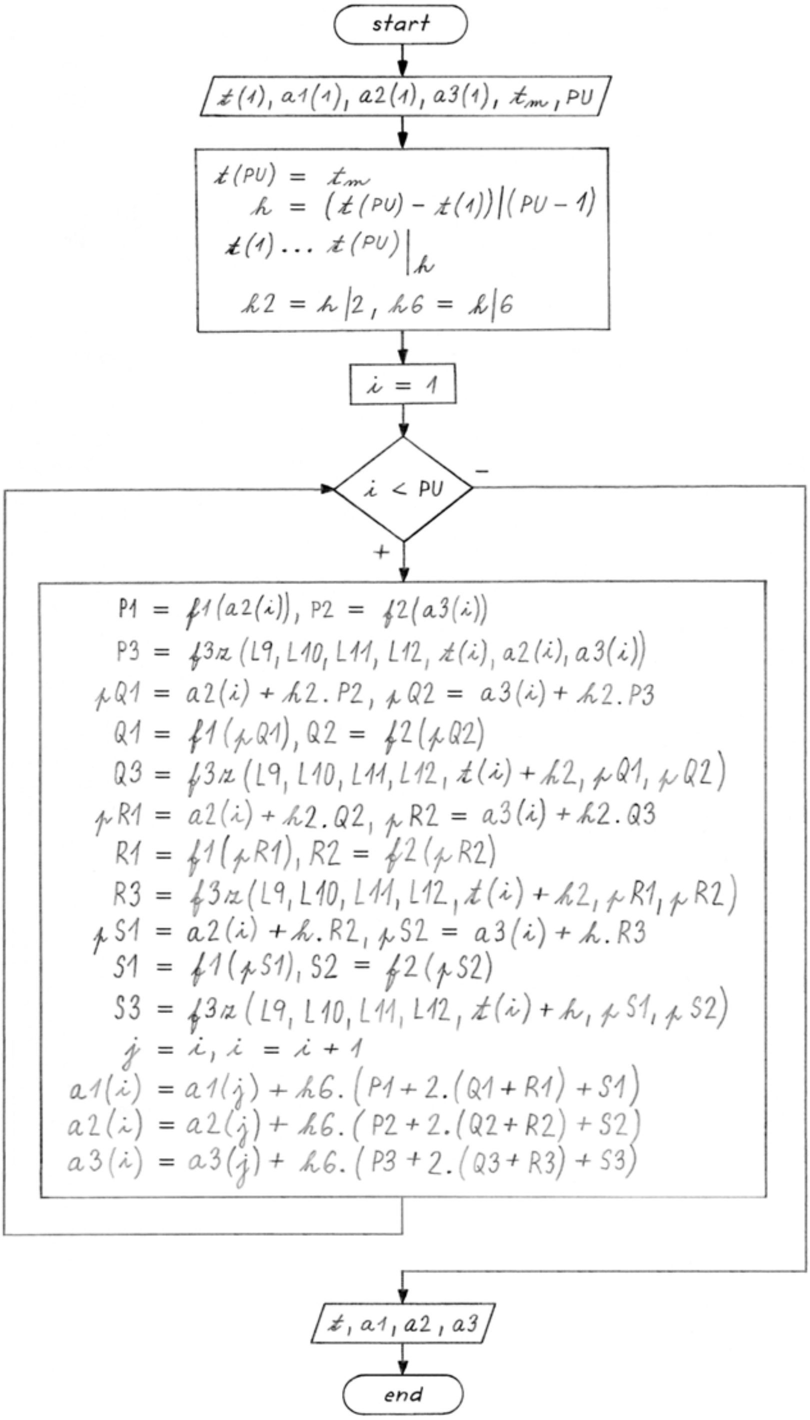
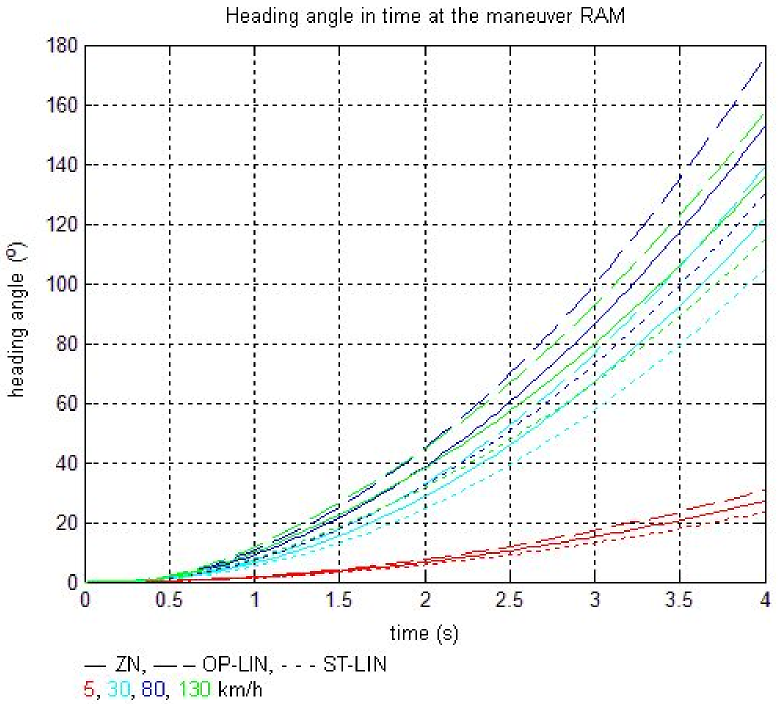


