AN APPROACH FOR FINDING THE OPTIMAL LOCATION OF PUBLIC LOGISTIC CENTRES IN THE CZECH REPUBLIC

Václav CEMPÍŘEK, Andrea SEIDLOVÁ, Jaromír ŠIROKÝ, Miroslav SLIVONĚ

Department of Transport Technology and Control

1. Introduction, problem formulation

The problem of optimal location of public logistic centres falls into multifacility location problems. This particular problem can be characterized as follows: all the new facilities (logistic centres) are of the same type and all the existing facilities (customers) are of the same type, so it does not matter which logistic centre serves which customer. This class of problems is called multifacility location-allocation problems: the location problem (decision-making about location itself) and allocation problem (determining which centre will serve which customer) are solved simultaneously.

Such type of problem can be solved using graph theory (set of nodes and set of links which represent existing transport network) or in geometrical plane. Regarding data available, the problem was solved as planar location problem with Euclidean distances.

The basic problem can be formulated as follows:

- determine optimal number of logistic centres,
- find optimal location of given number of logistic centres and
- assign existing facilities to each logistic centre.
2. Input data analysis and processing

In advance of solving the problem, it is necessary to perform analysis of input data to find out which data are required and which data are available. The only information relevant to amount of existing facilities' weight was the amount of export and import of goods among 14 administrative regions of the Czech Republic (one of those being the capital city of Prague). The data were structured as follows: road transport of goods, railway transport of goods and inland water transportation of goods (between each couple of regions). It is obvious that such data are insufficient. It was necessary to identify existing facilities (i.e. customers), obtain the locations of existing facilities and express some representation of their weight (i.e. amount of demand for offered services).

It was obvious, that each region will represent one customer (except capital city of Prague which will be merged together with Central Bohemia Region). Such structure of territory is quite rough but more accurate data were not available. The amount of export and import among particular regions was transformed into total amount of flow of goods for each region (sum of the amount of goods which flows in and the amount of goods which flows out). The total flow of goods will represent the weight of the region.

\[ F_{TOTAL}^i = \sum F_{IN}^i + \sum F_{OUT}^i \]  

where:

- \( F_{TOTAL}^i \) ......................... total amount of flow of goods
- \( \sum F_{IN}^i \) ......................... sum of amount of goods imported to region \( i \)
- \( \sum F_{OUT}^i \) .........................sum of amount of goods exported from region \( i \)

Subsequently, the locations of customers (i.e. regions) were established as medians calculated on the basis of number of commercial subjects situated in particular districts of each region (this data is available on the websites of the Czech Statistical Office). The number of commercial subjects was referred to the geographical coordinates of relevant district town. These coordinates were taken from digital maps; the geographical coordinates (WGS 84) were converted into planar ones. Planar coordinates of median of each region were calculated using Weiszfeld's algorithm (see bellow) which is implemented in software LOLA (freeware developed by Technical University of Kaiserslautern). The weights (i.e. total flows of goods) were concentrated in coordinates of medians.
3. The location-allocation problem

To find optimal solution of location-allocation problems is rather difficult task. It is difficult to avoid some local minima when treating both the location and allocation aspects simultaneously. There have been some heuristic approaches intended for solving this type of problems. Such approaches, if used with knowledge of their limitations, can be quite efficient.

One of these methods is known as alternate location-allocation method (ALA). The idea of this approach is as follows:

(a) given fixed locations of new facilities, find a minimum cost allocation of existing facilities to new facilities,

(b) given fixed allocations of existing facilities to new facilities, find a minimum total cost location for each new facility.

The steps (a) and (b) alternate until convergence. It is important to recognize that ALA is a heuristic procedure with no guarantee of optimality. The obtained solution is dependent on the initial fixed location of new facilities, and it is suitable to try solving the problems several times using different sets of initial locations. The solution obtained must satisfy necessary condition for optimality: for the given assignment of existing facilities to new facilities, the solution cannot be improved by changing locations; for the given locations, the solution cannot be improved by altering the assignment of existing facilities to new facilities.
The iterative solution procedure can be schematically described as follows:

![Flowchart](image)

**Fig. 2 The iterative solution procedure.**

**Decision-making about number of logistic centres and their initial location**

Decision about number of logistic centres must be done before the solution itself begins. The task can be solved for several variants and for each variant perform comparison of direct cost (the value of objective function (3), i.e. amount of tonkilometres) with indirect cost (approximated cost on build up the logistic centres and keeping them operational). The problem of optimal location was solved for 2, 3, 4, 5 and 7 logistic centres.

