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FALL OF NON-SPHERICAL PARTICLES THROUGH A POWER LAW FLUID

Ivan MACHAČ[1], Zdeněk LECJAKS and Bedřich ŠIŠKA
Department of Chemical Engineering, University of Pardubice,
CZ-532 10 Pardubice

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The free fall of cylinders and rectangular prism through pseudoplastic and elastic polymer solutions has been investigated experimentally in creeping flow regime. The terminal falling velocities were measured in perspex cylindrical test columns. The influence of rheological properties of fluid and wall effects on the particle terminal falling velocity have been evaluated. The relationships based on the power law viscosity model have been proposed for prediction of the wall effect correction factor and for prediction of the terminal falling velocity of short cylinders and prisms.

Introduction

With respect to its practical and theoretical meaning, great attention has been paid to investigation of dynamics of the fall of an individual solid particle through non-Newtonian fluids over the last several decades. An extensive review of the literature

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[1] To whom correspondence should be addressed.

dealing with this problem is given in Chabra's book [1]. However, the main body of knowledge concerns the fall of spherical particles and the problem of the fall of non-spherical particles has been very little explored up to now.

Our present experiments, concerning the influence of wall effects and rheological behaviour of liquid on the terminal falling velocity of non-spherical particles of different shape, should contribute to fill this gap. The present paper summarizes the results of experimental investigation of the fall of cylindrical and prismatic particles in non-Newtonian polymer solutions in creeping-flow region.

Theoretical

The magnitude F_D of the drag force acting on a particle, density ρ_s, moving at relative velocity u through a fluid, density ρ, can be expressed with the use of the drag coefficient c_D as

$$F_D = c_D A_n \frac{u^2}{2} \rho \tag{1}$$

where A_n is the area of particle projection into a plane perpendicular to the direction of motion.

For a particle falling at its terminal velocity u_t, the drag force is balanced with the gravitational and buoyancy forces, so that the coefficient c_D is related to the terminal falling velocity u_t by expression

$$c_D = \frac{4}{3} \frac{g d_v(\rho_s - \rho)}{\rho u_t^2} \left(\frac{d_v}{d_n} \right)^2 \tag{2}$$

where d_v is the equal volume sphere diameter (diameter of sphere of the same volume as the particle) and d_n is the diameter of the circle with area equal to A_n.

If the drag coefficient c_D is known, the particle terminal velocity u_t can simply be calculated from Eq. (2). For a sphere falling through an unbounded fluid, the drag coefficient depends on rheological behaviour of the fluid and on the flow regime. For the creeping motion through a fluid obeying power law viscosity model (power law fluid)

$$\eta = K \dot{\gamma}^{n-1} \tag{3}$$

the drag coefficient is expressed as (see e.g. Ref. [1])

$$c_D = \frac{24}{Re_n} X(n) \tag{4}$$

where

$$Re_n = \frac{d_v^n u_t^{2-n} \rho}{K} \tag{5}$$

is the particle power law Reynolds number and $X(n)$ is the drag correction function which depends on the power law index n.

If Eqs (2) and (4) are compared, we have

$$u_{t,o} = \left[\frac{g d^{1+n} (\rho_s - \rho)}{18 K X(n)} \right]^{1/n} \tag{6}$$

for spherical particles ($d_v = d_n = d$).

The relation between terminal falling velocity u_t of a non-spherical particle and terminal falling velocity $u_{t,o}$ of spherical particle (of the same volume, $d = d_v$) is often expressed by means of the dynamical particle shape factor φ as

$$\varphi = \frac{u_t}{u_{t,o}} \tag{7}$$

Substituting $u_{t,o}$ according to Eq. (6) into Eq. (7), the following relationship

$$u_t = \left[\varphi^n \frac{g d_v^{1+n} (\rho_s - \rho)}{18 K X(n)} \right]^{1/n} \tag{8}$$

results for calculation of terminal falling velocity of non-spherical particle moving through a power law fluid in creeping flow region.