***Fig. 4-1** One routine created for computations*

(additional quantities are not mentioned in section List of symbols)



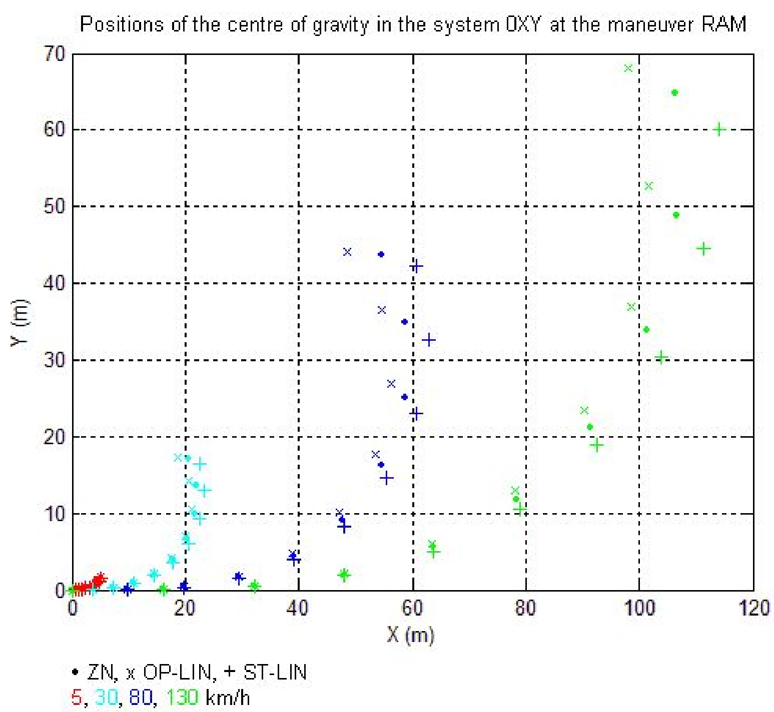


Fig. 4-2 *Two graphical outputs*

the equations

$$\dddot{\varepsilon} = f_1(t,\dot{\varepsilon},\ddot{\varepsilon}), \tag{4-3}$$

that have in all cases one and only solution for the given initial conditions, were extracted.

These third-order equations, after transformation into systems of first-order equations, were solved by the one-step Runge - Kutta method of sixth order - Huťa´s formula. The formula was chosen from group of candidates on basis of an accumulated error analysis.

With that computation of remaining dependences was finished.

