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MODELLING THE CAKE FILTRATION ON A ROTARY NUTSCHE IN OSCILLATORY MODE

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As an alternative to a classical Nutsche with a flat filtration surface, a rotary Nutsche ("RN Filter") has been developed. It is constructed like a circular drum with the bottom half being used as the filter area. The drum is placed on wheels and simple rope drive enables the drum to oscillate. During a batch filtration period, the filter area of RN filter decreases and equations of filtration taking into account the variability of filter face area should be used for the prediction of rotary Nutsche filtering performance. Such equations for filtration in the static (non-oscillatory) mode have been presented earlier. Herein, the model modified for filtration in an oscillatory mode is presented and the results of its numerical solution for filtration of diatomite and cellulose suspensions are compared with those obtained in testing the filtering ability of a pilot plant cylindrical rotary Nutsche.

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Introduction

For industrial separations of more coarse suspensions, the vacuum or pressure Nutsches are often used. As an alternative to a classical Nutsche with a flat filtration surface, a rotary Nutsche ("RN Filter") has been developed by Pierson [1]. It is constructed like a circular drum with the bottom half being used as the filter area. The drum is placed on wheels and simple rope drive enables the drum to oscillate. This arrangement should possess several advantages in comparison with the classical Nutsche.

During a batch filtration period, the filter area of RN filter decreases and equations of filtration taking into account the variability of filter face area should be used for the prediction of rotary Nutsche filtering performance. Such equations for filtration in the static (non-oscillatory) mode, along with their numerical solution for filtration of suspensions of diatomite and cellulose, have been presented in our previous paper [2].

Herein, the model modified for filtration in an oscillatory mode is presented and the results of its numerical solution are compared with those obtained in testing the filtering ability of a pilot plant cylindrical rotary Nutsche [3].

Rotary Nutsche Performance Modelling

Governing Equations of Filtration

A batch suspension filtration at constant pressure difference is considered on a cylindrical filter cloth attached to the inner wall of oscillating drum with radius R and length L. The drum oscillates with the frequency $f = 1/T = \omega/2\pi$ the central angle corresponding to the oscillation amplitude is ϑ_d. The scheme of filter drum geometry is shown in Fig. 1.

Regarding the instantaneous mutual positions of the filter cloth and the suspension level, the filtration period can be divided into four phases

1^st^ phase, $\varphi_0 \geq \varphi \geq \varphi_m + 2\vartheta_d$

During this phase of filtration, the whole filter cloth is immersed in the suspension. We suppose that the volume V_{s0} of suspension in the drum at the beginning of filtration, the volume V_s of suspension at variable central angle φ the volume V_f of filtrate, the volume V_{fc} of filter cake, the filter face area A, and the increase Δh of the filter cake height can be expressed by the following relationships

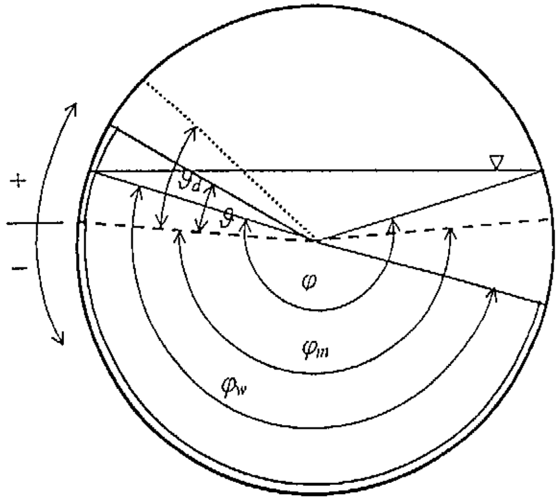

Fig. 1 Filter drum geometry

$$V_{s0} = \frac{R^2 L}{2}\left(\varphi_0 - \sin\varphi_0\right) \tag{1}$$

$$V_s = \frac{(R-h)^2 L}{2}(\varphi - \sin\varphi) \tag{2}$$

$$V_s = V_{s0} - \left(V_f + V_{fc}\right) \tag{3}$$

$$V_{fc} = \frac{V_f x}{(1-\varepsilon)\rho_s} \tag{4}$$

$$\Delta h = \frac{\Delta V_{fc}}{A} \tag{5}$$

$$A = (R - h)L\varphi_m \tag{6}$$

The central angle ϑ corresponding to the instantaneous displacement of the drum from its initial position ($\vartheta = 0$), is given as

$$\vartheta = \arcsin(\sin\vartheta_d \sin\omega\tau) \tag{7}$$

and the volume flow rate of filtrate is expressed by the filtration equation

$$\frac{dV_f}{d\tau} = \frac{CA}{2\left(\dfrac{h(1-\varepsilon)\rho_s}{x} + V_0\right)} \tag{8}$$

where C and V_0 are filtration constants related to the unit filtration area [3].

