IDENTIFICATION OF VISCOSOUS DAMPING COEFFICIENT OF HYDRAULIC MOTORS

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Katedra jakosti, provozní spolehlivosti a diagnostiky v dopravě

1. Introduction

Diagnostics methods are based on the measurement of various parameters and on the evaluation of their changes in longer intervals. In some cases it is possible to obtain only the values local of a few synchronously acting parameters by a straight measurement. The effects of the few synchronously acting parameters are often expressed by one non-local argument, which depends on a few partial parameters. If the characteristics of the real construction taken in an experiment do not correspond with requirements set in advance, it is necessary to change some partial parameter. The best way to do so is to use a simulation model and to change numeric values of the partial parameters in the simulation model first. After optimisation in the simulation model, we can change the partial parameters in a real construction. To build a simulation model, we have to know the numeric values and the functional dependence of every parameter. To make changes in a concrete construction, we need to know which physical material characteristics and which construction proportions influence (and how) the analysed non-local argument. This article focuses on the identification of the viscous damping coefficient, which has a straight effect upon the magnitude of damping forces. The viscous damping coefficient is a non-local dynamic parameter, which dependson the properties of the used fluid and on construction dimensions of the analysed hydraulic element.
2. Basic definitions

Describing dynamic aspects of a transmissive component, the resistance against a straight motion of an energy medium is expressed by a dumping force, which is linearly dependent on the velocity of motion. Damping force.

\[ F_b = b \cdot v \]  

(1)

Coefficient of proportionality \( b \) is called a viscous friction coefficient or a viscous damping coefficient or just a damping coefficient. Its dimension follows from the definition:

Damping coefficient:

\[ b = \frac{F_b}{v} \text{ [Ns/m]} \]  

(2)

The hydraulic force in hydraulic circuits, can be expressed as a product of the fluid pressure \( p \text{ [MPa]} \) and the area \( S \text{ [m}^2\text{]} \), on which the liquid acts. The total hydraulic driving force is then: \( F_H = p \cdot S = \sum \Delta p_z \cdot S \).

The hydraulic damping force is one component of all drag forces and its value is:

\[ F_{bh} = \Delta p_b \cdot S = b \cdot v = b \cdot \frac{Q}{S} \]  

(3)

where \( \Delta p_b \) means the pressure attenuation on hydraulic resistance against motion and it is:

\[ \Delta p_b = \frac{b \cdot v}{S} = \frac{b}{S^2} \cdot Q = R_b \cdot Q \]  

(4)

where \( R_b = \frac{b}{S^2} \text{ [N.s/m}^5\text{]} \) is a linear hydraulic resistance against motion.

There is also a quadrature hydraulic resistance \( R_K \), where the pressure attenuation is \( \Delta p_R = R_K \cdot Q^2 \).

Analysing hydraulic qualities of hydraulic components in technical practice, there are problems with the identification (setting the numeric value) of the damping coefficient \( b \). If we identify the value of the damping coefficient \( b \) correctly, there are no problems with setting the hydraulic resistance against motion because the numeric value of active place can be found easily. We can gain the value of the damping coefficient by a calculation based on the derived analytic relations or by the evaluation of measurements.

3. Analytic relations for hydraulic resistance against motion

A linear fluid motor will be chosen here as a representative of hydraulic elements. (Hydraulic cylinder). There are two basic resistances against motion in hydraulic cylinder. The first one is the resistance against motion of a piston with a piston bars. It is caused by the viscous friction of the piston in the cylinder and the piston bar in the caulking. This is the resistance against the motion of the solid energy bearer. The second one is the
resistance against the motion of a fluid energy bearer. Its reciprocal value is the leakage data throughput rate.

The leakage rate through the packing should be zero. Any oil drop must not escape through the packing into the surrounding area. Cuffs or other special sealing components usually seal the caulking. The same sort of sealing components can be used also for sealing a piston in a cylinder. By appropriate circuitry of an output and an input line in hydraulic shock absorbers, a leakage resistance is used for producing the damping force. The leakage data throughput rate in hydraulic cylinders with a piston sealed by cups is assumed to be zero. And the identification of the leakage resistance is not a part of this article.