Problem III

From the systems

$$F_3(t,\dot{\varepsilon},\delta,\dot{\delta},\varphi_2) = 0, \tag{4-4}$$

$$F_4(t,\dot{\varepsilon},\ddot{\varepsilon},\delta,\varphi_2) = 0 \tag{4-5}$$

the equations

$$F_5(t,\delta,\dot{\delta},\ddot{\delta},\varphi_2,\dot{\varphi}_2) = 0 \tag{4-6}$$

were extracted.

Through setting

$$\delta(t) = 0, \tag{4-7}$$

$$\dot{\delta}(t) = 0, \tag{4-8}$$

$$\ddot{\delta}(t) = 0 \tag{4-9}$$

the equations

$$\dot{\varphi}_2 = f_2(t,\varphi_2), \tag{4-10}$$

that have one and only solution for the given initial conditions, were expressed.

These equations were solved analytically.

With that computation of remaining dependences was finished.

Positions of the centre of gravity in the reference coordinate system *0XY* during a drive were computed at ten instants of time uniformly spaced on the time interval. To evaluate integrals the rectangular method was used.

5. Results

Problem I

Dependence LIN x mode ZN

Model in the arrangement OP-LIN /ST-LIN/ turns at both drive maneuvers and all running velocities *more* /*less*/ than in the arrangement ZN.

Trajectory of the centre of gravity of the model in the arrangement OP-LIN /ST-LIN/ diverts at both drive maneuvers and all running velocities in final of process in direction *indoor* /*outdoor*/ from curve that belongs to the arrangement ZN.

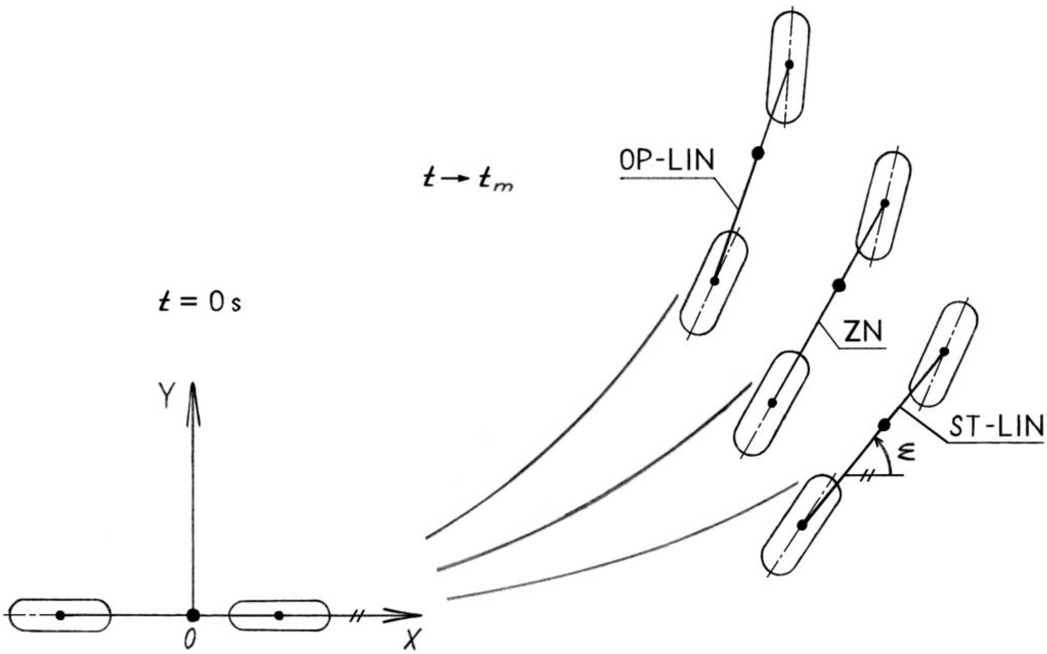


Fig. 5-1 *Problem I, dependence LIN x mode ZN*

Dependences LIN, NELIN_NAD, NELIN_POD, SIN

Mode OP: If the dependence with *higher* values of the back Wheel angle in the beginning of steering is used, *higher* values of yaw acceleration are, as observed, reached.

