THE BALLAST PICK-UP PROBLEM IN HIGH SPEED TRAINS

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When a high speed train overpasses a critical speed produces a wind speed close to the track large enough to start the motion of the ballast elements, eventually leading to the rolling of the stones and, if they get enough energy, they can jump and then initiate a saltation-like chain reaction. In this contribution a mathematical model of the initiation of the motion is presented. The dynamics of the stone is formulated taking into account both gravity and aerodynamic forces. These aerodynamic forces are the result of the train-generated-gust impinging on the stones. Although they are intrinsically non steady, under some conditions a cuasi-steady aerodynamic model can be employed. Conditions for the start of the motion are obtained in terms of the Tachikawa number (a kind of Froude number). The aerodynamic characteristics are included in terms of the zero aerodynamic moment line of the stone, and the aerodynamic moment coefficient.

Key words: high speed trains, ballast, saltation, modelization

1 Introduction

The flight of objects carried by the wind, in different configurations, has received a strong interest by the scientific community since old times. In this regard, three related problems can be outlined: eolian erosion [1], flying debris [2], and ballast pick-up by high speed trains debris [3]. In the case of a high speed train, when it overpasses a critical speed produces a wind speed close to the track large enough to start the motion of the ballast elements, eventually leading to the rolling of the stones and, if they get enough energy, they can jump and then initiate a saltation-like chain reaction. This chain reaction appears when the stones that have jumped return to the track floor and impact to the resting stones, transmitting them the momentum that the flying stones have obtained from the wind generated by the train. These high energy impacts give impulse to the hit stones, and some of them start to move, feeding the chain reaction. Sometimes these flying stones reach an height which is larger than the lowest parts of the train, striking them (and the track surroundings) producing considerable damage that are to be avoided e.g. by limiting the maximum allowed operational train speed. In section 2, the mathematical model is presented and the conditions for the starting and continuation of the motion are discussed. As the problem is a non-linear one, in order to attain some useful analytical results, a first order solution is obtained that is valid for small amplitude motion, solution which allows us to analyze the influence of the initial conditions in the starting of the motion. A more detailed presentation can be found in [4].

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2 Motion equations

The geometry and the nomenclature of the problem to be considered is displayed in Fig. 1. A stone, which is lying on a horizontal wall, can rotate around the point A, which is the rear contact point, due to the action of the aerodynamic loads produced by a time-dependent incoming flow $U(t)$, while restrained by the action of the gravity forces. A material line AC, otherwise called “chord”, which should be clearly outlined in the stone, is used as a reference for the several distinguished directions to be considered. The line LMN, the reference for the aerodynamic loads, is the direction of the mean wind that produces zero aerodynamic moment with regard to point A. The dynamics of the stone is described by the equation of the angular acceleration produced by the applied torque:

$$\ddot{\theta}_L = \frac{1}{I}\left[\frac{1}{2}\rho_a A_{Fp} R U^2(t) c_m(\theta_L) - M_p g d_{cm\mathrm{A}} \cos\theta_{cm}\right] \tag{1}$$

where I is the moment of inertia with regard to point A, ρ_a air density, A_{Fp} plan form area, R characteristic stone dimension (e.g. radius of the sphere circumscribing the stone), c_m coefficient of aerodynamic moment with regard to point A, M_p stone mass, g acceleration of gravity, $d_{cm\mathrm{A}}$ distance between the centre of mass and the pivoting point A, $\theta_{cm} = \theta_L + \delta_{Lcm}$, $\theta_L = \beta + \delta_{\mathrm{CL}}$, and $U(t) = U_0\, f(t)$. U_0 is the time-averaged incoming speed. The first term inside the brackets is the aerodynamic moment, with regard to point A, assuming a quasi-steady behaviour, thus neglecting unsteady effects, which is valid if the residence time is much less than the characteristic time of change of the boundary conditions.

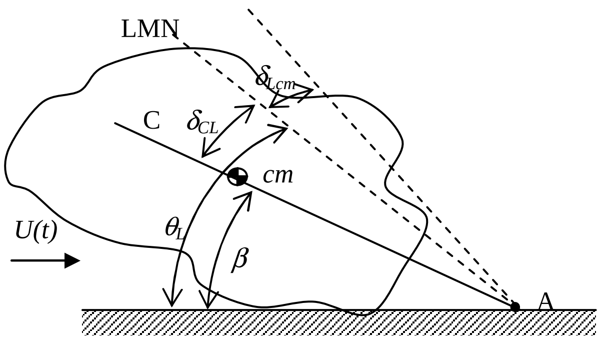

Fig. 1: Sketch of the configuration considered. Definition of angles. LMN: zero moment line. cm: centre of mass. AC: chord of the body.

