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COST MODELLING IN CHEMICAL PRODUCTION

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The paper deals with costs and their models as one of instruments for revitalization in chemical companies. The first part treats the construction of the technological production and consumption functions and their transformation into the cost and revenue functions. Then these functions are used for optimizing the economic results of a particular production. The technological functions are obtained usually by experimental measurements or by derivation from the regularities of chemical reactions. The cost and revenue functions and their parameters are then derived by simulation and/or regression analysis. In the second part, possibilities of utilization of the cost input-output models for creation the system and dynamic cost calculations in chemical multistep production are described. Two examples taken from practice are given as an illustration of the indicated possibilities.

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Introduction

A radical restructuring, usually combined with revitalization, has been a current and urgent task in a lot of our industrial companies recently. The aim of these processes is to support the company's competitiveness in demanding markets and to increase the effectiveness of production and the sales.

The pressure on savings of every sort and on the cost reduction is an integral part of restructuring and subsequent revitalization. The product cost calculations provide the basic information for decision-making on costs of individual products and services and on costs of total output.

At the same time the research conducted in a few chemical companies recently has pointed at a lot of drawbacks and problems in making up and utilization of costing:

a) A costing system, its objective, assignments and users are not explicitly defined.
b) Particular elements of costing system are not connected to a common database; the software utilization for calculations is fractional and non-systematic.
c) Periodicity of costing making is not solved according to the real requirements. Annual resulting costing is mostly made up late and is not consistently used for the control of planning costing.
d) The way of an indirect costs allocation is not transparent enough and since managers as users do not know it, they cannot use and interpret cost calculations correctly.
e) It is usually worked with the costing of particular products separately; connections that ensue from system interconnection of an individual production are not respected.
f) Incomplete and dynamic cost calculations that enable optimizing of production assortment and quantity of production on the basis of a contribution, break-even point analysis and cost functions are used only sporadically.

The *cost models* can play an important role just in solution to two last mentioned problems. In our paper we deal with the possibilities of their utilization in chemical production.

Cost Functions

Cost functions are the main instrument of the dynamic costing. These functions express the dependence of costs on the volume of production and possibly also on further factors that have a causal relation to the amount of costs.

The so-called *technological functions* could be the basis of the optimizing of production costs. In general interpretation, the functions express quantitative dependences of inputs and outputs of the production (technological) process on certain parameters of this process, for example, temperature, pressure, concentration, *etc*.

Chenery B.H. has been considered to be the founder of technological method of the cost functions derivation [1]. Ferguson's technological consumption function of fuel for an airplane has been considered first practical application [2]. Ferguson expressed this relationship as a function of 18 technological variables.

The determination of the technological function for a certain particular production process is mostly very laborious and often even a methodically difficult matter. It requires close collaboration of engineers and economists. The individual steps necessary for determination of technological function include:

- to take the production process to partial processes, possibly to phases and to production operations,
- to identify inputs and outputs at each partial process, and to explore their mutual impacts,
- to find out what technical and technological properties are important and determining for the process examined,
- to formulate a hypothesis on mutual dependences of input and output variables and
- to verify this hypothesis by empirical examination, measuring and assessment.

Technological functions are sometimes understood as a certain development and generalization of consumption functions. For example, we can find in Ref. [8] the following comparison of technological method with Gutenberg method according to analysis of factors which influence costs: unlike Gutenberg method, based on the consumption function that expresses dependence of consumption only on one independent variable, the technological method operates with more than one variable.

Our conception of technological function is, however, even wider in view of experience in chemical production. In particular we suppose that the consumption function does not need to express the dependence of consumption of production factors on one variable only. For example, the consumption of raw material (and its utilization) often depends on its properties (composition, purity, concentration, *etc.*) but also on parameters of technological process (temperature, pressure, reaction time, *etc.*). However, it is possible to do more in the direction to become widespread. In our opinion it is possible to put even the natural production functions into the technological functions. These natural functions express dependence of outputs (products or services) on input factors and on technical or technological quantities and parameters of the process. The point is that we could consider the natural production functions as the inverse to consumption functions and *vice versa*. For example, the same technological quantities and parameters that affect consumption of raw material also indirectly affect the product yield and *vice versa*.

