INFORMATION MINING FROM THIN EVALUATION SCALES

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Abstract: *Different scales are used at evaluation of various objects and effects. Problems arise above all in economy and other social sciences, where it is impossible to carry on evaluation by the exact measuring devices, but a scale and self evaluation are formatted at least in part intuitively. The paper deals with both possible approaches to the description of objects and events and problems, rising when thin evaluation scales are used.*

Keywords: *thin scale, variability of the event, confidence interval, empirical distribution function, approximation by continuous distribution*

1. Description of the event and its variability

The basic problem of every scientific area including economics is a description of the objects and events that are investigated. We consider the objects and events already existing (past and current), or future, that are estimated or designed. We will use only the term event[1] in the following text. The description is evaluated from the point of relation presented in the figure.1.

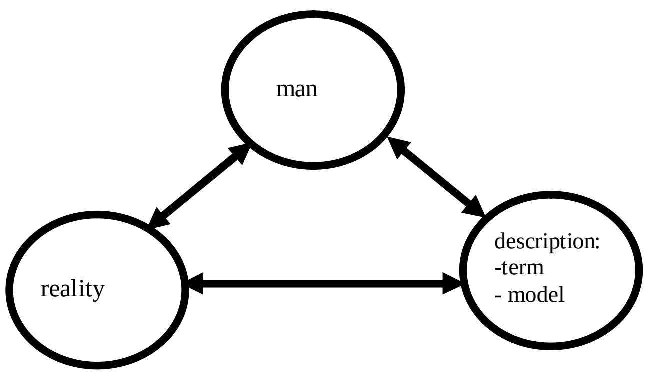

Fig.1 Relation: reality – description – man

Description of an event is always simplified both in perception, and at model creation. It is demanded at the same time, so that the description to the nines represents reality. We have however usually disposable only limited quantity of information with certain variability at description of a reality. Possibility of verification of the information is important regarding to this variability. We can verify the information by repetition of the observation or experiment in some cases. Repeatability does not exist in the area of technical and natural sciences and our chances are relatively limited. The task of the paper is not the general problem of information gathering. We only remind that proportion of simplification of the description is given by the quantity of the information and by their variability. The situation is demonstrated schematically in the figure 2.

[1] The event is more likely considered to be the materials event in the static state, for example the car. The event includes the objects in the certain system and dynamics. It is possible to consider it more general.

A difference between the model and reality is given partly thereby, how the model is created and further by indeterminateness of the particular properties of the model.

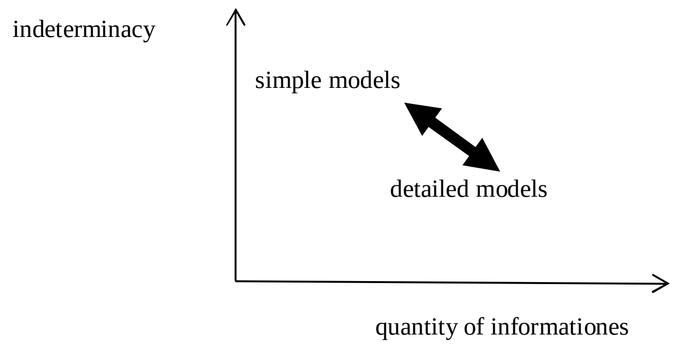

Fig.2 scheme of details of the description or models

It is important, at the model creation, to include:

- substantial elements,
- substantial relationships,
- important limits (separability).

Knowingly determined properties can be theoretically considered to be substantial or significant. Practically are significant only such properties that can be sufficiently identified and described. In this fact consists above suggested fundamental problem of the description and simulation of the events.

The events can be described with verbal terms (so-called nominal) and quantities cardinal and ordinal. As far we want to quantify the objects, we have to convert the verbal terms into quantities. For example performance of the formula can be expressed binary 1 – 0, or with a formation of a verbal appreciation with 5 levels, that is converted to scale according to the table 1. A linear scale and constant intervals were used in the example. A nonlinear scale degressive or progressive can be used as well.

Tab. 1 Example of a conversion of verbal evaluation to quantity

very good	2
good	1
I haven't opinion	0
bad	-1
very bad	-2

At the description always rise, let us say, exist variability that is due:

- variability of the very event,
- variability of information (accuracy of investigation or measuring),
- investigator, man, who creates a model description.