Then, the dependences $V_f = V_f(\tau)$, $V_s = V_s(\tau)$, $\varphi = \varphi(\tau)$, $V_{fc} = V_{fc}(\tau)$, $h = h(\tau)$, $A = A(\tau)$, and $\vartheta = \vartheta(\tau)$, which characterize the filtration period course, can be determined by solving the system of differential equations comprising equation (8) and the equations

$$\frac{d\varphi}{d\tau} = \frac{dV_s}{d\tau}\frac{2}{(R-h)^2 L(1-\cos\varphi)} + \frac{2(\varphi - \sin\varphi)}{(R-h)(1-\cos\varphi)}\frac{dh}{d\tau} \tag{9}$$

$$\frac{dV_s}{d\tau} = -\left(\frac{dV_f}{d\tau} + \frac{dV_{fc}}{d\tau}\right) \tag{10}$$

$$\frac{dV_{fc}}{d\tau} = \frac{dV_f}{d\tau}\frac{x}{(1-\varepsilon)\rho_s} \tag{11}$$

$$\frac{dh}{d\tau} = \frac{1}{A}\frac{dV_{fc}}{d\tau} \tag{12}$$

obtained by differentiation of Eqs (2) – (5), along with Eqs (6) and (7). At the same time, the corresponding initial conditions are given as

$$\begin{aligned} \text{for } \tau = 0: \quad & V_f = 0,\ V_s = V_{s0},\ \varphi = \varphi_0,\ V_{fc} = 0,\ h = 0, \\ & \vartheta = 0,\ A = A_0 = RL\varphi_m \end{aligned} \tag{13}$$

2^{nd} phase, $\varphi_m + 2\vartheta_d > \varphi \geq \varphi_m$

In this phase, the variable filter face area can be expressed as

$$A = (R - h)L\varphi_w \tag{14}$$

Here φ_w is the central angle of the wetted part of filter face area given by the expressions

$$\text{a) if } \frac{\varphi - \varphi_m}{2} \geq |\vartheta| \qquad \varphi_w = \varphi_m \tag{15}$$

and the solution of filtration is governed by equations (6) – (12)

$$\text{b) if } \frac{\varphi - \varphi_m}{2} < |\vartheta| \qquad \varphi_w = \frac{\varphi_m + \varphi}{2} - k_2(\omega)|\vartheta| \tag{16}$$

Here $k_2(\omega)$ is a kinetic correction coefficient taking into account the possible overflowing the filter cloth with suspension owing to drum oscillations.

Then, the system of equations for solution of filtration in this phase contains the differential equations (8) – (12) and the equations (7), (14), and (16); the corresponding initial conditions are given as

$$\begin{aligned} \text{for } \tau = \tau_1: \quad & V_f = V_{f1},\ V_s = V_{s1},\ \varphi = \varphi_1,\ \varphi_w = \varphi_{w1},\ \vartheta = \vartheta_1, \\ & V_{fc} = V_{fc1},\ h = h_1,\ A = A_1 \end{aligned} \tag{17}$$

3^{rd} phase, $\varphi_m > \varphi \geq \varphi_m - 2\vartheta_d$

In this phase, the central angle φ_w of the wetted part of filter face area is given by the expressions

$$\text{a) if } \frac{\varphi_m - \varphi}{2} \geq |\vartheta| \qquad \varphi_w = \varphi + k_{31}(\omega)|\vartheta| \tag{18}$$

$$\text{b) if } \frac{\varphi_m - \varphi}{2} < |\vartheta| \qquad \varphi_w = \frac{\varphi_m + \varphi}{2} - k_{32}(\omega)|\vartheta| \tag{19}$$

Here $k_{31}(\omega)$ and $k_{32}(\omega)$ are the corresponding kinetic correction coefficients for 3^{rd} phase of filtration.