The second term in (1) is just the moment of the gravity forces, with regard to point A. Eq. (1) can be rewritten in dimensionless form by using a dimensionless time $T = t/t_c$, based on a characteristic time t_c

$$t_c = t_{crg} = \sqrt{\frac{I}{M_p g d_{cm\mathrm{A}}}} \tag{2}$$

where t_{crg} is the characteristic time of the rotational motion of the body due to the effect of gravity forces, thus obtaining

$$\theta_L'' = K_0 f^2(t) c_m(\theta_L) - \cos\theta_{cm} \tag{3}$$

where dX/dT = X’, and

$$K_0 = \frac{1}{2}\frac{\rho_a A_{Fp} U_0^2}{M_p g}\frac{R}{d_{cm\mathrm{A}}} \tag{4}$$

K_0 is the Tachikawa number including the factor R/d_{cmA}, referred to the mean velocity U_0. The motion equation (3) is a highly non-linear one, due to both terms in the right-hand-side of the equation and the solution should be obtained by using numerical integration. However, for gusts of small intensity compared with the averaged speed U_0, there is the possibility of analyzing the solution by using a linear approximation. The conditions to be fulfilled at a time instant t_0 for the body to start the rotation around point A are: a) equilibrium of applied moment, that is,
$K_0 f^2(t_0) c_m(\theta_{L0}) = \cos\theta_{cm0}$, from (3), and b) positive acceleration $\theta_L'' > 0$ when $t > t_0$. Consider that the body is at rest, with the zero moment line at an angle $\theta_L(t_0) = \theta_{L0} = \beta_0 + \delta_{CL}$ and therefore $\theta_{cm}(t_0) = \theta_{cm0} = \theta_{L0} + \delta_{Lcm} = \beta_0 + \delta_{CL} + \delta_{Lcm}$. Thus

$$f^2(t_0) = \frac{\cos\theta_{cm0}}{K_0 c_m(\theta_{L0})} = \frac{\cos(\beta_0 + \delta_{CL} + \delta_{Lcm})}{K_0 c_m(\beta_0 + \delta_{CL})} , \tag{5}$$

which in the limit for $\theta_{L0} \sim \theta_{cm0} << 1$, that is, for high wind velocity ($K_0 >> 1$), is

$$f^2(t_0) \approx \frac{1}{K_0 c_m(\theta_{L0})} . \tag{6}$$

Ec. (3) can be rewritten, assuming a linear variation of $c_m(\alpha)$ versus the angle of attack, that is, $c_m(\theta_L) = c_{m\alpha}\theta_L$ ($c_{m\alpha}$ is the slope of the curve of variation of aerodynamic moment coefficient vs angle of attack) as follows

$$\theta_L'' = K_0 f^2(t) c_{m\alpha}\theta_L - \cos(\theta_L + \delta_{Lcm}) , \tag{7}$$

At the starting instant, $t = t_0$, is $\theta_L'' = \theta_L' = 0$ and therefore conditions (5) or (6) holds. Even though the stone starts to move at t_0 (which is the classical assumption employed in erosion studies), the successful continuation of the motion will not be guaranteed. Actually, only if the gust lasts enough time and the intensity is large enough the stone will obtain enough energy to overcome the restoring effect of gravity forces. In this case, as the stone centre of mass reaches the upper position, the gravity restoring torque disappears, and, if the rotation angle further increases, the restoring character will change to a destabilizing action, leading to a continuation of the motion (here called “successful motion”).

However, if the aerodynamic forces are not able to impulse the stone to reach this higher position, the stone will turn back towards its initial position and the motion will be frustrated. The determination of the relationship between the parameters involved that defines the limit of the initial conditions leading to either a frustrated motion or a successful one is based on the equation of motion. In a general case, the determination of initial conditions leading to successful motions implies the numerical integration of (7) with a suitable definition of the wind variation $f(t)$.

However, before entering this complex problem, it is possible to obtain some very helpful information by analyzing a simplified version of the problem obtained by linearization of (7).