Otherwise variability could be structured only to variability of a discovery method regardless, whether it is caused by investigator or measuring device, and to variability of a event. If variability is described by square of standard deviation (variance), then providing independency holds true

$$s_v^2 = s_j^2 + s_m^2 + s_c^2 \qquad (1)$$

where s_v is the resulting standard deviation, s_j is standard deviation of the event, s_m is standard deviation of investigation (measuring or method), s_c is standard deviation caused by the man. To determine the variability s_c is very difficult, it depends on the subjective description of the model. Components s_j and s_m are usually used with regard to complications how to determine the component s_c.

If we've possibility to repeat the experiment on the assumption of the constant properties of the event, then we can experimentally ascertain the error of the method. The upper estimate of the error of the estimate of the parameter μ on the assumption of normal probability distribution of population, is

$$\varepsilon = t_{\alpha,n-1} \cdot \frac{s_m}{\sqrt{n-1}} \qquad (2)$$

where 1-α presents the probability, that this estimate will not be overrun and n is number of used measurements.

The estimate of the confidence interval for the mean of the random variable X is

$$I_{1-\alpha} = \left\langle \overline{X} - t_{\alpha,n-1} \cdot \frac{s_m}{\sqrt{n-1}};\ \overline{X} + t_{\alpha.n-1} \cdot \frac{s_m}{\sqrt{n-1}} \right\rangle, \quad \text{where} \quad t_{\alpha,n-1} = F_{t_{n-1}}^{-1}\left(\frac{2-\alpha}{2}\right) \qquad (3)$$

Repeatability cannot be generally assumed when problems of the economic practice are solved and variability of the method must be estimated. The values of the random variables in the economic practice can be determined in two ways:

- by rigorous measurement,
- by subjective evaluation.

When rigorous measurement is applied, the variability is determined according to used method and other factors. For example a length measured with common meter has error ±0,5 mm, profit is approximated to 100Kc, then the error is ±50 Kc etc.

When subjective evaluation is practiced, for example a profit evaluation, a future state evaluation etc. a variability depends on a discrimination ability given by human psyche. The social-psychological research deals with subjective evaluation. For example Nakonecny [1] discusses a measurement of a position. Hayesova in [2] states in pg. 112 so-called Likert scale with 5 points[2]. A lot of attempts about subpartion of the scale were done; it is described in Nakonecny in [1] on pgs. 38 and 39. A method with the scale about 11 points is introduced. When this method is applied, the statements in which the reviewers mostly differ are removed and after adjustment 5 till 9 points scale is created. Haysova [2] pg.113 and Nakonecny [1] pg. 39 state so-called semantic differential Osgooda that uses 7 points for specific positions.

It results from the signed extensive psychological and sociological research, that subjective scales should include 5 till 10 points. Such thin scales conform to natural thinking about possibility of reception and strong individual difference of visions and opinions of individuals. The absolute error of the record at 5 till 10 points scale is 0,5. The relative errors in the middle 2,5 of the 5 points scale is ±20%, in maxima ±10%. The relative error in the

[2] The typical example is the appreciation of students´ knowledge.

middle of the 10 points scale is ±10%, in maxima ±5%. The mentioned relative errors even at 5 points scale answer to the common economic estimations.

2. Thin scales in general

We are in principle interested in not only characteristic of position, but even characteristics of variability, at objective description of the changeable, uncertain events. Our decision is unrolled from the all confidence interval. When subjective evaluation is practiced, variability is determined both with variability of the event, and with subject of evaluation (knowledge, position etc.).

The group evaluation is necessary at decision making about public matters. It is implemented either among citizen, or in the expert groups[3]. Information range[4] is given by the number of the points of the scale multiplied by the number of the informants. We impoverish for the range of the information source if the results of the analyses are limited, let us say, rounding only to the points of the scale.

The scales with the range about 10 points are called thin. If normality of the population distribution is presumed, or at least symmetry, than we suffice with average and standard deviation as statistical characteristics of the sample. The presented presumptions are more likely extraordinary in the economical problems and especially at subjective evaluation. Practical frequency distributions are discrete and mostly unsymmetrical. We've possibility to excavate more accurately values at the thin scales, than provide points of the scale. For example, in the five points scale, we will have frequencies of the individual points according to the figure 3.