The technological functions include two fundamental forms in our conception:

- Production functions – express the dependence of production volume at a certain quality on quantity of consumed production factors and on further technical (technological) parameters of process, for example, temperature, pressure, viscosity, *etc*.

  The production function can formally be written as

$$Q = f(x_1, x_2, ...x_i, ...x_n, a_1, ...a_h, ...a_k) \tag{1}$$

where x_i – separate production factors (inputs) and quantities, for $i = 1, 2, ...n$,

a_i – parameters of the production technological function, for $h = 1, 2, ...k$

- *Consumption functions* – express dependence of consumption of separate production factors (raw materials, energy, working time, *etc.*) on certain technological parameters and on the quantity of demanded products.

  The consumption function for factor *i* can be, in general, written as follows

$$S_i = f(q_1, ...q_j, ...q_m, b_1, ...b_h, ...b_k) \tag{2}$$

where q_j – volumes of products, for $j = 1, 2, ...m$

b_h – parameters of technological function, for $h = 1, 2, ...k$.

It has been determined in a number of production processes that consumption of individual production factors are mutually dependent, often indirectly. In practice this means that the better utilization of first factor the worse consumption of second factor and *vice versa*. So, for example, it is possible to improve the utilization of raw material in a batch by extending the reaction time in discontinual chemical production. However, this usually means a worse time utilization of apparature, i.e. further production factor. These substitute dependences of consumption functions of individual factors can be the basis for the optimizing of total cost functions.

- In some production processes it could be favourable to express the technological function as a function of *losses*. We can use it if there are losses gradually in separate phases within the technological process and these losses can be balanced and specified in certain way.

The function dependences and parameters of functions are then deduced either from a qualitative analysis, from an analogy to similar examined processes or from a statistical analysis on the basis of some experimental verifications, laboratory tests, *etc*. A simple and multiple regression analysis is the most frequently used statistical method.

If we evaluate the technological consumption functions according to a given purpose at suitably chosen price, we will receive the cost functions. A compound cost function could be received if we join more partial cost functions

$$N = f(S_1, \ldots S_i, \ldots S_n, p_1, \ldots p_i, \ldots p_n) \qquad (3)$$

where S_i – the total consumption of factor i according to consumption function,

p_i – factor price i, for $i = 1, 2, \ldots n$ production factors.

In this case the cost function is a compound function and through the consumption of production factors expresses the dependence of costs on physical quantities of technological process.

By a different, often opposite, course of consumption and cost functions we can make an attempt on the optimizing of relevant costs. As an illustration let's state the following example:

Case 1

Stibnite (ore) is a basic raw material in production of antimony. The ore is first calcined and the antimony dioxide obtained is reduced by carbon in the next stage. The reaction can be described as follows

$$Sb_2S_3 + 5\,O_2 \rightarrow 2\,SbO_2 + 3\,SO_2 \qquad SbO_2 + 2\,C \rightarrow Sb + 2\,CO$$

The ore is ground before calcination. The grain size has influence on antimony losses in ash and in flue gasses. Surface of raw material becomes smaller with increasing grain size resulting in imperfect transformation of sulphide to dioxide and higher losses of antimony in the ash.