2.1 Linear approximation

The abovementioned linearization can be carried out in the case that the angles are small, the wind velocity high enough, and the gust intensity is small compared to mean wind speed, that is $f(t) = 1 + \varepsilon \sin\omega t = 1 + \varepsilon\sin\Omega T$, $\varepsilon << 1$, and Ω is the dimensionless angular frequency

$$\Omega = \frac{2\pi}{t_{cn}/t_{crg}} , \tag{8}$$

where $t_{cn} = 2\pi/\omega$ is the period of the sinusoidal gust. In the case that $\varepsilon << 1$, then $f^2(t) \approx 1 + 2\,\varepsilon \sin\Omega T$ and the starting condition (5) can be written as follows

$$1 + 2\varepsilon \sin \Omega T_0 = \frac{\cos(\theta_{L0} + \delta_{Lcm})}{K_0 c_{m\alpha} \theta_{L0}}, \tag{9}$$

that is

$$K_0 c_{m\alpha} \theta_{L0} = \frac{\cos(\theta_{L0} + \delta_{Lcm})}{1 + 2\varepsilon \sin \Omega T} \approx (1 - 2\varepsilon \sin \Omega T) \cos(\theta_{L0} + \delta_{Lcm}), \tag{10}$$

leading to

$$\varepsilon \sin \Omega T_0 = \frac{1}{2}\left(1 - \frac{K_0 c_{m\alpha} \theta_{L0}}{\cos(\theta_{L0} + \delta_{Lcm})}\right), \tag{11}$$

This condition states that for the starting of the motion to occur, in the frame of this simplified analysis, the RHS terms should be also small enough. In fact (11) defines the condition that should fulfil the starting instant $T = T_0$.

The motion can be studied by considering small amplitude deviations θ from a mean value θ_{Lm} such that $\theta_L = \theta_{Lm} (1 + \varepsilon\theta)$, where θ_{Lm} is the solution of the equilibrium $(\theta_L'' = 0)$ when $\varepsilon = 0$, that is

$$K_0 c_{m\alpha} \theta_{Lm} - \cos(\theta_{Lm} + \delta_{Lcm}) = 0, \text{ or } \quad \theta_{Lm} \approx \frac{1}{K_0 c_{m\alpha}}, \tag{12}$$

provided that $\theta_{Lm} + \delta_{Lcm} << 1$. Eq (7) then can be rewritten as

$$\theta_{Lm} \varepsilon \theta'' = K_0 c_{m\alpha} \theta_{Lm} \varepsilon(\theta + 2 \sin \Omega T) + \sin(\theta_{Lm} + \delta_{Lcm}) \varepsilon \theta_{Lm} \theta, \tag{13}$$

and, neglecting O(ε^2) terms one obtains

$$\theta'' - k^2 \theta = h \sin \Omega T, \tag{14}$$

where $k^2 = K_0 c_{m\alpha} + \sin(\theta_{Lm} + \delta_{Lcm})$ and $h = 2K_0 c_{m\alpha}$. The homogenous solution of (14) contains exponential terms that lead to fast increasing values (or decreasing values) of θ_L with the exception of a particular solution

$$\theta = \theta_s \sin\Omega T \qquad ; \quad \theta_s = \frac{-h}{\Omega^2 + k^2}, \tag{15}$$

and the minus sign represents a 180º delay between the excitation term and the response. Note that (15) is the forced response to the driving term in the RHS in (14), which is an oscillation like the incoming flow fluctuation, but with an amplitude reduced by the factor $h/(\Omega^2 + k^2)$, and delayed 180º. Therefore, the high frequency terms involved in the wind gusts do not excite a noticeable response. In fact, (14) represents a kind of strange low pass divergent filter with a cut frequency at $\Omega = k$. The general solution of (14) is

$$\theta(T) = C_1 \exp(kT) + C_2 \exp(-kT) - \frac{h}{\Omega^2 + k^2} \sin \Omega T, \tag{16}$$

Note that, to avoid a positive exponential growth, a stability region can be defined by stating that $C_1 \leq 0$. Negative values of exponential term represent a sudden decrease of the angle towards the support (frustrated motion). In the case that the stone is at rest at $T = T_0$, that is $\theta_0' = 0$, the stability condition is reduced to

$$\frac{h}{\Omega^2 + k^2}\left(\sin \Omega T_0 + \frac{\Omega \cos \Omega T_0}{k}\right) + \theta_0 \leq 0, \tag{17}$$

which can be rewritten as

$$|\theta_0| \ge |\theta_{0\lim}| = \frac{2}{\sqrt{1+X}} \quad ; \quad X = \frac{2\Omega^2}{h} \cong \frac{\Omega^2}{K_0 c_{m\alpha}}, \tag{18}$$

with $\theta_0 < 0$, which means that θ_{L0} (defined as $\theta_{L0} = \theta_{Lm}(1+\varepsilon\theta_0)$) should be smaller than θ_{Lm} by an amount $\varepsilon|\theta_0|$. That is, in physical variables the condition (18) can be rewritten as

$$\theta_{L0} \le \theta_{L\lim} = \frac{1}{K_0 c_{m\alpha}}\left(1 - \frac{2\varepsilon}{\sqrt{1+X}}\right) = \frac{M_p g d_{cm\mathrm{A}}}{\frac{1}{2}\rho_a A_{Fp} U_0^2 R c_{m\alpha}}\left(1 - \frac{2\varepsilon}{\sqrt{1+X}}\right), \tag{19}$$