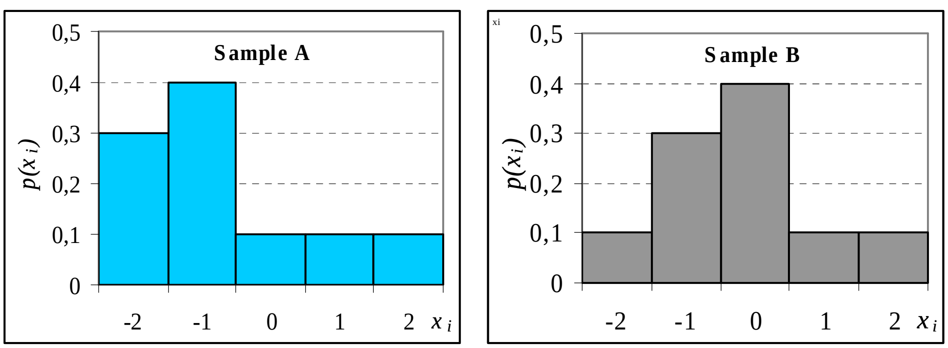

Fig.3a,b Examples of the relative frequency distribution, sample A and sample B

If location of the samples is evaluated by average, we obtain $\bar{x} = -0{,}14$ at the sample A and $\bar{x} = -0{,}05$ at the sample B. It is evident in the case A that this distribution is skewed. Obtained values of median are $\tilde{x} = -1$ for the sample A and $\tilde{x} = 0$ for the sample B. These values of average and median probably don't represent very well the position of the sample. There exist different methods when the discrete values of the class sign and corresponding frequency distribution are approximated by a continuous function.

The principle, and at the same time weak point of this approach, is choice of the model of probability mass function at single class intervals.

[3] The subjective evaluation realized by the expert group provide, under assumption that this group is suitably compounded from experts, high-quality results less influenced with group interests.
[4] The content is given by probability of the information.

2.1 Classical solution

Classical way of solution consists in construction of the empirical distribution function. Random sample of the range *n* from distribution *F* is considered again. *x* is arbitrary real number. m_x indicates a number of values of the sample, that are less than number x, $x \in$ R.

The quotient $\frac{m_x}{n}$ expresses relative frequency of the values of the statistical character, that belongs to the random sample and that are less than the value *x*. This frequency is function of the variables *x*, it is indicated $F_n(x) = \frac{m_x}{n}$ and it is called empirical distribution function. This function can be considered as estimate of the population distribution function *F*. If $(x_1, x_2, ..., x_n)$ is any realization of the random sample and if $x_1 \leq x_2 \leq .... \leq x_n$, then empirical distribution function F_n is more often expressed in the form:

$$F_n(x) = \begin{cases} 0 & x \leq x_1 \\ \frac{i}{n} & x_i < x \leq x_{i+1} \quad i = 1,2,...,n-1 \\ 1 & x > x_n \end{cases}$$

The empirical distribution function for the model of the sample A (figure 3) is illustrated in the figure 4.

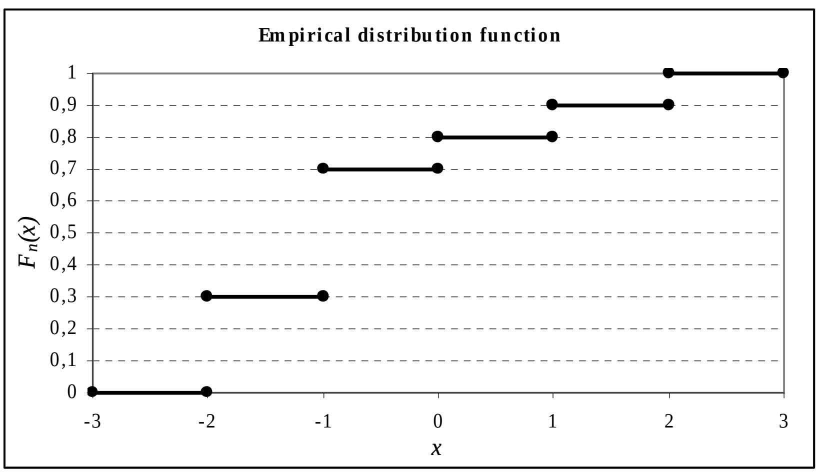

Fig. 4 Empirical distribution function

We are able to construct this distribution function even without presumption about probability distribution. It provides approximated estimation of the population distribution function.

2.2 Alternative solution

A model with constant probability density function in separate intervals with width 1 and middle in five points from example in figure 3 is suggested in the following part of this paper. For example interval $x \in (0{,}5; 1{,}5)$ is considered for the point 1. Numeric value of the density is identical to the discrete relative frequency. The task is to form the distribution function *F* and to find the value *x* for example for median, where *F*(*x*) = 0,5 and for limits of 0,95% confidence interval. Distribution function is constructed to be linear in separate intervals. The example corresponding to frequencies from the picture 3A is stated in the figure 5.