In this phase, the filtration course is described, as it was in the second phase, by the differential equations (8) – (12) and by the equations (7), (14), and (18) or (19); the corresponding initial conditions are given as

$$\begin{aligned} \text{for } \tau = \tau_2: \quad & V_f = V_{f2},\ V_s = V_{s2},\ \varphi = \varphi_2,\ \varphi_w = \varphi_{w2},\ \vartheta = \vartheta_2, \\ & V_{fc} = V_{fc2},\ h = h_2,\ A = A_2 \end{aligned} \tag{20}$$

4^{th} phase, $\varphi < \varphi_m - 2\vartheta_d$

In this phase, the central angle of the wetted part of filter face area is supposed to be approximated by the expression

$$\varphi_w = \varphi + k_4(\omega)|\vartheta| \tag{21}$$

with the kinetic filter face area correction factor $k_4(\omega)$.

Then, the system of equations for solution of filtration in the 4th phase contains the differential equations (8) – (12) and the equations (7), (14), and (21); the corresponding initial conditions are given as

$$\begin{aligned} \text{for } \tau = \tau_3: \quad & V_f = V_{f3},\ V_s = V_{s3},\ \varphi = \varphi_3,\ \varphi_w = \varphi_{w3},\ \vartheta = \vartheta_3, \\ & V_{fc} = V_{fc3},\ h = h_3,\ A = A_3 \end{aligned} \tag{22}$$

Solution Procedure

For numerical solution of the proposed model for filtration on a RN filter in the oscillatory mode, the special programme, based on the Runge–Kutta method of 4th order with automatic time step control, was compiled using Delphi software. Usability of the model was verified by comparison of the calculated and experimental V_f data obtained for filtration of suspensions of diatomite and cellulose on a prototype of the pilot rotary Nutsche [3].

The values of filtration parameters corresponding to the conditions of the testing the filtration ability of the pilot plant rotary Nutsche were

RN Nutsche: R = 0.21 m, L = 1.3 m, φ_m = 3.15 rad ≅ 180°, A_0 = 0.86 m, V_{s0} = 180 l, (φ_0 = 5.73 rad ≈ 328°), ϑ_d = 0 (static mode) or 0.698 rad (40°), $T = \infty$ (static mode), 1.5, 2, and 8 s.

Diatomite suspension filtration: x = 50 kg m^{-3}, V_0 = 0.240 m.

Cellulose suspension filtration: x = 25 kg m^{-3}, V_0 = 0.124 m.

Since the quality of filter cake was in an individual filtration experiment variable (see [3]), the mean cake porosity ε was determined for each filtration from the balance of the filtrate volume V_f and the filter cake volume V_{fc} (Eqs (3) and (4)) at the end of a filtration test. The corresponding values of filtration constants C were evaluated from the beginning course of each filtration test to receive a good fit of experimental V_f data in this starting period. The obtained values of ε and C, which were used in the following simulation calculations of filtrations, are summarized in Table I.

Table I Filtration parameters

Filtration	Solid	T, s	ε	$C \times 10^4$, m^2 s^{-1}	k_2	k_{31}	k_{32}	k_4
F 11	Diatomite	∞	0.75	3.8	0	0	0	0
F 12		1.5	0.62	4.0	0.5	3	0	1.8
F 13		2	0.65	4.2	0.8	1	0.1	1
F 14		8	0.70	3.5	(1	0	1	0)
F 21	Cellulose	∞	0.75	1.40	0	0	0	0
F 22		1.5	0.72	1.70	(0	3	–3	1.8)
F 23		2	0.88	1.40	(1	0	1	0)
F 24		8	0.88	1.30	1	0	1	0

The values of kinetic coefficients k_2, k_{31}, k_{32}, and k_4, which need to be determined experimentally, were evaluated from the results of simulation calculations to receive a good fit of experimental V_f data for all four phases of fil-

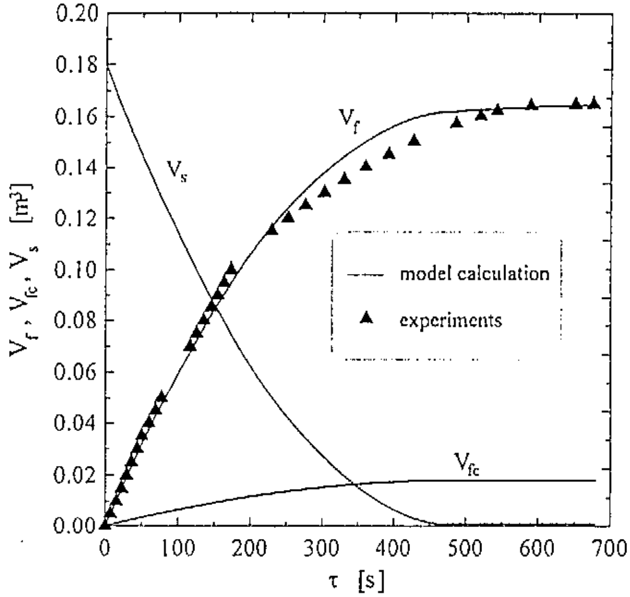

Fig. 2 The dependences of the filtrate volume V_f, suspension volume V_s, and filter cake volume V_{fc} on the time τ of filtration for diatomite suspension; static mode