On the basis of analogy to similar processes it is possible to approximate the dependence of losses of antimony in ash by the function

$$Sb_{(\text{in ash})} = 0.89 \times 1.41^x, \quad \text{where } x \text{ is the grain size in mm for } x \in \langle 1;\ 10 \rangle$$

On the other hand, losses of antimony grade down with increasing grain size as a consequence of its going off in flue gasses according to quadratic equation

$$Sb_{(\text{in flue gasses})} = 20.94 - 2.63x + 0.1125x^2$$

If each kg of antimony in ash or in flue gasses is evaluated as a loss in profit 200,- CZK, we will obtain two partial cost functions

$$N_1(\text{CZK}) = 200(0.89\times1.41^x)$$

and $$N_2(\text{CZK}) = 200(20.94 - 2.63x + 0.1125x^2)$$

their sum is then compound cost function

$$N_C(\text{CZK}) = 200(0.89\times1.41^x + 20.94 - 2.63x + 0.1125x^2)$$

Courses of all three cost functions are shown in Fig. 1.

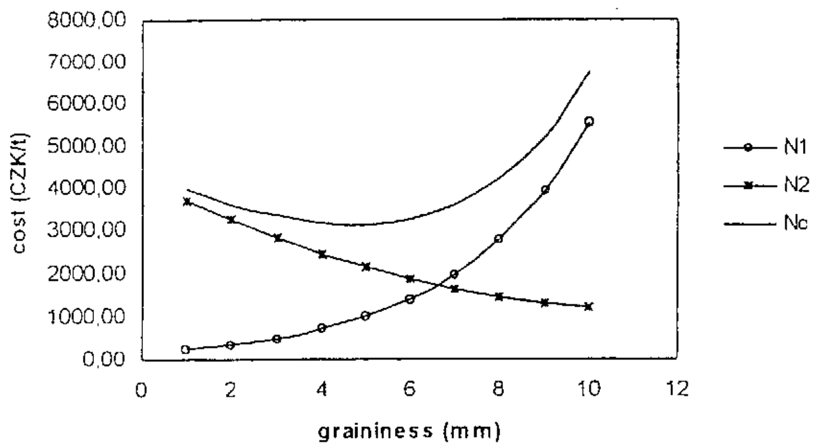

Fig. 1 Dependence of costs on grain size

It follows from simulated course of this function that it would be optimal to set and keep grain size in the interval of at 4 – 6 mm.

The technological method enables to determine the minimum costs and/or losses only for the costs that are directly connected with consumption of production factors. So, it does not make possible to determine the optimum of the other costs, for example, rent, insurance premium, interests, *etc.*

In practical application the use of this method is suitable particularly during development of new products and technologies when it is possible to install the optimizing procedure of technological costs directly into project and technological preparation of new production.

Even the revenue functions can be used for optimizing of economic results. The revenue function can continue the natural production function. Therefore, in this case, we will get the revenue function from the evaluation of *outputs* of production function (i.e. outputs in natural expression) at a purposely chosen price. Then a difference between revenue and cost function is a *net contribution* function. We can obtain functions of contribution or profit functions in individual cases according to the costs included in the cost function.

It is possible to find more different technological functions especially in chemical production and then a great number of them can be a potential source of cost savings and profit-cost optimizing of these kinds of production. In practical utilization a teamwork of technologists and economists is not only desirable but an absolutely essential condition of success.

Cost Input-Output Models

Principles of system costing are often broken in multistep production that is very frequent in chemical industry. The requirement of system costing is based on the fact that it is impossible to make up and assess costing of individual products separately but with respect to every further products. Every change of produced quantity or any cost items of one product can have an influence on costing of the next or even all products in the system.

This approach requires to specify unambiguously the system bounds (group of products that pertain to a system) and, hence, even to define inputs and outputs of system. Then it is possible to differentiate external costs that are caused by a consumption of external sources from interial costs that are caused by a consumption of internal sources. The basic problem is in transformation of external costs into the internal costs that could change their character. This change could misrepresent the costing. Next we demonstrate how the cost input-output model could contribute to solving the above-mentioned problems.

Cost input-output model is a modification of balance input-output model; Leontief [5] has been considered to be the founder of this model applied on state economic level.

