CALCULATION OF A MUNICIPALITY PERCENTAGE SHARE IN THE SHARED TAXES YIELD BY MEANS OF GENETIC PROGRAMMING AND ITS REGIONAL COMPARISION

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Abstract: *The application of the genetic programming method is executed in this contribution in order to determine the percentage share of a concrete municipality in the shared taxes yield in the Czech Republic. Results obtained by using the genetic programming method are compared with the real data publicised in the Ministry of Finance of the Czech Republic notices. The comparison is done both for the size of municipality category for the entire Czech Republic and for the space – according to individual regions for the period from 2008 to 2010. The objective of this contribution is to verify the accuracy of the results obtained by the genetic programming method and the possibility of their utilization in practical usage, in particular for the prediction of the share of the individual municipality in the shared taxes Šeld.*

Keywords: *Tax Assignment to Sub-national Government Level, Shared Taxes, Genetic Programming, Regional Self-governments, Regional Comparison*

1. Introduction

Tax Assignment to sub-national Government Level („RUD" in Czech - Tax Assignment to sub-national Government Level) for municipalities is a widely discussed topic in the Czech Republic. There are two associations which strive for the change in the current system, these are the Union of Towns and Villages („SMO" in Czech) and the Association of Local Administrations („SMS" in Czech). The Ministry of Finance of the Czech Republic (MFCR) asked, in year 2008, a consortium of universities working under the umbrella of the Economic University (VŠE) to elaborate a study on *„Analysis of financing state administration and local administrations"*. The objective of this study was to gather information fundamental for creating proposals leading to change in relevant RUD legislation [2]. The objective of such changes should not have been to increase the municipalities´ share in the total gross tax revenues re-distributed according to the RUD, but it should be more the correction of some heavily criticized disproportions inbuilt in the current system [6, 7]. Financial crises have caused a dramatic decline in tax collection in which both the national budget and the local administrations (regions and municipalities) have a share, thus any efforts to change the construction of shared tax re-distribution to municipalities and changes in RUD legislation are currently not in the centre of attention.

However, it may be expected that the already fading financial crises impacts on national budgets and the essential consolidation of public finance will bring the question of optimal local administrations financing back into attention. This subjected article is a contribution to the discussion over this topic. The objective of this contribution is to propose and design an algorithm for re-distribution of shared taxes to municipalities by application of the genetic programming method and to compare this with the current shared taxes re-distribution system.

2. Existing valid tax assignment to sub-national government level

The effective RUD legislation – Act on Tax assignment of selected taxes yields to sub-national independent administrations and to certain state funds (Act no. 243 from year 2000 on RUD), has been in effect since year 2001. This Act sets the rules for re-distribution of tax yields among the state, regions and municipalities. During the period of its validity the Act was several times up-dated, the last up-date was done in year 2008, and it was published as Act No. 377/2007 Coll. effective from January 1, 2008.

According to the valid existing legislation municipalities get the following shared taxes yields allocations [2]:

- 21,4 % of the natural person income tax from dependent activities collection;
- 21,4 % of the national legal entities tax collection (excluding taxes paid by municipalities themselves);
- 21,4% of the national natural person income tax collected by reduction tax;
- 21,4% of the national tax collection from the VAT;
- 21,4% of the national natural person income tax from independent business activities tax collection (only 60% of this national tax collection is re-distributed).

Municipalities receive only 30% of the yield from natural person’s income tax according to the natural person place of residence. This portion of the tax is linked to the municipality and it works as a motivation element towards promoting business activities in municipalities. 10% of the national yield of this tax belongs to the state, and only the remaining 60% of the tax yield is assigned to be re-distributed among the national budget, regional budgets and municipal budgets.

Next to the above-mentioned shared taxes municipalities get also exclusive tax revenues – these are real estate tax and the legal entity tax paid by municipalities. Detailed diagram of the valid RUD is showed in Appendix.

The concrete amount from the national gross shared taxes yield is allocated to individual municipalities based on three criteria:

- Total area of the municipality – criterion weight is 3% (the share of the municipality is defined as the share of this municipality area in the total Czech Republic municipalities´ area). The usage of this area criterion gives advantage to those municipalities that have lower population density. It also compensates increased expenditures for repair and maintenance of local communications and expenditures for transportation services. This criterion is also advantageous for those small municipalities who cannot, if willing so, integrate with neighbouring municipalities due to local geographic conditions.