The fundamental problem of application of Eq. (8) to prediction of a particle terminal falling velocity is the determination of corresponding values of shape factor φ and correction function $X(n)$. In the creeping flow region, factor φ depends on quantities characterizing the geometry of the particle and the rheological behaviour of the fluid. To simplify the calculation of terminal falling velocity, we can suppose that the quantity φ^n can for a given particle be expressed by means of dynamical shape factor φ_N, characterizing the fall of the particle through Newtonian fluid, as

$$\varphi^n = \varphi_N(G_i)Y(n) \tag{9}$$

Factor φ_N is expected to depend only on geometrical parameters G_i; $Y(n)$ is the correction function of dynamical shape factor for the fall through power law fluid. Therefore, Eq. (8) can be transformed into

$$u = \left[\frac{g d_v^{1+n}(\rho_s - \rho)}{18KB(n)}\right]^{1/n} \tag{10}$$

where

$$B(n) = \frac{X(n)}{Y(n)} \tag{11}$$

is the overall terminal falling velocity correction function for a power law fluid.

Substituting Eq. (10) into Eq. (2), we find the following expression for the drag coefficient of the fall of non-spherical particle through a power law fluid

$$c_D = \frac{24}{Re_n}\left(\frac{d_v}{d_n}\right)^2 \frac{B(n)}{\varphi_N} \tag{12}$$

which transforms to Eq. (4) for a spherical particle.

The value of the factor φ_N, which results from the solution of the fall of non-spherical particles through a Newtonian fluid, can be found in the literature (see e.g. Refs [2–5]), experimental determination of the function $B(n)$ for the fall of short cylinders and prisms is dealt with in this paper.

Experimental

The terminal velocities were measured of cylinders [6] and square prisms falling through glycerol and water solutions of polymers in cylindrical columns. The experiments were performed in creeping flow region.

Particles

Twenty four 24 types of cylindrical particles made of duralumin ($0.2 \leq h/d \leq 5$, $\rho_s = 2800$ kg m^{-3}), 26 types of square prisms made of aluminium ($0.25 \leq c/a \leq 5$, $\rho_s = 2638$ kg m^{-3}), and 26 types of particles of the same geometry made of PVC ($\rho_s = 1379$ kg m^{-3}) were used in our terminal velocity measurements.

Liquids

The test liquids were glycerol and water solutions of methylcellulose Tylose MH 4000, hydroxyethylcellulose Natrosol 250 MR, polyethylenoxide Polyox WSR 301, and polyacrylamide Kerafloc A 4008. The shear rate-shear stress dependencies were measured using the rotary cylindrical rheometer Rheotest II. The parameters of the power-law model along with other characteristics of test liquids are given in Table I.

The solutions of glycerol have Newtonian behaviour, polymer solutions exhibit a different degree of shear thinning and elasticity. The solution of Tylose

Table I Characteristics of the test liquids, 20 °C

Liquid	ρ, kg m^{-3}	n	K, Pa s^n	$\dot{\gamma}$, s^{-1}	Particles
Glycerol	1260	1	1.4	-	
Tylose 1.7%	1003	0.863	1.24	1.5 – 27	
Natrosol 1.6%	1003	0.697	2.28	1.5 – 27	cylinders
Polyox 0.8%	1001	0.527	0.956	1.5 – 27	
Kerafloc 0.3%	1000	0.309	1.98	1.5 – 27	
Glycerol	1260	1	1.422	-	
Natrosol 1.6%	1003	0.848	1.840	0.37 – 1.46	rectangular
		0.713	2.095	1.5 – 27	prisms
Kerafloc 0.3%	1000	0.311	2.477	1.5 – 81	

is slightly shear thinning, the solution of Natrosol exhibits a greater measure of shear thinning and solutions of Polyox and Kerafloc are highly shear thinning. It followed from additional dynamic tests with oscillating stresses, carried out on Haake rheometer RS 150, that along with the growth of shear thinning the degree of polymer solution elasticity increased as well. The solutions of Polyox and Kerafloc are already evidently elastic and show the rod-climbing (Weissenberg) effect.