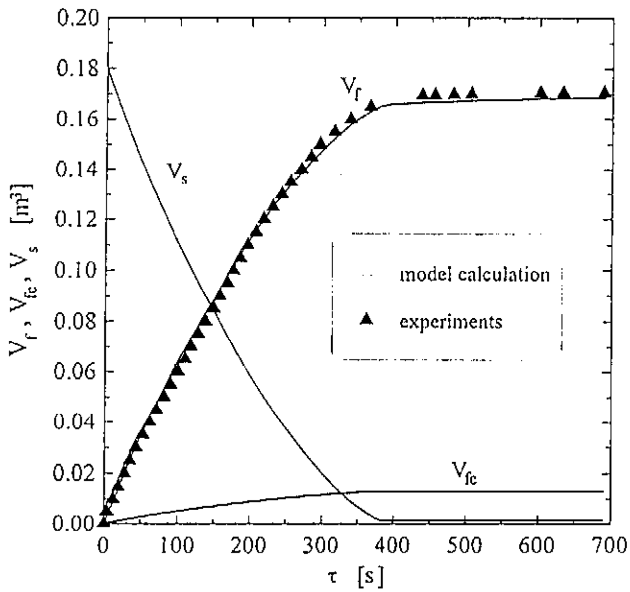

Fig. 3 The dependences of the filtrate volume V_f, suspension volume V_s, and filter cake volume V_{fc} on the time τ of filtration for diatomite suspension; oscillation period T = 2 s

tration course. They are given in Table I as well. If there is no overflowing the filter cloth with suspension owing to drum oscillations, $k_2 = k_{32} = 1$ and $k_{31} = k_4 = 0$. If the overflowing the filter cloth takes place during filtration period, the values of coefficients k_2 and k_{32} should decrease and the values of coefficients k_{31} and k_4 should increase. At the same time, the value of filter face area is limited by the condition $A < A_0$.

The starting calculation time steps were chosen as 0.1 s for the first phase of filtration and 0.01 s for the other phases of filtration.

Results and Discussion

The examples of dependences $V_f = V_f(\tau)$, $V_s = V_s(\tau)$, and $V_{fc} = V_{fc}(\tau)$, which were calculated using kinetic coefficients given in Table I, are shown in Figs 2 and 3 for filtration of the diatomite suspension and in Figs 4 and 5 for filtration of the cellulose suspension. In these figures, the calculated dependences $V_f = V_f(\tau)$ are also compared with the experimental ones resulting from the pilot rotary Nutsche testing [3].The corresponding dependences $\varphi = \varphi(\tau)$ and $h = h(\tau)$ are shown in Figs 7 and 8.

For the suspension of diatomite, an acceptable accordance between the model calculation and experimental V_f data has been obtained for all test filtra-

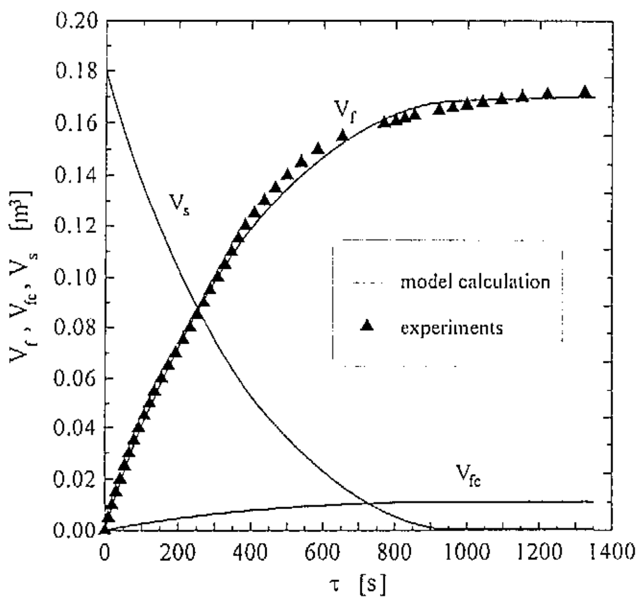

Fig. 4 The dependences of the filtrate volume V_f, suspension volume V_s, and filter cake volume V_{fc} on the time τ of filtration for cellulose suspension; static mode