- Simple number of inhabitants – criterion weight is 3% (the share of the municipality is defined as the municipality simple number of inhabitants in the total number of Czech Republic inhabitants).
- Number of inhabitants adjusted by gradual transitions between municipality size categories coefficients – criterion weight is 94%. Only the part of the number of inhabitants which falls into the relevant number of inhabitant's interval (Table 1) is calculated by the given coefficient of gradual transitions. This ensures that the shares of individual municipalities create a continuous curve with any jump steps in between individual size criteria. This methodology is not used for Prague, Brno, Ostrava and Plzen)[1].

The calculation algorithm is defined in the following way. First the share of the capital city Prague, the share of city Brno, Ostrava in shared taxes are found, then the total share in shared taxes is found for municipalities in the Czech Republic. The share of a concrete municipality (it is announced each year in the MF CZ by-law) is then defined as the multiple of the number of inhabitants of the municipality and the relevant coefficients of gradual transitions in the sum of these multiples for all municipalities (without Prague, Brno, Plzen and Ostrava).

Table 1: Gradual transition coefficients and multiples of gradual transitions

Municipalities with number of inhabitants from - to	Gradual transitions coefficients	Gradual transitions multiple
0 – 300	1,0000	1,0000 x number of inhabitants in municipality
301 – 5 000	1,0640	300 + 1,0640 x number of inhabitants in a municipality that are above the number 300
5001 – 30 000	1,3872	5 300,8+1,3872 x number of inhabitants in a municipality that are above the number 5 000
30 001 – a more	1,7629	39 980,8 +1,7629 x number of inhabitants in a municipality that are above the number 30 000

Source: [2]

3. Genetic Programming

The genetic algorithm (GA) transforms a population of individual objects, each with an associated value of fitness, into a new generation of the population. The Darwinian principle of survival and reproduction of the fittest and analogue of naturally occurring genetic operation such as crossover (sexual recombination) and mutation is using for the GA.

[1] These towns have their own re-calculation coefficients.

A special group that evolve separately, but which draw from GA is genetic programming (GP), where GP is an extension of the GA in which the genetic population contains computer programs. GP makes use of the same techniques as a GA, but it implements over acceptable data structure (N–ary tree). The node of the tree contains entity from two sets (the set of primitive function and the set of terminals) [3, 4, 8].

A functions can be arithmetic (+, -, *, /, etc.), algebraic (sin, cos, exp, log, etc.), logical classical or fuzzy (not, and, or, etc.), conditional operator (If - Then - Else, etc.). A terminal symbol (A, B, C, etc.) can be input variable of program, integer, real, logical, …, constant, function without arguments having secondary effect.

In case of GP are definitions next basic operations: crossover, selection and mutation [3, 5]. Advantage GP in comparison with GA is, that GP is obtained not only common model for solving problems, but also description how is problems solution (particular analyst representation). The basic flowFig. for GP is in [14].

3.1 The design of models for calculation of GP share

For the process of searching for a formula for calculation of the percentage share of a municipality in shared taxes with using GP the following attributes have been utilized:

- Common number of inhabitants of a given municipality (O),
- The total number of the Czech Republic inhabitants (CO),
- Total area of municipality (U),
- Total area of the Czech Republic (CU),
- Percentage share of municipality in shared taxes (P).

Design of model for the P calculation is described in Fig. 1.

The result of the GP is the following function that replaces the standard method of calculation of percentage share of CZ individual municipalities in shared taxes:

genG$_P$ = ((((O+O)+(((O+(-4212+(O+O)))+(O+((((O+O)+O)+O)+O)))+(O+O)))+
((O+O)+(O+O))))/(((U+CO)-(((((O+(-3880-(-9978*((-4212+(((O+(O+O))+(-4212+
(O+O)))+U))+4212))))+(((U+(-4212+((((((((6568+(-9978+(-5704*(O-(-8872*(O+
O))))))-O)+((-8872-(-1560*(((((((((2084+((6568-((O+(-4212+(O+O)))*((((O+(O+
(2084*CO)))--3880)+((O-(-1560*((CU-((O+(-4212+(O+O)))*(((((-3880*(-9978*
(3340+(O+O))))+CO)-(-1560*((((CO+(CO+((-3880*CO)+(-3870*(-9978+CO)))))-
O)+(-8872*CO)))/(O)))+CO))/(O))))/(O)))+O)))/(O)))+((-8872*((-3880+(O+O))+O))*
((O+(((((O+O)+(((-2218+-3880)+O)+(O+(((O+(-3880+(O+(((O+(-4212+(O+(((O+
O)+(((-1868+-3880)+O)+(O+(O+O))))+O))))+O)+O))))+O)+O))))+O)+O)+O))+(O+
O)))))+(O+(-8872*(-9978*(3340+((4756+(O+3016))-(O+O)))))))-O)+((O-((-4212+
(O+O))*(((O+(4756+(-9978*CO)))+(O*(CO+(CO-(-9978*((O+(O+O))+O))))))))/((O+
U))))+O))+(-5860+(-4212*(-9978*((4756+(O+3016))-(O+O))))))--3880)+(O+CO))--
4212))/(O)))+(-4212*(O-(-8872*(O+O))))))+-4212)-O)+-8872)-O))/(O)))+O)+
3016))+(((-5704-(-9978*((-212+((O+(O+O))+O))+(O+O))))+O)+-5704)))/(O)+(CU+
(((((4756+(O+3340))+O)+O)+((3016+(O+3340))+O))+O))))) (1)

Equation (1) can be consequently used in, for example, table calculator for the realization of the stated calculation. In case of need this equation can be simplified by the usage of basic mathematical operations.

This function contains 4 input attributes (O, CO, U, CU) and 11 various constants generated by programme (for example 4212, 3880, 9978, 5704 etc.).

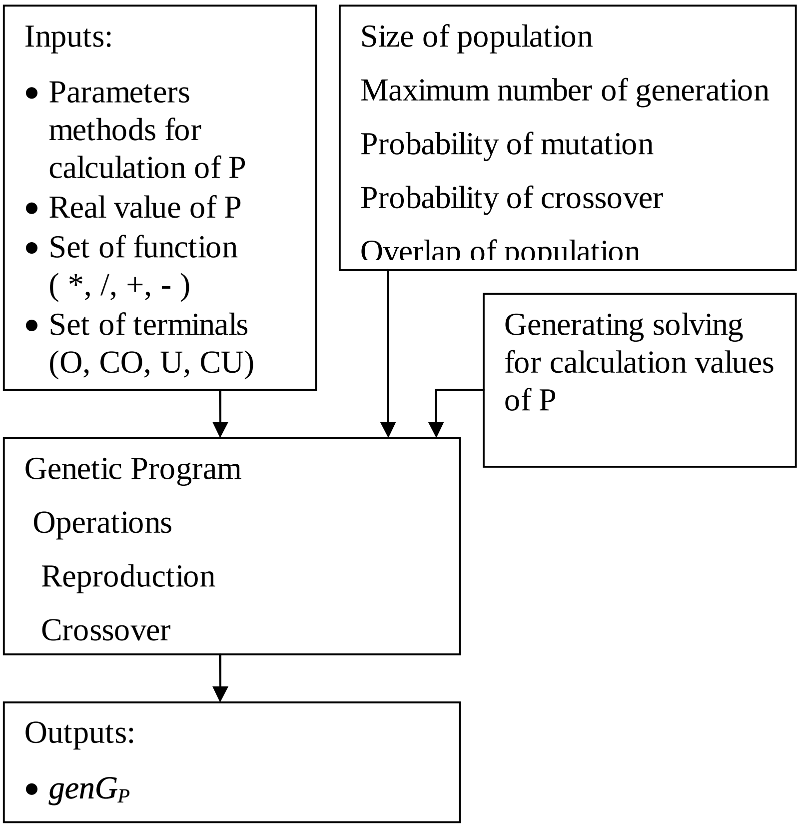

Fig. 1: FlowFig. for design of models for calculation P

[Source: elaborated with using 12]

3.2 Comparison of Results in years 2008-2010

The resulting function for the calculation of P (1) was applied to the input values (values O, CO, U, CU) and results were compared with values P listed for year 2008 in [11].