Initial location of logistic centres was chosen randomly and each location-allocation problem was solved several times using different sets of initial locations.

**The allocation problem**

Solution of allocation problem is trivial: given fixed locations of logistic centres, find for existing facility (i.e. region’s median) the closest logistic centre. It means for each existing facility to find:

\[
\min_i \sqrt{[(x_i - a)^2 + (y_i - b)^2]}
\]  

(2)
The location problem

If all the logistic centres are relative independent, the original multifacility location problem was reduced to single-facility problem, because every new facility is assigned to set of existing facilities. Now is possible to use Weiszfeld's algorithm to compute planar coordinates of median.

Weiszfeld's algorithm is intended for solving planar single-facility location problems, specifically minisum location with Euclidean distances. The algorithm itself is quite simple and easy to implement on a computer. Consider \((x, y)\) denoting the location of the new facility, \((a_i, b_i)\) denoting the location of existing facility \(i\), \(w_i\) denoting the weight of facility \(i\) and \(m\) denoting the number of existing facilities. The cost when new facility serves existing facility \(i\) can be expressed as product of Euclidean distance between the new facility and facility \(i\) and the weight of facility \(i\). The total cost expression is given by:

\[
f(x, y) = \sum_{i=1}^{m} w_i \cdot \sqrt{[(x - a_i)^2 + (y - b_i)^2]}
\]

where:
- \(w_i\)......................... weight of facility \(i\)
- \(x, y\)......................... planar coordinates of new facility
- \(a_i, b_i\)......................... planar coordinates of existing facility \(i\)
- \(m\)......................... number of existing facilities

The basic idea is transformation \(W(x, y)\), that transforms a point \((x, y)\) in the plane into another point in the plane. The initial point \((x_1, y_1)\) can be chosen randomly, the transformation can be used to compute \((x_2, y_2) = W(x_1, y_1)\), then can be computed \((x_3, y_3) = W(x_2, y_2)\) etc. The limit of this sequence of points is the optimal solution to the problem. Instead of actually computing the limit, in practice the computation stops when the distance between two successive computed points is sufficiently small. The choice of an initial point does not influence the success of the algorithm (it can be verified by computational experience).

The transformation \(W(x, y)\) is derived as follows:

Computing the partial derivatives of the total cost expression (3):

\[
\frac{\partial f(x, y)}{\partial x} = \sum_{i=1}^{m} \frac{w_i}{\sqrt{[(x - a_i)^2 + (y - b_i)^2]}} \cdot (x - a_i)
\]
\[
\frac{\partial f(x,y)}{\partial y} = \sum_{i=1}^{m} \frac{w_i}{\sqrt{[ (x-a_i)^2 + (y-b_i)^2 ]}} \cdot (y-b_i)
\]

For convenience, denote:

\[
\gamma_i(x,y) = \frac{w_i}{\sqrt{[ (x-a_i)^2 + (y-b_i)^2 ]}}, \quad \Gamma = \sum_{i=1}^{m} \gamma_i(x,y), \quad i = 1, ..., m
\]

Setting the partial derivatives to zero and solving for \(x\) and \(y\) gives \(W(x, y)\):

\[
W(x,y) = \sum_{i=1}^{m} \frac{\gamma_i(x,y)}{\Gamma(x,y)} \cdot (a_i, b_i) \quad (4)
\]

As well as in the case of computing medians of regions, LOLA program had been used to compute particular iterations. LOLA is able to find the particular medians simultaneously (it solves “multifacility” problem with fixed assignment to existing facilities).

4. Results of computation

Planar coordinates were transformed into geographical to enable their location in digital maps. It is important to emphasize that the results must be adaptively corrected on the basis of confrontation with real conditions (connection to transport infrastructure, industry etc.).

The variant with 5 logistic centres was evaluated as the most suitable. The resulting location of logistic centres was acceptable with regard to real conditions and also their density (when considering density of existing of planned logistic centres in Germany or Slovakia) was reasonable. The direct comparison of direct cost with indirect cost was not performed – it was impossible to estimate the indirect cost.