Cost input-output model is derived from the product balance input-output model that can be applied on a firm as well intracompany level. Vysušil [9,10] deals with cost input-output models on general level, Gros [3], Machač [6] deal with applications of this models in chemical production.

A balance input-output model for a production system (group of products, production unit, company, *etc.*) Where the n elements (intermediates and products) are produced and where material flows exist among them can be described by a system of balance equations

$$x_i = a_{i1}x_1 + ...a_{ij}x_j + ...a_{in}x_n + y_i \tag{4}$$

where a_i – coefficients of direct consumption of i^{th} product an j^{th} product,
x_i – volumes of total production of particular products for $i = 1, 2, ...n$ and
y_i – volumes of final production of j^{th} product for $i = 1, 2, ...n$.

Balance equations show how much of total production of particular products and intermediates is assigned to it's own need and how much of the final production remains for sales or increase of supply.

If we use the matrix form, the system of balance equations becomes

$$\boldsymbol{x} = \mathbf{A}\boldsymbol{x} + \boldsymbol{y} \tag{5}$$

Then we can solve two basic types of problems for balance planning:

1. The vector $\boldsymbol{y}$ is derived from the set vector $\boldsymbol{x}$

$$\boldsymbol{y} = (\mathbf{I} - \mathbf{A})\boldsymbol{x} \tag{6}$$

where $\mathbf{I}$ is identity matrix

2. The vector $\mathbf{x}$ is derived from the set vector $\mathbf{y}$

$$\boldsymbol{x} = (\mathbf{I} - \mathbf{A})^{-1}\boldsymbol{y} \tag{7}$$

Elements of the matrix $(\mathbf{I} - \mathbf{A})^{-1}$ are known as coefficients of the full consumption. They play an important role in the cost models.

The model (4) is, from a system point of view, open on its way out only. If we want to extend a balance bonds even to the external inputs, we have to extend the model with the balance of direct raw materials and other materials.

Thus the equation of the consumption of the i^{th} raw material is

$$b_{i1}x_1 + ...b_{ij}x_j + ...b_{in}x_n = b_i \tag{8}$$

for $i = n + 1, ..., n + m$ raw materials,

where b_{ij} – consumption coefficient of the i^{th} raw material on product j,
b_i – total consumption of the i^{th} raw material on all products

If we denote a matrix of consumption coefficients as $\mathbf{B} = [b_{ij}]$, we figure out the vector of the total consumption of raw materials

$$b = \mathbf{B}x \tag{9}$$

or after substitution $\mathbf{G} = \mathbf{B}(\mathbf{I} - \mathbf{A})^{-1}$ we will obtain

$$b = \mathbf{G}y \tag{10}$$

where the elements of **G** matrix are coefficients of the full consumption of raw materials on intermediates and products.

Cost Input-Output Model for Incomplete Costs

The simplest cost model starts from the widespread balance model, and it is established on the following assumptions. External costs are only direct and variable costs and internal costs include only the consumption of intermediates and products on each other. This consumption is evaluated at cost of external direct inputs.

The result of the model calculation is to ascertain just this evaluation of internal consumption of intermediates. Any cost items are variable in this model and thus the model always correctly reacts to changes of manufactured or sold amount of particular products.

A consumption of raw materials, intermediates and products on certain, eg. j^{th}, product is found in the j^{th} column of the matrices **B** and **A**. This is why the cost equations start from the transposed matrices $\mathbf{B}^{\mathrm{T}}$ and $\mathbf{A}^{\mathrm{T}}$.

The cost equation for the j^{th} product is

$$n_j = \sum_i a_{ij} n_i + \sum_i b_{ij} c_i \tag{11}$$

where c_i – acquisition prices of raw materials and of the other direct external sources,
n_i – internal unit costs of consumed intermediates and products.