The deviation ratio of calculation AP of the resulting function was evaluated according to the following relation:

$$A_P=(P-genG_P)/P. \qquad (2)$$

The results of comparing Ap of the resulting analytical function genGP for data from year 2008 according to the size of individual municipalities are stated in Table 2.

From Table 2 issues that function created by means of GP provides results with sufficient accuracy in comparison with the standard way of calculation. The accuracy of calculations and the improvement of the prediction capability of function genGP of result GP has been proven also by the comparison of actual shares of individual municipalities published in MFCR notices for years 2009 a 2010 [10, 9] with calculations results achieved by using equation (1) for function P value. The average deviation error for the individual years is the lowest in year 2009 (AP=0,885), for year 2010 the deviation ratio is AP=1,001. Higher inaccuracies in calculations have been demonstrated only in the category of the smallest municipalities with number of inhabitants lower than 300 inhabitants: (in year 2008 1,939), in the following years the deviation in the calculation accuracy is lower also for this category of municipalities (in year 2009 AP=1,803 and in year 2010 AP=1,930). With these municipalities the created function assumes higher share in the P value.

Table 2: Evaluation of the Calculation A_P

Number of Inhabitants	2008		2009		2010	
	Number of Munici-palities	A_P	Number of Munici-palities	A_P	Number of Munici-palities	A_P
299 to 0	2452	1,939	2422	1,803	2404	1,930
499 to 300	1135	0,760	1127	0,630	1118	0,752
999 to 500	1311	0,446	1329	0,314	1345	0,432
4 999 to 1 000	1072	0,172	1091	0,037	1104	0,154
9 999 to 5 000	141	0,043	142	-0,088	142	0,024
19 999 to 10 000	70	0,097	70	-0,024	69	0,079
29 999 to 20 000	27	0,098	27	-0,020	27	0,079
39 999 to 30 000	10	0,093	10	-0,020	10	0,074
49 999 to 40 000	5	0,199	5	0,094	5	0,182
99 999 to 50 000	16	0,162	15	0,065	15	0,149
199 999 to 100 000	1	0,133	2	0,032	2	0,114
TOTAL	6240	1,026	6240	0,885	6241	1,001

[Source: own proceeding]

Table 2 and Fig. 2 again illustrate the results of the comparison of accuracy of forecasting the shares of individual municipalities in the shared taxes yield for the individual size categories for the given years.

3.3 Results Comparison by Regions

When comparing results obtained by the application of function (1) for the calculation of the share of a municipality in the shared taxes yield by regions we can see that even in the individual regions (Fig. 3) the results do not differ from the results obtained for the individual size categories for the entire Czech Republic (Fig. 2). The best prediction (the lowest deviation ratio) from the actual shares was reached in year 2009, somewhat worse results have been obtained for prediction P in year 2010. In Fig. 3 there is illustrated the size of the deviation ratio AP in the individual years (2008 to 2010) for the Czech Republic regions (the CR) and the average deviation ration for year 2008. In this graph we can see that the highest value of the deviation rate AP is in the South Bohemia, Hradec Kralove, Plzensky and Vysocina regions.

Comparing data in Fig. 3, 4 and Table 3, 4 we can state the following conclusions. The deviation rate AP from the average value Ap from year 2008 in the Czech republic regions framework corresponds with the frequencies of municipalities in the individual size categories (Table 3, 4 and Fig. 4). From Fig. 4 it is clear that the highest percentage representation of municipalities in the smallest size category (0 – 299) is in the four above stated regions with the highest deviation ratio. From this it is clear that a specific calculation of the municipality share in the shared taxes yield with this category is not fully accepted by the equation (1). This causes the growth of the deviation ratio in those regions where is the highest number of municipalities with number of inhabitants from 0 – 299.