Mode ST: If the dependence with *higher* values of the back Wheel angle in the beginning of steering is used, *lower* values of yaw acceleration are, as observed, reached.

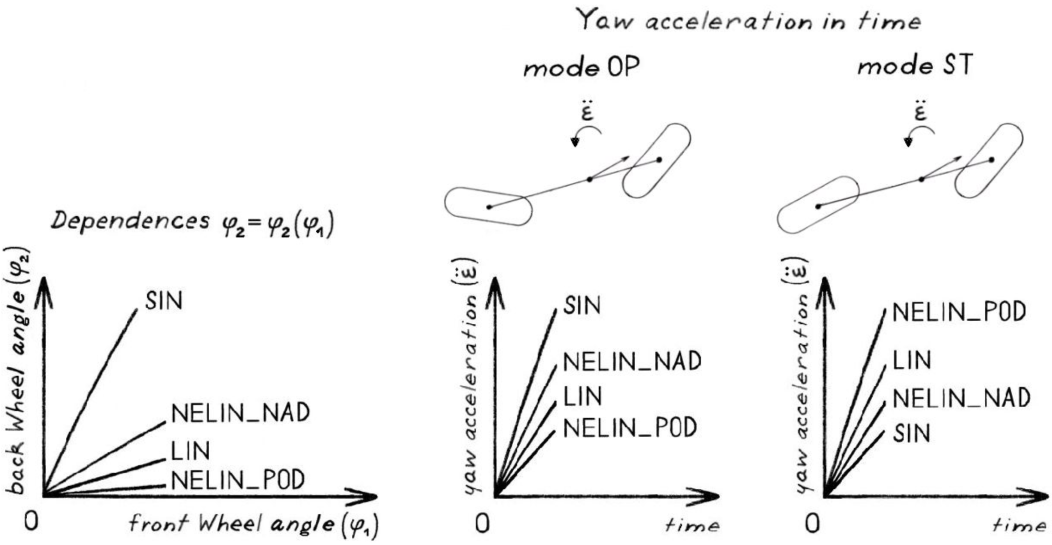


***Fig. 5-2** Problem I, dependences $\varphi_2 = \varphi_2(\varphi_1)$*

Problem II

Model at using the κ_2 value (the *higher* one) turns at both drive maneuvers and all running velocities *less* than at using the κ_1 value.

Trajectory of the centre of gravity of the model at using the κ_2 value diverts at both drive maneuvers and all running velocities in final of process in direction *outdoor* from curve that belongs to the κ_1 value.

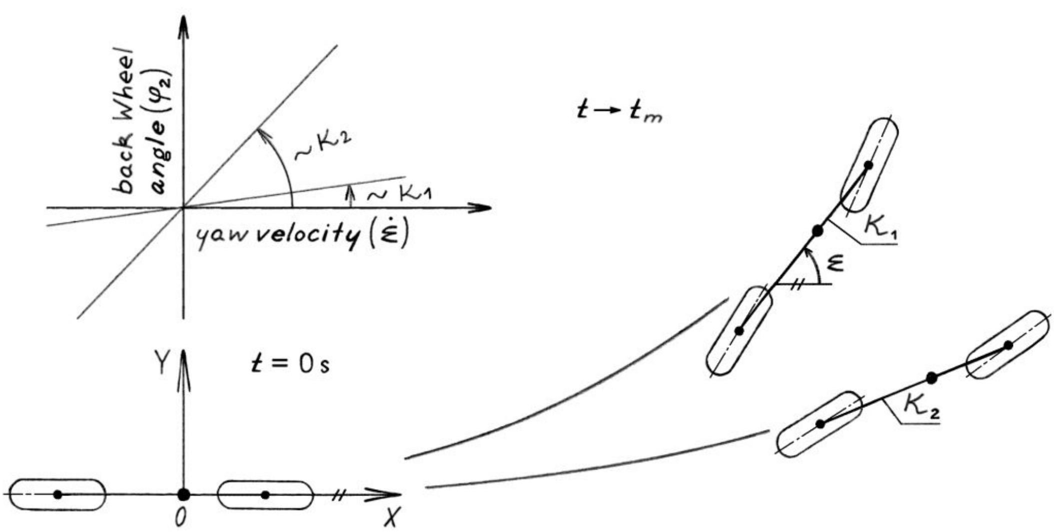


***Fig. 5-3** Problem II, positions of the model*

Back Wheel of the model in both cases is at the maneuver PUL in the instant of time t_m at all running velocities turned (front Wheel is not turned).