Test Vessels

Five perspex cylindrical columns of diameters D = 190, 140, 80, 40, and 20 mm were used in our particle settling experiments. The height of the columns was approximately 1 m.

Terminal Falling Velocity Measurements

The measurements of terminal velocities of particles were performed for two basic orientations of particles designated as vertical and horizontal. For the vertical orientation of cylindrical particles, the cylinder axis of revolution is identical with that of the test column; for the horizontal orientation, the particle axis of revolution is perpendicular to the test column longitudinal axis. For the vertical orientation of prisms, the square base of particle is perpendicular to the test column longitudinal axis (to the direction of fall of particle); the horizontal orientation is characterized by perpendicularity of a side wall of prism to the particle fall direction.

During a particle fall through Newtonian liquid (glycerol), the both orientations vertical and horizontal were stable. During fall through non-Newtonian liquids (polymer solutions), only one orientation was stable, the vertical one for particles characterized by the ratio h/d (or c/a) > 1 and the horizontal one for h/d (or c/a) > 1.

The terminal falling velocities were determined from the mean time of particle fall through two test column sections of 10 cm length. The stop watch reading of 0.01 s accuracy was used for timing the particles. The values of Reynolds number Re_n ranged from 0.001 to 0.3.

Results

Wall Effects

The extent of wall effects on retardation of particle terminal velocity was expressed using the wall factor

$$F_W = \frac{u_t}{u_{t,\infty}} \tag{13}$$

The values $u_{t,\infty}$ of terminal falling velocity in an unbounded fluid was evaluated making use of extrapolation of experimental dependencies of u_t on the ratio d_v/D_e. Examples of these dependencies are given in Fig. 1 for the fall of vertically oriented prisms. The effective diameter D_e of the column with a falling particle has been defined as

$$D_e = D - z \tag{14}$$

where z is the longest dimension of the particle projection into a plane perpendicular to the direction of particle fall. The effective diameter D_e fulfils the limiting requirements: $F_W = 1$ for $z \ll D_e$ and $F_W = 0$ for $z = D$, that is for $D_e = 0$.

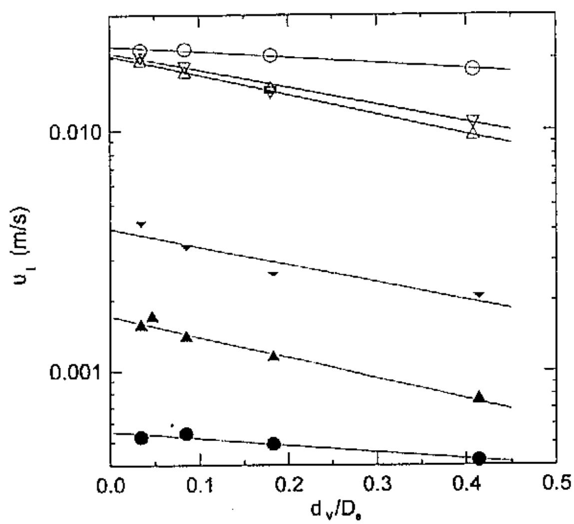

Fig. 1 Example of experimental dependencies of particle terminal falling velocity u_t on the ratio d_v/D_e for the fall of vertically oriented prisms ($c/a = 2$): (▲, △) glycerol; (▼, △) Natrosol; (●, ○) Kerafloc; solid symbols – Novodur particle; open symbols – Al particle

Analyzing the experimental data obtained, we can conclude that the relationship

$$F_W = \frac{1}{1 + c(n)\dfrac{d_v}{D_e}} \tag{15}$$

can be used for prediction of dependence of wall factor F_W on the rheological behaviour of the fluid and on the system geometry.