3.1 Hydraulic resistance against piston motion in a cylinder

Analytic relations for the damping coefficient and the hydraulic resistance against piston motion in a cylinder will be derived by application of following simplifying assumptions.

- a piston with length \( \ell \) and with radius \( r_1 \) is placed in a cylinder with radius \( r_2 \), with gap \( 2\delta = d_2 - d_1 \), constant over the whole perimeter.
- the assembling gap forms an annulus of width \( \delta \) which is fulfilled with fluid of dynamic viscosity \( \mu = \nu \cdot \rho \). The width of the annulus \( \delta \) is much smaller than theme an radius of the gap \( r = 0,5(r_1 + r_2) \). There is an inequality \( \delta \ll r \).
- the piston moves by a constant speed \( \nu \). The fluid forms a wetting couple with the piston and the cylinder material. The fluid adheres on the piston surface and has here the same speed \( \nu \) as the piston. The fluid adhering on the wall on the cylinder has zero speed. The distribution of the fluid speed across the annular gap of width \( \delta \) is linear.
- there is the same (zero) fluid pressure in front of the piston and behind it. Output and input hydraulic lines are disconnected and brought together into a conservation reservoir with zero relative compressive.

Basic dimensions of the piston and the cylinder are designated on picture 1. (Shown).
Fig. 1 Moving piston in a cylinder with fluid.

Newton’s relation is valid for the stress in fluid caused by a viscous friction.

\[
\tau = \frac{F_b}{S_1} = -\mu \cdot \frac{dv}{dr}
\]  
(5)

With linear distribution of the fluid speed through the gap, the following formula is valid for the strain rate

\[
\frac{dv}{dr} = \frac{v}{\delta}
\]  
(6)

The friction surface which affects the shear stress \( \tau \) is \( S_1 = 2 \cdot \pi \cdot r \cdot \ell = \pi \cdot d \cdot \ell \).

Friction force:

\[
-F_{b1} = \tau \cdot S_1 = S_1 \cdot \mu \cdot \frac{v}{\delta} = \pi \cdot d \cdot \ell \cdot \rho \cdot v \cdot \frac{v}{\delta} = b_1 \cdot v
\]  
(7)

The negative sign at the friction force expresses the fact that the friction force acts against the piston motion whose velocity is \( v \). There is dependence for the viscous damping coefficient derived from the formula for the friction force.

\[
b_1 = \pi \cdot \rho \cdot v \cdot \frac{d\ell}{\delta}
\]  
(8)

The diameter \( d \) is, according to Fig. 1 an average of the diameters \( d_1 \) and \( d_2 \), which can also be expressed by following formula \( d = d_1 + \delta = d_2 - \delta \), as \( \delta << r \), nominal values of the diameter of piston and cylinder.

With moving piston, the viscous damping coefficient is directly proportional dependent on the density of the working fluid \( \rho \), its kinematic viscosity \( v \), the diameter of the cylinder \( d \), of the piston length \( \ell \) and reciprocal \( \delta \). To set the numeric values of the viscous damping coefficient \( b_1 \), it is useful to define to establish relative constructional parameters, which are constant for a certain hydraulic cylinder.

\[
k_\delta = \frac{d}{\delta}, \quad k_\ell = \frac{\ell}{d}
\]  
(9)

Formula (8) will get the following form after this substitution:

\[
b_1 = \pi \cdot \rho \cdot v \cdot k_\delta \cdot k_\ell \cdot d
\]  
(10)
According to formula (10), the value will of the viscous damping coefficient increases linearly with the cylinder diameter \( d \). Values of constructional parameters can be in a range of:

\[
k_\delta = 2 \cdot 10^3 \div 4 \cdot 10^4 \quad k_\ell = 0,5 \div 1,5
\]

(11)

Lower values \( k_\delta \) can be found at cylinders with very small diameters, the highest values \( k_\delta \) are valid for cylinders of the largest diameters.