General problem is to find the value x for determined $F(x) = H$. Individual intervals corresponding to class characters are marked generally with the index i, $i \in \langle 1; n \rangle$, the explicit interval then j. It is valid in the interval j

$$F_j(x) = \sum_{i=1}^{j-1} p_i + p_j (x - x_{jD}) \tag{4}$$

where p_i and p_j are probability density functions in individual intervals, x_{jD} is lower limit of the interval j, x_{jH} is upper limit of this interval.
Let's select the interval j in which is $F(i)=H$, also

$$\exists j \in \langle 1; n \rangle \,\Big|\, \left[F(x_{jD}) \leq H < F(x_{jH}) \right] \tag{5}$$

The value of the distribution function $F(x)=H$ is also in the interval j and for the searched x_H holds true

$$x_H = x_{jD} + \frac{H - \sum_{i=1}^{j-1} p_i}{p_j} \tag{6}$$

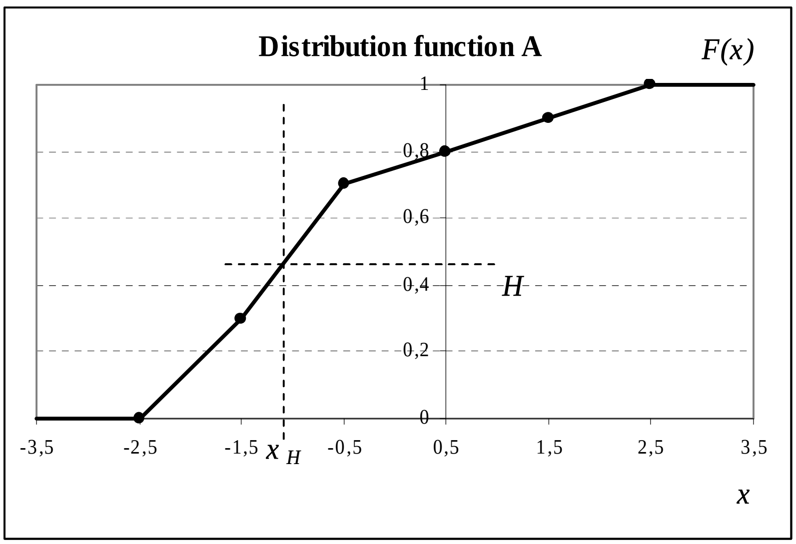

Fig. 5 Determination of the value x

According to the formula (6) we can determine median and characteristics of variability. In economics, sociology and other social sciences variability has importance to illustrate above all oscillation of properties of the evaluated event and person making evaluation. The confidence limits have in this case other meaning than „credibility limits" and determination of them offer much interesting possibilities with different predicative interpretation.

Assessment of the error of x is disputable from the standpoint of merit in case of subjective evaluation. To use the Gaussian approach can be here rather misleading. Practical decision-making situation of subjective evaluation in thin scale consists in selection of the point j of uncertainty in the sets $\{(j-1); j; (j+1)\}$.[5] The uncertainty is determined with the intersection of the sets corresponding to these points.

3. Thin scales and approximation with normal distribution

[5] The decision making situation is simpler at the end points of the scale, it covers only two points.

The following simulated example shows probability of decision making at five-point scale.

We supposed, that majority of persons who make evaluation evaluate objectively according to property of the assessed object. But we supposed as well, that there have been persons making evaluation with tendency to negative conclusions, in the same way anyhow persons making evaluation with opposite tendency. Under these presumptions we can consider, that approximation with normal probability distribution is possible. We simulated five samples, in sequence from $N(1;\ 0{,}65)$, $N(2;\ 0{,}6)$, $N(3;\ 0{,}55)$, $N(4;\ 0{,}6)$, $N(5;\ 0{,}7)$ distributions and smoothed the continuous probability density function (figure 6).

It is evident in the picture 6, which way and with which probability can person making evaluation commit an improper evaluation. The greater is a variability of evaluation, the greater is a possibility of mistakes. We consider for example „ the middle" Gaussian curve with the mean 3. The object is evaluated really with the degree 3 of the evaluating scale with certain probability. It is, as a rule, if $x \in \langle 2{,}5;\ 3{,}5)$.