For the free fall of cylindrical particles, the expression

$$c(n) = -1.186 + 6.074n - 2.209n^2 \tag{16}$$

has been found for determination of dependence of coefficient $c(n)$ upon liquid flow index n. Evaluating experimental data for the free fall of prisms gave the relationship

$$c(n) = -1.575 + 8.455n - 4.228n^2 \tag{17}$$

The mean relative deviation of experimental values of F_W and those calculated according to Eq. (15) was 5%, maximum value 7.1% was observed for the fall of prisms through solution of Kerafloc.

Terminal Falling Velocity in Unbounded Liquid

Making use of the extrapolated data $u_{t,\infty}$, the values of the function $B(n)$ were calculated from Eq. (10) for all individual particle-liquid combinations investigated. At the same time, very good agreement was found between the experimental values of the dynamic shape factor φ_N (determined from Eq. (7) considered for a Newtonian fluid ($\varphi = \varphi_N$) using data $u_{t,\infty}$ obtained for glycerol and data $u_{t,o}$ calculated according to the Stokes law) and the values predicted according to relationships proposed for short cylinders and prisms by Heiss and Coull [3]. The mean relative deviation of experimental and predicted data was 3.1% for cylinders and 7.9% for prisms. Therefore, calculating the function $B(n)$, the values φ_N were substituted to Eq. (10) determined according to relationships [3]

for vertical orientation

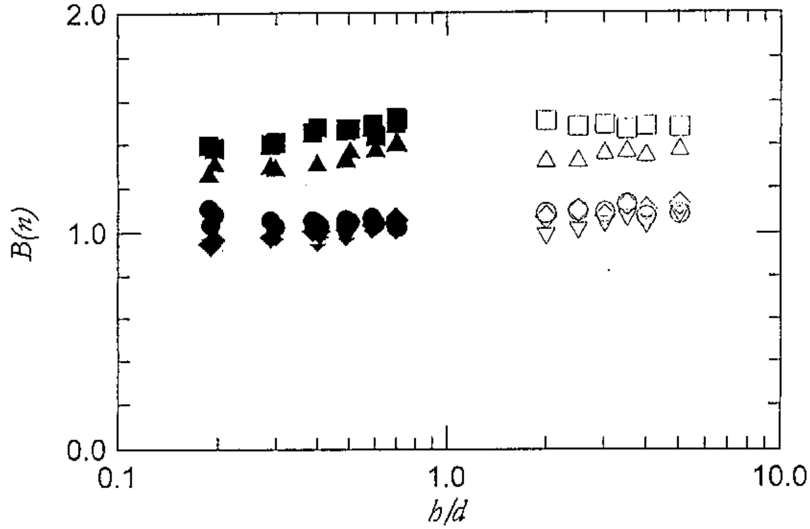

Fig.2 Correction function $B(n)$ for cylinders: (●, ○) glycerol; (▼, △) Tylose; (♦, ◇) Natrosol; (■, □) Polyox; (▲, △) Kerafloc; solid symbols – horizontal orientation; open symbols – vertical orientation