To illustrate the calculation we can choose \( k_\delta = 10^4, k_\ell = 1 \). If the working surfaces are lapped, the constants \( k_\delta \) and \( k_\ell \) can be even higher.

For the control slide with two composition rings, the total length of the composition rings can be \( \ell = (2 \div 3) \cdot d \).

Mineral oil is usually used as working fluid. Its specific weight can be \( \rho = 900 \ [kg \cdot m^{-3}] = 900 \ [N \cdot s^2 \cdot m^{-4}] \). The kinematic viscosity of hydraulic oils depends on a sort of oil and working temperature. The working temperature of different hydraulic devices varies usually 25°C to 90°C. In all cases, hawed, the kinematic viscosity ( in the range of possible working temperatures, from ) be the following range :

\[
\nu = (16 \div 36) \ [mm^2 \cdot s^{-1}] = 10^{-5} \ (16 \div 3,6) \ [m^2 \cdot s^{-1}]
\]

(12)

There is a logarithmic chart of function - cinematic viscosity of hydraulic fluid dependent on temperature on picture two (following catalogue data from the producer of hydrostatical converters REXROTH m.b.h.).

![Fig.2 Function of hydraulic oil viscosity on temperature.](image)

For kinematic viscosity ranging according to Fig. 2, the dynamic viscosity will be:
\[ \mu = \rho \cdot \nu = 10^{-2} \cdot (1.44 \div 3.24) \text{ [N} \cdot \text{s} \cdot \text{m}^{-2}] \]

And the calculation constant will be:
\[ \pi \cdot \rho \cdot \nu = 10^{-2} \cdot (4.5 \div 10) \text{ [N} \cdot \text{s} \cdot \text{m}^{-2}] \]

By substitution \( k_\delta = 10^4 \) formula (9) will become:
\[ b_1 = 100 \cdot (4.5 \div 10) \cdot d \text{ [N} \cdot \text{s} \cdot \text{m}^{-1}] \quad (13) \]

A miniature hydraulic cylinder with piston diameter \( d = 10 \text{ mm} \) can have the viscous damping coefficient \( b_1 = 4.5 \div 10 \text{ [N} \cdot \text{s} \cdot \text{m}^{-1}] \).

A hydraulic cylinder with piston diameter \( d = 100 \text{ mm} \) will have the viscous damping coefficient \( b_1 = 45 \div 100 \text{ [N} \cdot \text{s} \cdot \text{m}^{-1}] \).

Higher values correspond to higher viscosity with lower temperatures. The shaded area of working temperatures and viscosities on Fig. 2 is a recommendation for manufacturers of hydraulic generators, which are sources of hydraulic power in hydraulic systems. It is obvious the Fig. 2 that VG 100 oil is assigned to high working temperatures. Using this oil for machinery working steadily with oil temperature \( 20^\circ \text{C} \) its working viscosity will be \( \nu = 400 \text{ mm}^2 \cdot \text{s}^{-1} \), which is much less than recommended values.

According to Fig. 2, starting viscosity for all mentioned oils is \( \nu_{\text{max}} = 1000 \text{ [mm}^2 \cdot \text{s}^{-1}] \).

With lower temperatures the numeric value of viscous damping coefficient \( b_1 \) calculated according to formula (8) will be far higher, as in the showed illustrative example. Also the constants \( k_\delta \) and \( k_\iota \) digger significantly.

Formula (8) was derived for a smooth piston without sealing elements. The easiest way of sealing of a piston with cups is depicted in a simple way on Fig. 3.

![Fig. 3 The generation of hydraulic fluid betwixt cuff.](image)

A fluid volume in the space between cuffs is \( \pi \cdot d \cdot \ell \cdot \delta \). In a cut on Fig. 3, the fluid between cups rolls so that on the surface of the piston it has the same speed \( \nu \) as the piston and on the surface of the cylinder it has zero speed. The fluid closed between cups
causes viscous friction, which determines the value of viscous damping coefficient following formula (8).