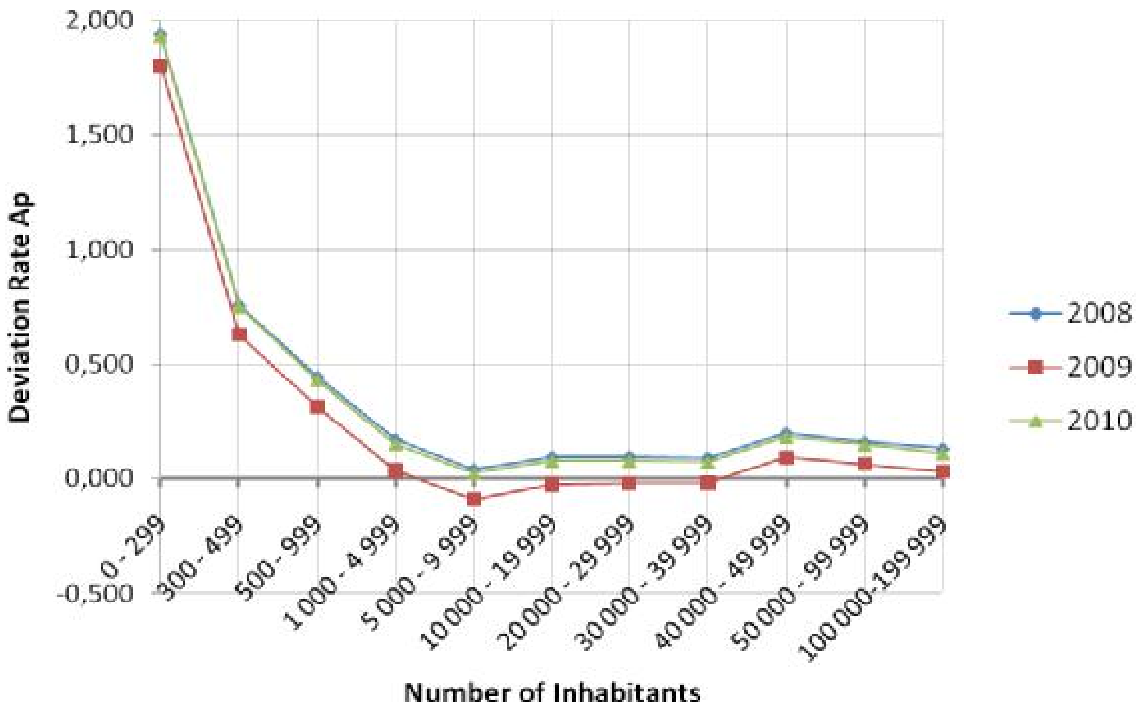

Fig.2: Deviation Ratio Error AP according to municipalities size in years 2008-2010

[Source: own proceeding]