The transposed vectors $\boldsymbol{n}^{\mathrm{T}}$ and $\boldsymbol{c}^{\mathrm{T}}$ are obviously in matrix form as follows

$$n^{T} = n^{T}A + c^{T}B \qquad (12)$$

The matrix Eq. (12) is solved with respect to n

$$n^{T} = c^{T}B(I - A)^{-1} = c^{T}G \qquad (13)$$

It is obvious from the Eq. (13) that vector n^{T} depends only on the evaluation of external inputs and on the comprehensive coefficients of the external inputs consumption on intermediates and products. Thus the costs of consumed intermediates are transformed external costs on input sources.

Then total variable costs of the total volume of products are

$$N(X) = n^{T}X_{d} = c^{T}GX_{d} \qquad (14)$$

where X_d is a diagonal matrix derived from the vector $\mathbf{x}$

As an illustration let us give an example:

Case 2

The consumption coefficients of intermediates and products, purchase prices of raw materials and sales requirements of products at a plant with a production of the dye intermediates are shown in Table I.

Table I Basic consumption, production and sales data

Intermediates	Consumption coefficients, $t\,t^{-1}$				Sales, t
	1.	2.	3.	4.	y
1. Phthalic anhydride	-	1.4	-	-	50
2. Benzoic acid	-	-	0.9	0.5	0
3. Sodium benzoate	-	-	-	-	100
4. Benzyl chloride	-	-	-	-	80

Table I – continued

Raw materials					Price CZK t^{-1}
1. Naphthalene	1.2	-	-	-	8000
2. Chlorine	-	-	-	0.8	2300

First we will solve the balance model. Matrix (**I** - **B**) is derived from matrix **A** (matrix of the consumption coefficients). Then the vector of total production $\boldsymbol{x}$ is obtained if the inverse of (**I** - **A**) is multiplied by the vector of sales $\boldsymbol{y}$.

	$(\mathbf{E} - \mathbf{A})^{-1}$			y	$= x$
1	1.4	1.26	0.7	50	232
0	1	0.9	0.5	0	130
0	0	1	0	100	100
0	0	0	1	80	80

The next matrix **G** (matrix of the comprehensive coefficients of consumption of raw materials) is calculated from the relationship $\mathbf{G} = \mathbf{B}(\mathbf{I} - \mathbf{A})^{-1}$, and the total consumption of raw materials in the manufacturing programme is obtained by multiplying **G** by vector **y**.

	G			$= b$
1.2	1.68	1.512	0.84	278.4
0	0	0	0.8	64

Now we can calculate $\boldsymbol{n}^{\mathrm{T}}$ (vector of unit variable costs) from Eq. (10) and vector **N** (the total variable costs of manufacturing programme $\boldsymbol{x}$) will be obtained after multiplication of the vector $\boldsymbol{n}^{\mathrm{T}}$ by the diagonal matrix $\mathbf{X}_{\mathrm{d}}$.

c^{T}	8	2.3			Total
$n^{\mathrm{T}} = c^{\mathrm{T}}\mathbf{G}$	9.6	13.44	12.096	8.56	X
$\mathbf{N} = n^{\mathrm{T}}\mathbf{X}_{\mathrm{d}}$	2227.2	1747.2	1209.6	684.8	5868.8

Further generalization of the model on a cost input-output model for complete costs requires to include further cost items in the model, namely *direct fixed costs* (if they exist) and *indirect costs*, which are common for all semifinished articles and products. In this case, it is necessary to choose the way of cost allocation so that the model can keep its dynamic character as well as possible and reflect the cost behaviour by different changes of final and whole production vector accurately. About this problem in greater detail, see Ref. [7].

Conclusion

In conclusion, let us add that introduction of the system costing need not be an insurmountable problem at present, when companies have powerful computers and at many a place even networks with information database at their disposal. The existence of quality basis of consumption standards (coefficients) and purposely established topology of all cost items are important objective conditions.

It is beyond dispute that cost models can effectively contribute to better decision-making on spending variety of resources and costs in companies and thus to increase their productivity and general effectiveness.

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