The object can be evaluated even with the degree 2 with probability $P(x < 2{,}5) = F_{\mu,\sigma}(2{,}5) = \Phi((2{,}5-3)/0{,}55) = 0{,}182$, where Φ indicates the distribution function of the standard normal distribution with the parameters 0 and 1. It is obvious that the probability of incorrect evaluation (mistake in evaluation 3 by evaluation 2) is 0,182. With regard to symmetry of normal distribution, the incorrect evaluation in the opposite tendency has the same probability

(mistake in evaluation 3 by evaluation 4) also as well 0,182.

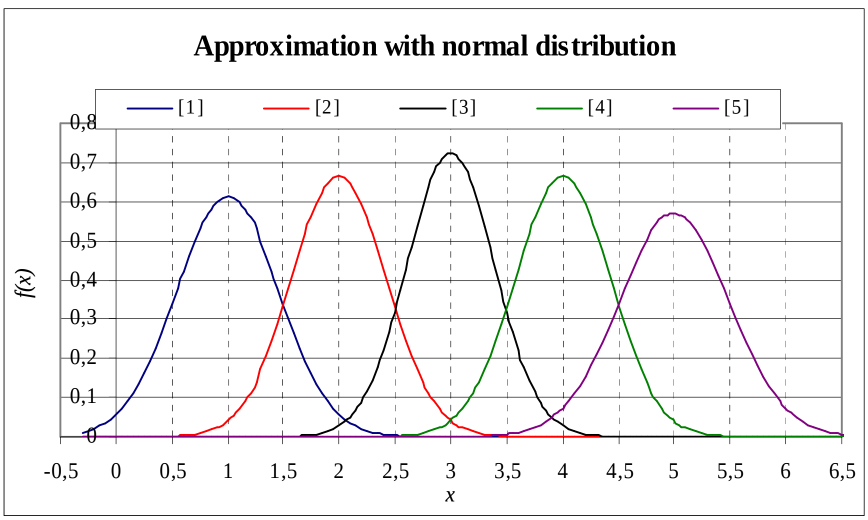

Fig. 6 Approximation with normal distribution – five points scale

Probability of improper evaluation can be determined in a similar way for all points of the scale. We can then estimate the total probability of improper evaluation on the basis of above mentioned calculation. It is necessary to pay attention to the end point values of the scale, because probability of incorrect evaluation used to be here lower.

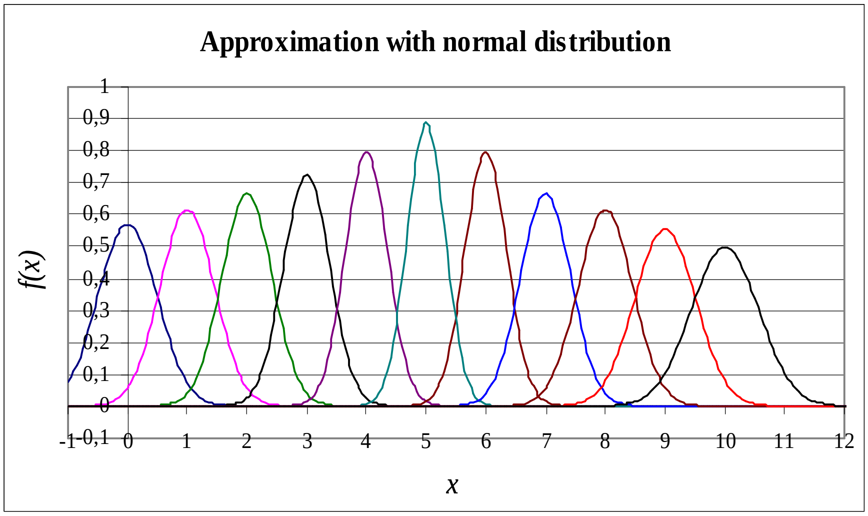

Fig. 7 Approximation with normal distribution – eleven points scale

Problem of subjective evaluation is illustrated in picture 7. Although the probability distribution for every scale item is almost ideal, we can see, that at greater number of degrees of the scale is the correct evaluation difficult. It is possible to confuse even about more than one step.

One important question emerges to what degree is a person making evaluation capable to distinguish individual levels of the scale and how many levels is capable to differentiate. As far as the targets of our observation is to determine confidence intervals for estimates of individual parameters, quantil values or to perform other statistical analyses, classical statistical method are used, described for example in [3].

4. Conclusion

Application of thin scales at evaluation carries along losses of information. The way of approximation of the discrete scale by the continuous function was suggested in the article. Second task was to draw attention to mistakes, which can arise at evaluation in extreme values of the thin scale. Problems of thin scales are very extensive and require far more detailed elaboration.

Remark

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