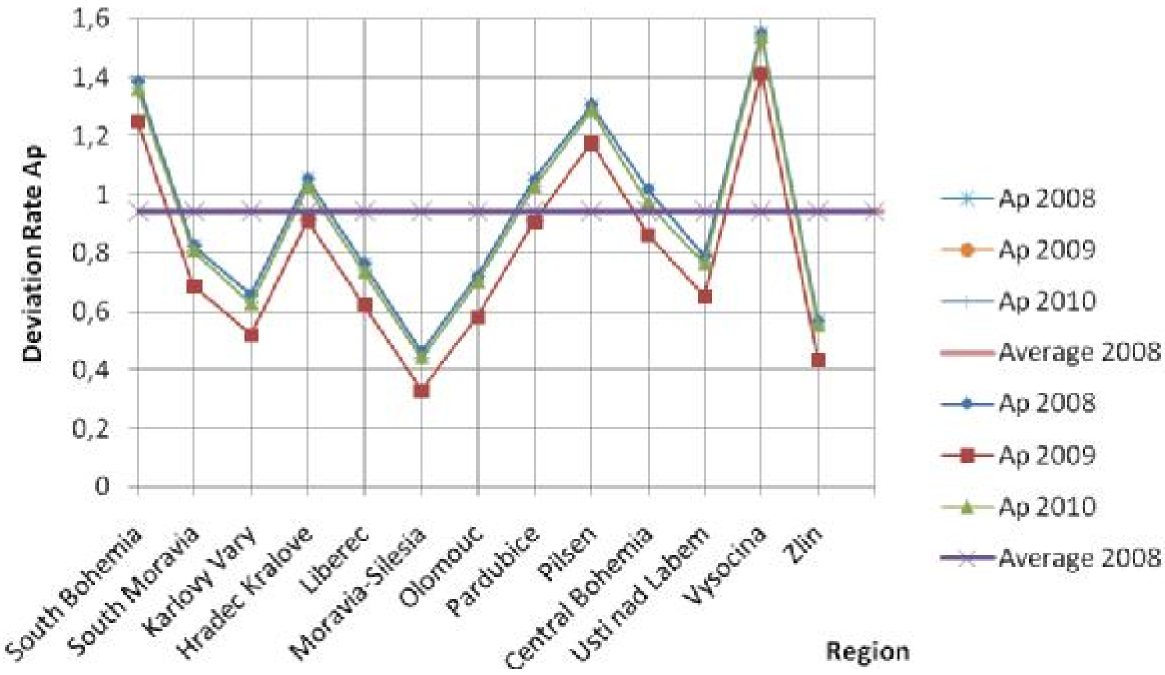

Fig. 3: Deviation Ration Error AP according to regions in years 2008-2010

[Source: own proceeding]

Table 3: The Frequency of Municipalities by Number of Inhabitants in Regions

Region	Number of Inhabitants											
	0 - 299	300 - 499	500 - 999	1 000 - 4 999	5 000 - 9 999	10 000 - 19 999	20 000 - 29 999	30 000 - 39 999	40 000 - 49 999	50 000 - 99 999	100 000 - 199 999	TOTAL
South Bohemia	335	110	78	79	13	2	3	1		1		622
South Moravia	194	120	180	155	13	4	4	1				671
Karlovy Vary	33	23	32	30	6	4	1	1		1		131
Hradec Kralove	189	95	86	56	14	5	1	1		1		448
Liberec	67	27	63	43	10	2		1	1	1		215
Moravia-Silesia	38	37	77	114	17	4	5	2		4		298
Olomouc	101	75	101	107	3	6	1		2		1	397
Pardubice	193	96	87	58	7	8	1			1		451
Pilsen	264	74	76	73	8	4	1					500
Central Bohemia	436	248	258	164	20	14	1	2	1	1		1145

Usti nad Labem	104	79	80	65	9	9	3		1	4		354
Vysocina	449	96	94	47	10	4	2	1		1		704
Zlin	49	55	99	81	11	4	4			1		304
TOTAL	**2452**	**1135**	**1311**	**1072**	**141**	**70**	**27**	**10**	**5**	**16**	**1**	**6 240**

[Source: own proceeding]

Table 4: The Frequency of Municipalities by Number of Inhabitants in Regions in percentages

Region	**Number of Inhabitants**										
	0 - 299	**300 - 499**	**500 - 999**	**1 000 - 4 999**	**5 000 - 9 999**	**10 000 - 19 999**	**20 000 - 29 999**	**30 000 - 39 999**	**40 000 - 49 999**	**50 000 - 99 999**	**100 000 - 199 999**
South Bohemia	53,9	17,7	12,5	12,7	2,1	0,3	0,5	0,2	0,0	0,2	0,0
South Moravia	28,9	17,9	26,8	23,1	1,9	0,6	0,6	0,1	0,0	0,0	0,0
Karlovy Vary	25,2	17,6	24,4	22,9	4,6	3,1	0,8	0,8	0,0	0,8	0,0
Hradec Kralove	42,2	21,2	19,2	12,5	3,1	1,1	0,2	0,2	0,0	0,2	0,0
Liberec	31,2	12,6	29,3	20,0	4,7	0,9	0,0	0,5	0,5	0,5	0,0
Moravia-Silesia	12,8	12,4	25,8	38,3	5,7	1,3	1,7	0,7	0,0	1,3	0,0
Olomouc	25,4	18,9	25,4	27,0	0,8	1,5	0,3	0,0	0,5	0,0	0,3
Pardubice	42,8	21,3	19,3	12,9	1,6	1,8	0,2	0,0	0,0	0,2	0,0
Pilsen	52,8	14,8	15,2	14,6	1,6	0,8	0,2	0,0	0,0	0,0	0,0
Central Bohemia	38,1	21,7	22,5	14,3	1,7	1,2	0,1	0,2	0,1	0,1	0,0
Usti nad Labem	29,4	22,3	22,6	18,4	2,5	2,5	0,8	0,0	0,3	1,1	0,0
Vysocina	63,8	13,6	13,4	6,7	1,4	0,6	0,3	0,1	0,0	0,1	0,0
Zlin	16,1	18,1	32,6	26,6	3,6	1,3	1,3	0,0	0,0	0,3	0,0