With viscous friction in fluid closed between cups, we must calculate radial forces $F_{R_1}$ and $F_{R_2}$, where the sealing cups touch the surface of the cylinder. Frictional force cups it is $F_T = F_R \cdot f$.

These not specified fiction forces of greasy friction are not generally dependent on the speed of piston motion but they are dependent on pressures $p_1$ and $p_2$ acting on the front of the piston.

A cylinder with one-sided piston bar will have a gland also sealed by at least two cups. There is fluid between them in the gland, which rolls between the piston bar and the body in the similar way as on Fig. 3.

The resultant viscous damping coefficient of hydraulic cylinder will consist of a sum of two components. The first component is formed from friction forces between a piston and a cylinder. Friction forces acting against the piston motion in the gland compose the second component. The second component will be lower than the first one because the piston bar diameter is smaller than the piston calibre. Hydraulic cylinders producers try to make the resistance against piston motion the lowest. Their calculation following derived formula (8) is burdened by a high inaccuracy, which is caused by undefined and uncalculated effect of sealing cups. The derived formula (8) provides good results only in calculation of viscous damping coefficient of control slides placed without sealing cups.

**Analytic description of resistance against piston motion**

With an active piston area $S_P = \pi d^2 / 4$ (on the side without a piston bar), the linear resistance against piston motion can be described by following formula:

$$R_b = \frac{b_1}{S_P^2} = \frac{16}{\pi} \cdot \rho \cdot v \cdot \frac{1}{\delta \cdot d^3}$$  \hspace{1cm} (14)$$

To overcome the resistance against piston motion a differential pressure $\Delta P_R = R_b \cdot Q$ will be used up and it reduces the value of pressure $p_1$ acting on the active piston surface $S_P$ in the course of piston motion.

The aktive piston surface on the side of the piston bar with diameter $d_3$ is:

$$S_P = \pi \cdot (d^2 - d_3^2) \hspace{1cm} S_P^2 = \frac{\pi^2}{16} \cdot (d^2 - d_3^2)^2$$ \hspace{1cm} (15)$$

And the hydraulic resistance

$$R_b = \frac{b_1}{S_P^2} = \frac{16}{\pi} \cdot \rho \cdot v \cdot \frac{d \cdot \ell}{\delta \cdot (d^2 - d_3^2)^2}$$ \hspace{1cm} (16)$$
4. Determination the viscous damping coefficient by measurement

The active piston surface is characterised by a viscous damping coefficient $b_1$ in hydraulic diagrams, as on Fig. 4.

A reversible distributor in a hydraulic circuit with blocked bores is in central zero position in an initial stationary state. In an arbitrary stationary state all power variables are constant, some of them beÁny zero. In an initial stationary state some of them are zero.

A distributor with a closed centre separates the hydraulic generator circuit from the hydraulic motor circuit.

The geometrical volumetric capacity of the hydraulic generator is $V_G$ and the revolutions are $n$. Total rate of flow $Q_G = V_G \cdot n \cdot \eta_{QG}$. In the circuit of hydraulic motor it $Q_1(0) = 0$, $n_1(0) = 0$ and a constant external loading force $Fz$ creates a constant initial loading pressure $p_1(0) = Fz / S_P$. Assuming the fluid is noncompressible, the pressure $p_2(0)$ on the back side of the piston is zero. The initial hydraulic gradient on the piston is then:

$$\Delta p(0) = \Delta p_Z = \frac{Fz}{S_P} = p_1(0) = \text{konst.} \quad (17)$$

Moving the distributor RP into position one, and up the switching the electric circuit to the left electromagnet control panel disturbs the initial stationary state. The switching is a skip. Ideal distributor moves by a skip in time $t_0$ equal zero from position zero to position one.