![Fig. 3 Proposed location of 5 logistic centres.](image-url)
### Tab. 1 Logistic centre Plzeň

<table>
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<th>LC No. 1:</th>
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<tr>
<td>WGS-84 coordinates:</td>
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<td>Plzeňský, Karlovarský</td>
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<td>Road:</td>
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### Tab. 2 Logistic centre Praha

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<tr>
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<td>Assigned regions:</td>
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<td>Road:</td>
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<tr>
<td>Air:</td>
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<td>Water:</td>
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<td>2, 36, nearby D11, R35</td>
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- (Praha – Česká Třebová – Olomouc – Ostrava) |
| Air: | Pardubice |
| Water: | Labe |

*Tab. 3 Logistic centre Pardubice*

### LC No. 4:

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<th>WGS-84 coordinates:</th>
<th>E 18° 17’ N 49° 50’</th>
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<td>Approximately matches:</td>
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- (Brno – Přerov – Ostrava) |
| Air: | Brno-Tuřany |

*Tab. 4 Logistic centre Brno*
LC No. 5:

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<td>Approximately matches:</td>
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<td>Assigned regions:</td>
<td>Zlinský, Moravskoslezský</td>
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<td>D47, R56</td>
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<td>Railway:</td>
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Tab. 5 Logistic centre Ostrava

5. Conclusions

The presented results are part of the task VLC2005CDVUP announced by Ministry of Transport “Concept of public logistic centres in the Czech Republic in context of importance strengthening of multimodal freight transport” with the solving period 2005-2008.

References


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Summary

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The text deals with an approach for finding the optimal location of public logistic centres in the Czech Republic. This particular problem can be characterized as follows: all the new facilities (logistic centres) are of the same type and all the existing facilities (customers) are of the same type, so it does not matter which logistic centre serves which customer. This class of problems is called multifacility location-allocation problems: the location and allocation problems are solved simultaneously. The problem was solved as planar location problem with Euclidean distance. In the case when the capacities of logistic centres are unlimited, only two types of data are
sufficient for the solution: coordinates of existing facilities (customers) and amount of their demand. The amount of demand was considered as the flow of goods among 14 administrative regions (road transport of goods, railway transport of goods and inland water transportation of goods). The chosen structure of territory is quite rough but more accurate data were not available. There were calculated several variants; as the most suitable (when considering density of existing of planned logistic centres in Germany or Slovakia) was evaluated the variant with 5 logistic centres.

Resumé

MOŽNÝ PŘÍSTUP K URČENÍ OPTIMÁLNÍ LOKACE VEŘEJNÝCH LOGISTICKÝCH CENTER
V ČESKÉ REPUBLICE

Václav CEMPÍREK, Andrea SEIDLOVÁ, Jaromír ŠIROKÝ, Miroslav SLIVONĚ

V textu je uveden možný postup řešení při hledání optimální polohy logistických center v České republice. Tento problém patří do třídy lokačních úloh. Úloha je v charakteristická tím, že je obecně umístěváno několik obslužných center stejného typu a tato centra budou využívána skupinou zákazníků rovněž stejného typu. Proto nezáleží na tom, které umístěné centrum slouží kterému zákazníkovi. Toto třídu problémů, nazývanou jako lokačně–alokační úlohy, lze řešit v roviněm prostoru. Úloha lokace (tedy rozhodování o umístění) byla řešena současně s úhlo vakace (tedy rozhodování o tom, které centrum bude sloužit kterému zákazníkovi). V případě, že kapacita navrhovaných center nebude omezená, lze v podstatě vystačit se dvěma typy údajů: souřadnicemi umístění obsluhovaných objektů (zákazníků) a velikostmi jejich požadavků. Velikost požadavků byla zadána jako velikost přepravních proudů mezi jednotlivými krajmi České republiky (silniční přeprava, železniční přeprava, přeprava zboží po vnitrozemských vodních cestách). Zvolené členění je poměrně dobrohodné, ale musí být bráno jako postačující, protože přesnější data nejsou k dispozici. Z několika variant řešení byla nakonec vybrána (vzhledem k hustotě existujících nebo plánovaných logistických center ve Slovenské republice nebo v Německu) varianta s 5 logistickými centry.

Zusammenfassung

MÖGLICHE LÖSUNG DER OPLIMALEN LOKATION VON GÜTERVERKEHRSZENTREN IN DER TSCHECHISCHEN REPUBLIK

Václav CEMPÍREK, Andrea SEIDLOVÁ, Jaromír ŠIROKÝ, Miroslav SLIVONĚ