[Source: own proceeding]

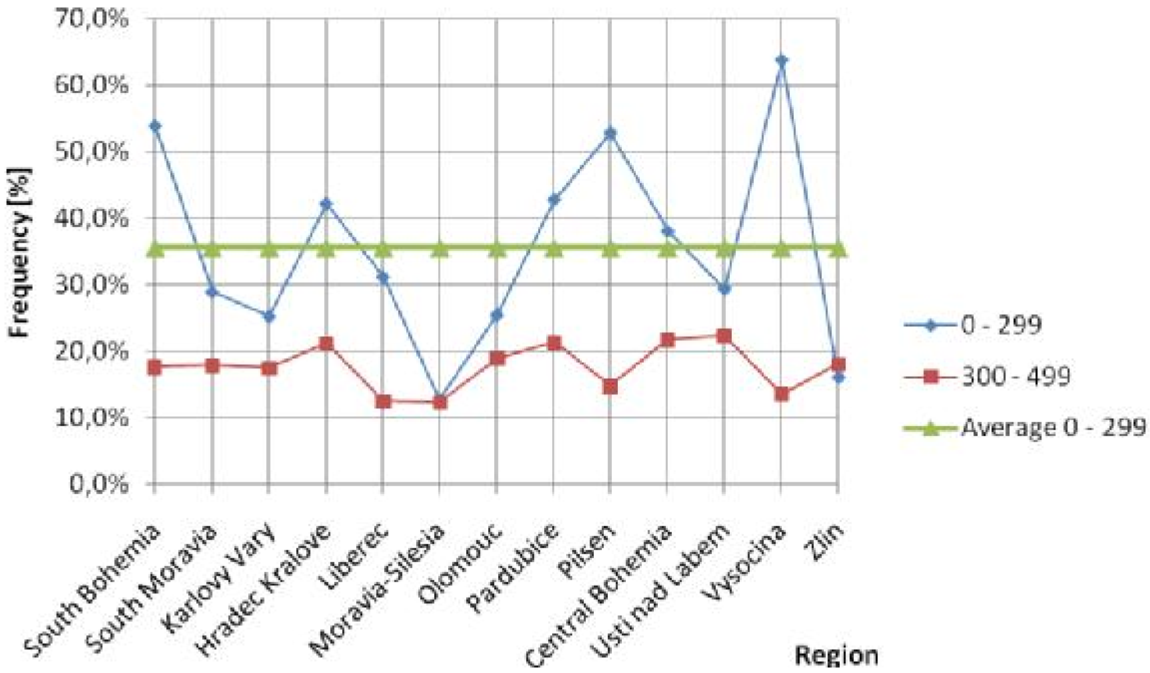

Fig. 4: The Frequency of Municipalities with the Lowest Number of Inhabitants by Regions in year 2008

[Source: own proceeding]

Conclusion

The advantage of using the derived function (1) for the prediction of the share of a concrete municipality in the shared taxes yield for the municipality representatives lays in its simplicity (despite the apparent complexity of the generated up function genGP) and as it has been proved above also in the sufficient accuracy of the prediction. For a municipality only four parameters must be known – size of municipality area according to cadastre measurement (U), size of the entire CR area (CU), next the number of municipality inhabitants (O) and the total number of the CR inhabitants (CO). When instituting the stated values to function (1) and with using the table processor, each municipality is able to forecast its share in the shared taxes revenues (P) with sufficient accuracy, with sufficient advance in time and without the need to use the quite complicated process given by the effective Act on RUD [1], eventually even before the publication of the relevant MF CR notice on the individual municipalities shares for the next fiscal year.

This contribution is focused only on a partial part of the system of municipal financing – tax yield allocation to municipalities. The objective however is also to show the utilization of state-of-the-art modelling methods in this area. The entire system of the RUD and municipal financial management must be seen and analyzed as a complex system [6, 7]. New method for municipal financing proposal must be based on deep analyses of municipal financial management on both the income and expenditures sides and in view of municipalities changing needs issuing from the impacts on financing in some services sectors.