The pressure rises by the skip from $p_1(0)$ to $p_\nu = p_{\text{max}}$. Supposing the pressure $p_2 = p_\tau$ will stay zero throughout the whole transitional process, hydraulic gradients can be calculated as follows:

$$\Delta p(t) = \Delta p(0) + \Delta p_D(t) = \Delta p_Z + \Delta p_D(t) \quad (18)$$

where $\Delta p_D(t) = p_{\text{max}} - p_1(0)$ means an initial step change of the hydraulic gradient.
During the transitional process the following equation of force equilibrium is valid:

\[ F_H = \Delta p(t) \cdot S_p = (\Delta p_Z + \Delta p_D(t)) \cdot S_p = m \frac{d\nu(t)}{dt} + b_1 \cdot \nu(t) + F_Z \]  

(19)

Considering that \( \Delta p_Z \cdot S_P = F_Z \) we can reduce static forces in formula (19) and the equilibrium of dynamic forces will stay:

\[ \Delta p_D(t) \cdot S_p = m \frac{d\nu(t)}{dt} + b_1 \cdot \nu(t) \]  

(20)

This formula can be rewritten as an equation of equilibrium of pressure decrements:

\[ \Delta p_D(t) = \frac{m}{S_p^2} \frac{dQ_1(t)}{dt} + \frac{b_1}{S_p^2} \cdot Q_1(t) = H \cdot Q_1(t) + R_D \cdot Q_1(t) = \Delta p_H(t) + \Delta p_R(t) \]  

(21)

We also have to include the equation of the rate of flow distribution

\[ Q_G = Q_V + Q_1 \]  

(22)

An ideal escape valve \( V_P \) keep pressure \( p_V = p_{\text{max}} \) constant, if the rate of flow is not zero. As soon as pressure \( p_1(t) \) drops under \( p_V = p_{\text{max}} \) level during the transitional process, the escape valve closes down and \( Q_V = 0 \), \( Q_1 = Q_G \) is valid for rates of flow.

From this moment the piston speed is \( \nu = Q_G / S_P \). This can happen when a hydraulic generator will be under-designed with a small geometrical volumetric capacity \( V_g \).

If the hydraulic generator is over designed with a big geometrical volumetric capacity, the escape valve will be opened during the whole transitional process, the rate of flow \( Q_V \) will not be zero, the rate of flow on the hydraulic motor entrance will be \( Q_1 = Q_G - Q_V \), pressure \( p_1 \) will keep the opened escape valve constant \( p_1 = p_V = p_{\text{max}} = \text{const} \).

In a new stationary state (in time \( t = t_U \)), all power variables are constant and the acceleration is zero. The resistance against acceleration is not working and following formula is valid for the equilibrium of constant forces:

\[ \Delta p_D(t_U) \cdot S_P = (p_1(t_U) - \Delta p_Z) \cdot S_P = b_1 \cdot \nu(t_U) \]  

(23)

Formula (23) calculated the value of the viscous damping coefficient.

Will be derived coefficient and can be influences the working life. The viscous damping coefficient is a global diagnostic parameter depending on used materials characteristics and on constructional admeasurements of analysed hydraulic element and amplitude dimension of mechanic parts oscillation.

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Literatura


Resumé

IDENTIFIKACE KOEFICIENTU ViskoZNÍHO TLUemenÍ HYDRAULICKÝCH VÁLCŮ

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Summary

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This article is about an identification of viscous damping coefficient of hydraulic elements. The viscous damping coefficient is one of the most important diagnostic parameters, influencing the driving characteristics of real devices and the amplitude of mechanic parts oscillation. Thus it influences the working life and the operational reliability too. Viscous damping coefficient is a global diagnostic parameter, which depends on used materials characteristics and on dimensions of the analysed hydraulic element. In this article the analytic relations for the calculation of viscous damping coefficient of hydraulic cylinders will be derived as will as, the methodology of experimental definition of its numerical value.

Zusammenfassung

KENNZEICHNUNG DES FLÜSSIGEN DAMPFUNGSKOEFFIZIENT DER HYDROZYLINDER

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