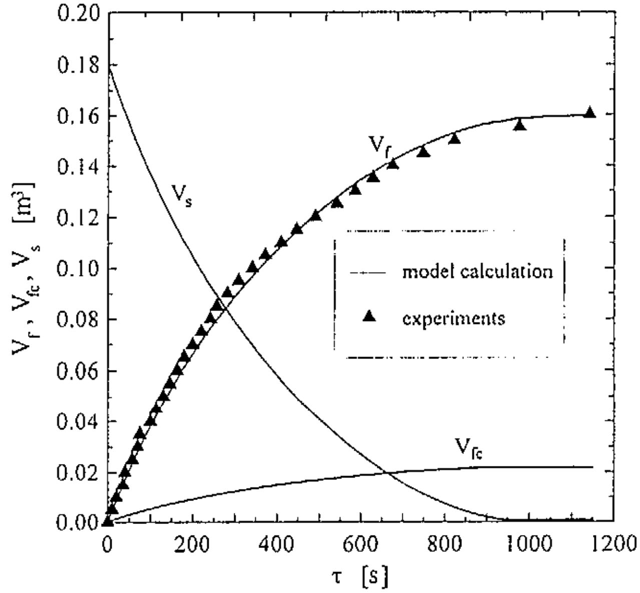

Fig. 5 The dependences of the filtrate volume V_f, suspension volume V_s, and filter cake volume V_{fc} on the time τ of filtration for cellulose suspension; oscillation period T = 8 s

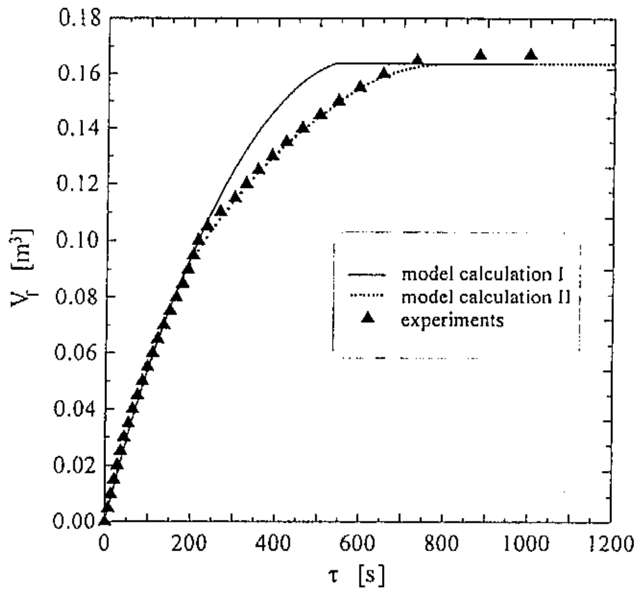

Fig. 6 Comparison of two different filtration model solutions; filtration of the diatomite suspension with oscillation period T = 8 s

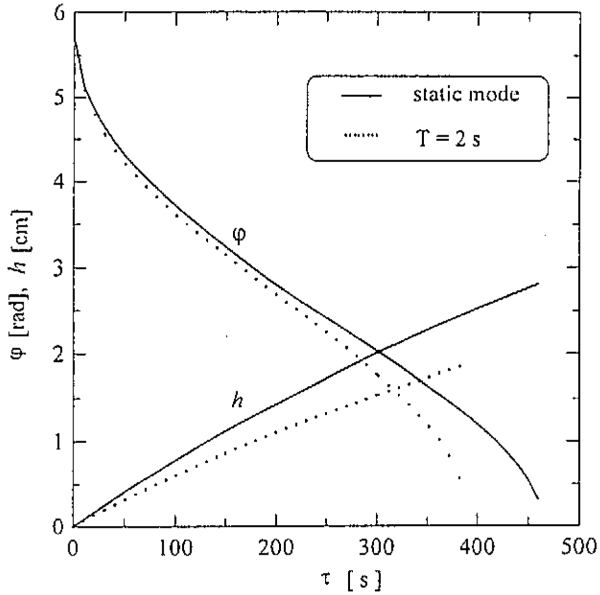

Fig. 7 The dependences of the central angle φ of suspension level and the filter cake height *h* on the time τ of filtration; diatomite suspension