The proposal of the design for re-distribution of shared taxes collections on some standards bases that would provide for the financing of the basic needs of inhabitants in municipalities, or for the financing of needs the municipality needs for its catchment area remains to be a question. In this area we see a major space for the utilization of multi-dimensional modelling methods.

Acknowledgment

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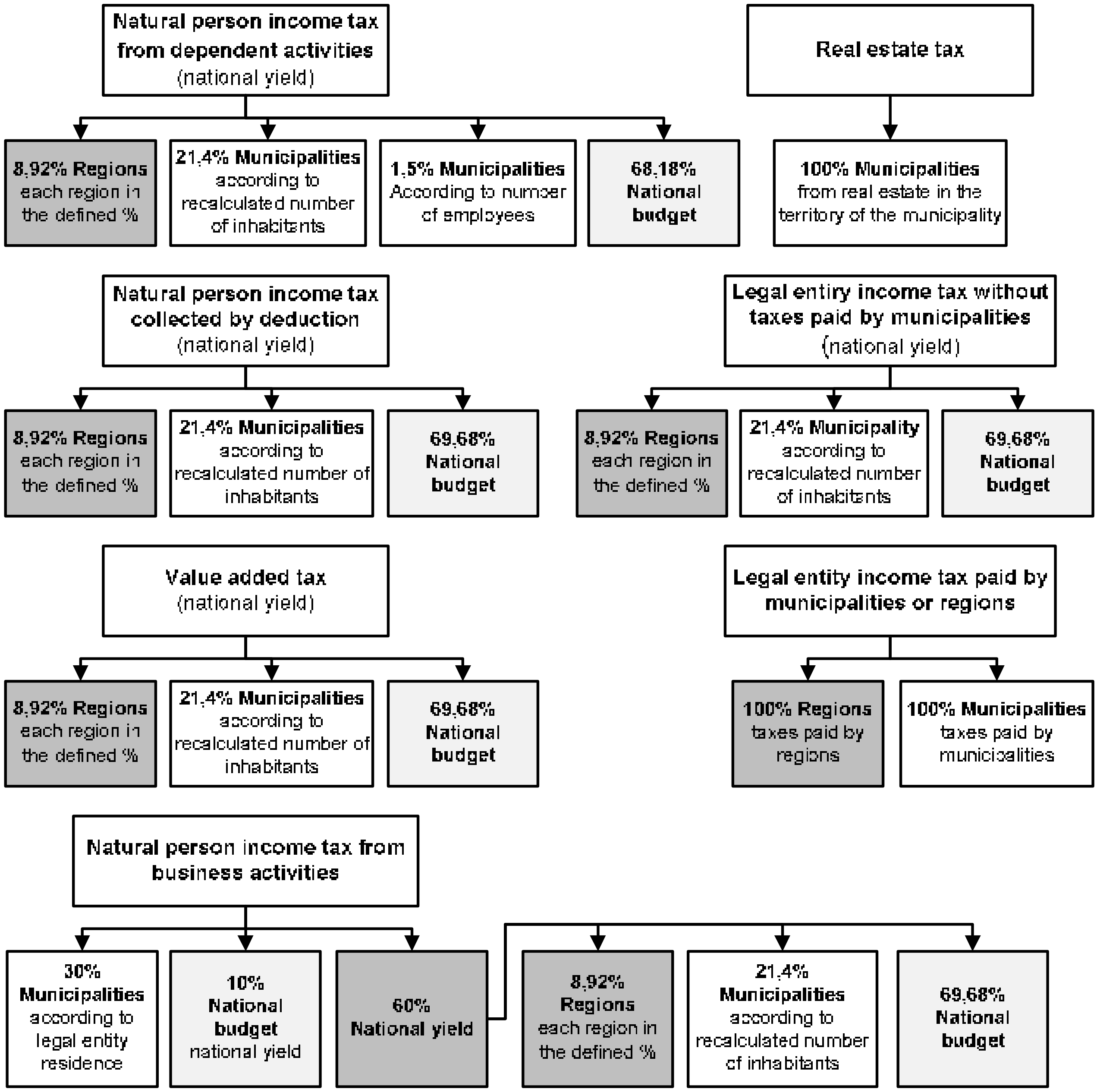
Appendix



Appendix RUD valid as of year 2008 (without National Transportation Infrastructure Fund, fees and fines)

[Source: 13]