Therefore, the larger the amplitude of speed variation (large ε) the larger is the difference between θ_{L0} and θ_{Lm}. If the angle at the initial position θ_{L0} is lower than θ_{Lm}, the aerodynamic force does not produce a moment large enough until the speed has overpassed the mean value by an amount $\varepsilon\sin\Omega T_{0.}$

The condition to maintain a continuous oscillation is $\theta_0 = \theta_{0\lim}$, which makes null the contribution of the positive exponential term ($C_1 = 0$). If $\theta_0 > \theta_{0\lim}$ then a positive contribution of this exponential term appears, leading to a divergent solution. Note that the linear analysis presented here is a conservative one in predicting the stable region, as the gravity force considered is larger than the real one, because of the assumption that β is small enough. The stabilizing torque due to gravity in the linear approximation is larger than the real one when β approaches $\pi/2$. The limit cases for the gust effect are:

- Limit L) long duration gust $X \to 0$ ($t_{cn} >> t_{crg}$), $\theta_{\mathrm{Llim}L} = \theta_{Lm}\,(1-2\,\varepsilon)$, and
- Limit S) short duration gust $X \to \infty$ ($t_{cn} << t_{crg}$), $\theta_{\mathrm{Llim}S} = \theta_{Lm} = 1 / (K_0\, c_{m\alpha})$.

In the case of long duration gusts, the effect is the same as that of a quasi steady flow, that is, the instantaneous speed can be taken as a permanent speed, and then the effect on aerodynamic moment is $1+2\varepsilon$, therefore, the limit value is the equilibrium at maximum speed. In the case of short duration gust, the limit angle is due to the mean value of the aerodynamic force, as the dynamical system (14) filters out the high frequencies of the gust. Eq. (19) can be rewritten as

$$\log\theta_{L\lim} = -\log K_0 c_{m\alpha} + \log\left(1 - \frac{2\varepsilon}{\sqrt{1+X}}\right) \cong -\log K_0 c_{m\alpha} - \frac{2\varepsilon}{\sqrt{1+X}}\log e, \tag{20}$$

The extreme cases L and S can be displayed as two straight lines in a logarithmic plot (see Fig.2). The limit case S is a fixed line while case L depends on the value of ε. The region between both lines represents suitable configurations for the starting of successful motion (if X is not known). The vertical distance between the two lines is $2\varepsilon \log e/(1+\mathrm{X})^{1/2}$ aprox.

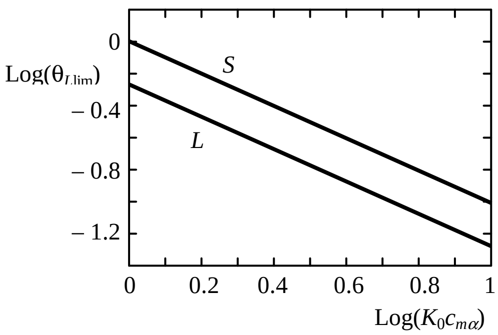

Fig. 7. Variation of the limit angle θ_{Llim} with the parameter $K_0 c_{m\alpha}$. Long- (limit L, $X = 0$); and short-duration gust (limit S, $X = \infty$).

3 Conclusions

The study of the effect of wind on bodies lying on a flat floor has been presented, the main parameters influencing the phenomenon of the starting of the motion have been identified, and the relationship among them that leads to a successful motion has been obtained. Two limits exits, for long and short duration gusts, respectively. The study covered in this paper has been carried out under technical and financial support from Talgo. The authors would like to thank David Perez Rodriguez and Emilio García for fruitful discussions and suggestions.

Reference literature

1. BAGNOLD, R.A., The physics of blown sand and desert dunes, Chapman and Hall, London. 1941
2. BAKER, C.J., The debris flight equations, J. Wind Eng. Ind. Aerodyn. 2007, 95, pp. 329–353.
3. KWON, H.B., PARK, C.S., An experimental study on the relationship between ballast-flying phenomenon and strong wind under high-speed train. Proceedings of 7th World Congress on Railway Research. Montreal, Canada, 2006, June 6.
4. SANZ-ANDRES, A., NAVARRO-MEDINA, F., The initiation of motion of a lying object caused by wind gusts, J. Wind Eng. Ind. Aerodyn. (in review).