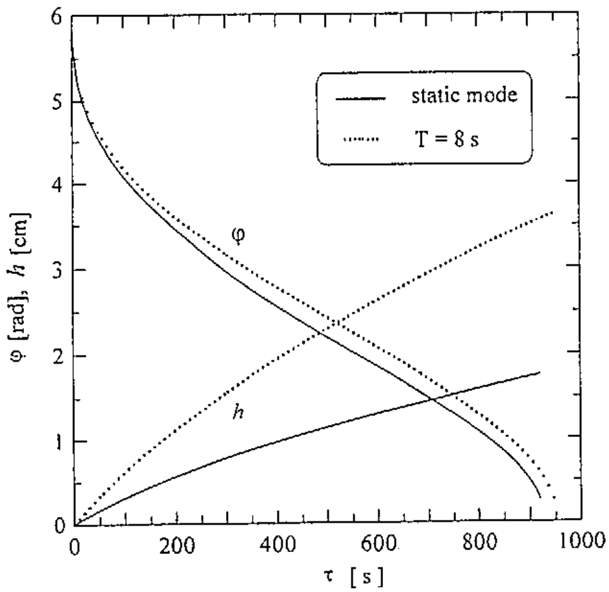

Fig. 8 The dependences of the central angle φ of suspension level and the filter cake height *h* on the time τ of filtration; cellulose suspension

tions except the experiment with the oscillation period $T = 8$. In this case, in spite of that no overflowing the filter cloth has been considered, the model calculation (calc. I) has over-estimated the filter performance in the third and forth phases of filtration. This discrepancy, shown in Fig. 6, can be explained by a possible decrease of pressure difference during these periods of filtration experiment. Nevertheless, the model yields a good filter performance prediction, if unreal values of coefficients $k_{31} = -1$ and $k_4 = -1$ (which represent an effective decrease of filtering area) along with the coefficient $k_{32} = 3$, are used for calculation (calc. II).

For the suspension of cellulose, an acceptable accordance between the model calculation and experimental V_f data has been obtained only for test filtrations with no oscillation mode and with oscillation period $T = 8$ s (Fig. 4). During filtration of cellulose suspension with short oscillation periods a destruction of non adhesive filter cake occurred and it led to an anomalous filtration course. Thus, for filtration with oscillation period $T = 1.5$ s, the proposed filtration model under-estimates the filter performance even if the filter face area is $A = A_0$ during whole filtration course. This indicates that this filtration test cannot be considered for evaluation of the proposed filtration model.

Figures 7 and 8 show a gradual decrease of the central angle φ indicating the suspension level in the filter, and the corresponding increase of the filter cake height h depending on filtration time τ. At the same time, the dependence $h = h(\varphi)$ for $\varphi < 3.15$ approximately represents the distribution of the filter cake height on the filter cloth at the end of filtration. However, for more precise prediction of the filter cake height distribution, an adaptation of the proposed filtration model will be necessary, taking into account the variable deposition of the filter cake on filtering area.

Conclusion

A numerical model was proposed for the solution of a constant pressure difference filtration on a cylindrical rotary Nutsche working in an oscillatory mode. In this model, the variability of the filter face area and the possible overflowing the filter cloth with suspension owing to drum oscillations have been taken into account.

The results were presented of the model solution for filtration of diatomite and cellulose suspensions in the oscillating regimes with various oscillation periods. The applicability of the model for these working conditions was confirmed by a sufficient agreement between the calculated and representative experimental V_f data.

Symbols

A filter face area, m^2
A_0 filter cloth area, m^2
C constant of filtration related to the unit filtration area, $m^2\,s^{-1}$
h filter cake height, m
$k(\omega)$ kinetic coefficients
L length of cylindrical filter face area, m
R radius of cylindrical filter face area, m
T oscillation period, s
V volume, m^3
V_0 constant of filtration related to the unit filtration area, m
x concentration of suspension, $kg\,m^{-3}$
ε filter cake porosity
ϑ central angle of the drum oscillation, rad
ϑ_d central angle of the drum oscillation amplitude, rad
ρ_s solid density, $kg\,m^{-3}$
τ filtration time, s
φ central angle, rad
φ_m central angle corresponding to the area covered with filter clothe, rad
φ_w central angle corresponding to the filter cloth area covered with suspension, rad
φ_0 central angle corresponding to the level of suspension at the beginning of filtration, rad
ω angular velocity, $rad\,s^{-1}$

Subscripts

f related to the filtrate
fc related to the filter cake
s related to the suspension
0 related to the beginning of filtration
1, 2, ... related to the corresponding filtration phase

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