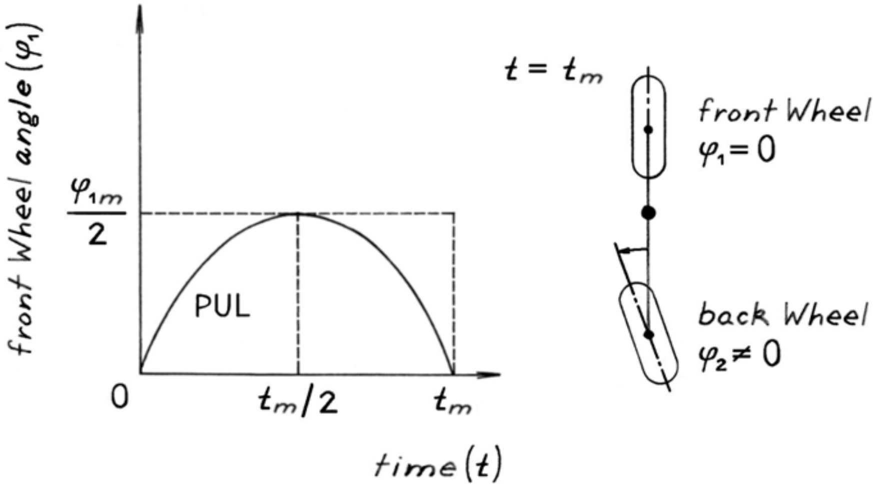


Fig. 5-4 *Problem II, Wheels turn in the end of maneuver PUL*

Problem III

Back Wheel angles reached in the instant of time t_m at the maneuver RAM at running velocities 80 and 130 km/h are of the *same* order as the maximum front Wheel angle used for computations.

Back Wheel of the model at the maneuver PUL is in the instant of time t_m at all running velocities also *turned*.

6. Conclusion

The study can be used as a foundation for more complex computer simulations based on the MBS-software (Multi-Body Systems, see 10.).

On the given level of approach the inverse problem of dynamics is offering to solve.

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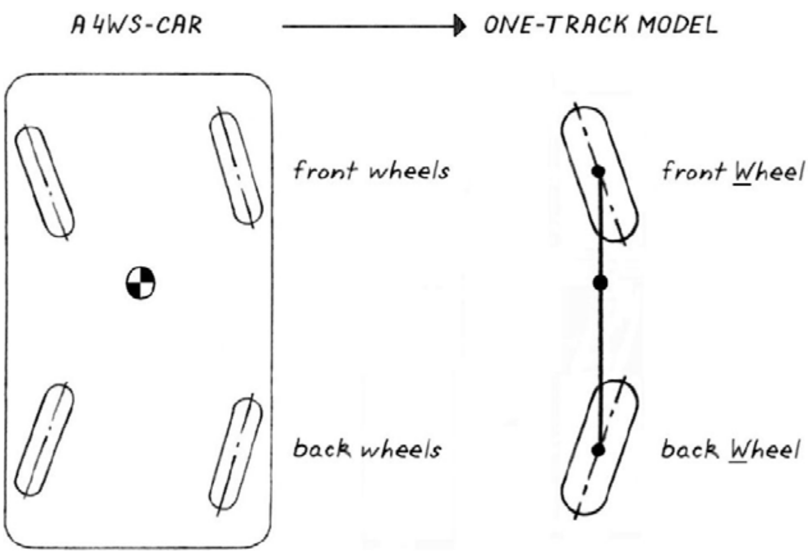
List of symbols

φ_1	rad	front Wheel angle
φ_2	rad	back Wheel angle
φ_{1m}	rad	maximum front Wheel angle
φ_{2m}	rad	maximum back Wheel angle
m	kg	mass
l_1	m	distance between the centre of gravity and rotation axis of the front Wheel
l_2	m	distance between the centre of gravity and rotation axis of the back Wheel
J_z	kg.m^2	moment of inertia about the z-axis
k_1	N/rad	directional stiffness of the front Wheel
k_2	N/rad	directional stiffness of the back Wheel
$p_0, ..., p_3$	1	coefficients of the dependence NELIN_NAD
κ_1, κ_2	s	values of the κ coefficient in the linear relation between the back Wheel angle and the yaw velocity
$0XY$		reference rectangular coordinate system

T	centre of gravity	
$Txy(z)$	rectangular coordinate system connected with the model	
t	s	time
t_m	s	instant of time marking boundary of the given interval
v	m/s	running velocity of the centre of gravity (running velocity)
a_d	m/s^2	centripetal acceleration of the centre of gravity
δ	rad	sideslip angle in the centre of gravity
$d\delta/dt$	rad/s	first-order derivative δ with respect to t
ε	rad	heading angle
$d\varepsilon/dt$	rad/s	yaw velocity
$d^2\varepsilon/dt^2$	rad/s^2	yaw acceleration
N_1	N	normal force acting on the front Wheel
N_2	N	normal force acting on the back Wheel

mode OP Wheels turned in converse sense
mode ST Wheels turned in same sense
mode ZN back Wheel being not turned

LIN, NELIN_NAD, NELIN_POD, SIN dependences of the back Wheel angle on the front Wheel angle
RAM, PUL drive maneuvers (excitation)



Summary

ON THE DRIVE DYNAMICS OF A 4WS-CAR

Martin WEINER, Miroslav TESAŘ

In the article study of drive dynamics of a 4WS-car (Four-Wheel Steering) based on the one--track model is resumed. Attention was focused on three single problems. At solving equations both analytical and numerical techniques were applicated. To effect calculations and plot outputs the programming language Matlab was used.

Resumé

K DYNAMICE JÍZDY VOZIDLA ŘÍZENÉHO VŠEMI KOLY

Martin WEINER, Miroslav TESAŘ

V příspěvku jsou shrnuty výstupy výpočtů chování čtyřkolového automobilu aktivně rejdujícího i svými zadními koly (používáno je označení 4WS z anglického Four-Wheel Steering).

Automobil byl představován rovinným jednostopým lineárním modelem. Zkoumány byly tři problémy.

Užito bylo jak analytických, tak numerických výpočetních metod, to podle charakteru úlohy. Procedury pro numerické výpočty byly sestaveny a realizovány ve výpočetním prostředí Matlab. Tohoto bylo také využito k zobrazení výsledků.

Zusammenfassung

ZUR FAHRTDYNAMIK EINES ALLRADGELENKTEN FAHRZEUGES

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Im vorgelegten Beitrag ist eine Studie der Fahrtdynamik eines allradgelenkten Fahrzeuges (die Bezeichnung 4WS aus dem Englischen Four-Wheel Steering wird benützt) beschreibt.

Als Ausgangsmodell eines solchen Fahrzeuges wurde das lineare Einspurmodell genommen.

Im allgemeinen, es wurde auf drei Probleme gezielt. Das erste Problem war Verhalten des Modells bei festgesetzten Verbindungen von Rädern. Im zweiten Problem, mit Motivation Gierbewegung zu bedrücken, war Drehwinkel des hinteren Rades linear abhängig von Gierwinkelgeschwindigkeit. Im dritten Problem ging es um die Aufgabe Schwimmwinkel zu eliminieren.

Im Rahmen der ganzen Studie wurden sowohl analytische als auch numerische Methoden verwandt. Numerische Rechnungen und Abbildungen von Ergebnissen wurden mit Hilfe der Programmiersprache Matlab gemacht.