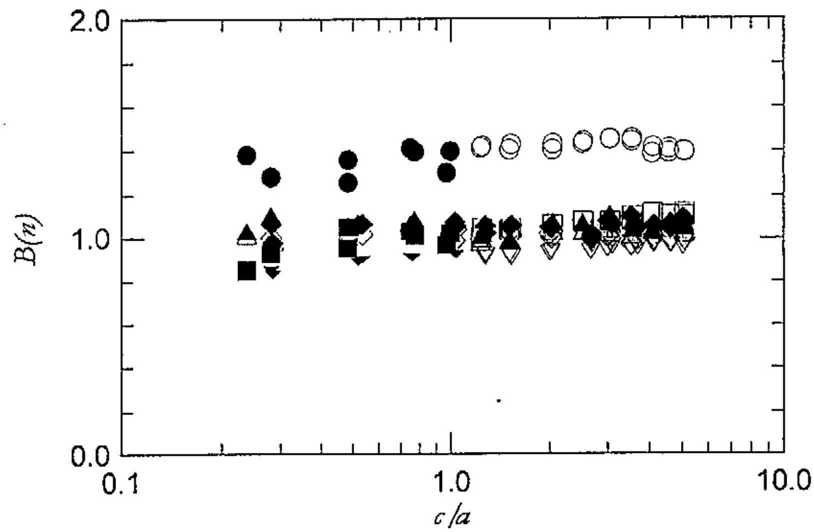

Fig.3 Correction function $B(n)$ for prisms: (▲, △) Al particle-glycerol; (♦, ◇) Novodur particle-glycerol; (■, □) Al particle-Natrosol; (▼, △) Novodur particle-Natrosol; (●, ○) Al particle-Kerafloc; (☆) Novodur particle (vertical)-Kerafloc; solid symbols – horizontal orientation; open symbols – vertical orientation

$$\log \varphi_{N,v} = -0.25 \sqrt{\psi \frac{d_v}{d_n} \left(\frac{d_v}{d_n} - 1 \right)} + \log \left(\frac{d_v}{d_n} \sqrt{\psi} \right) \tag{18}$$

and for horizontal orientation

$$\log \varphi_{N,h} = -\frac{0.270}{\sqrt{\psi} \left(\frac{d_v}{d_n} \right)^{0.345}} \left(\frac{d_v}{d_n} - 1 \right) + \log \left(\frac{d_v}{d_n} \sqrt{\psi} \right) \tag{19}$$

The resulting values of function $B(n)$ are shown in Fig. 2 for cylinders and in Fig. 3 for prisms. From Figs 2 and 3 it can be seen that for inelastic and moderately shear-thinning fluids ($n > 0.7$) the value of function $B(n)$ does not depend (within experimental error) on rheological behaviour of the fluid and $B(n) \cong 1$. The mean relative deviation of experimental values of terminal falling velocities and those calculated according to Eq. (10) with $B(n) = 1$ is 6.8% for the horizontal particle orientation (maximum value is 10.7% for the fall of prisms in solution of Natrosol) and 7.5% for the vertical particle orientation (maximum value is 15% for the fall of cylinders in solution Natrosol).

For strongly shear-thinning as well as elastic liquids (solutions of Polyox and Kerafloc), the values of $B(n)$ depend on rheological behaviour of the fluid. For the fall of metallic particles, the values $B(n) > 1$ have been evaluated (the arithmetic mean $\langle B(n) \rangle = 1.36$ for Kerafloc and $\langle B(n) \rangle = 1.46$ for Polyox). At the same time, $B(n) \cong 1$ for the fall of PVC prisms through solution of Kerafloc (measurements with Polyox have not been performed yet). This discrepancy can be explained by the fact that the rheological behaviour of Kerafloc solution is not sufficiently well described using the power-law viscosity model at very low shear rates reached in the fluid during the fall of PVC particles. For illustration, the values of ratio u_t/d_v, which are considered to be proportional to the mean fluid shear rate at the particle surface, ranged from 0.088 to 0.185 s^{-1} for the fall of PVC particles and from 0.5 to 7.5 s^{-1} for the fall of aluminium particles.

Conclusion

In cylindrical test vessels, the terminal falling velocities have been measured of cylinders and square prisms moving through glycerol and water solutions of polymers of various degree of shear-thinning and elasticity.

Simple relationships, which are based on the power-law viscosity model, have been proposed for evaluation of the wall effect corrective factor and the terminal falling velocity in an unbounded liquid.

It was found out that the value of function $B(n)$ needed for the terminal velocity calculation, does not depend on rheological behaviour of the liquid and can be considered equal to 1 for inelastic and moderately shear-thinning liquids. For highly shear-thinning and evidently elastic liquids, the value of function $B(n)$ depends on the measure of both shear-thinning and elasticity. For investigation of the fall of non-spherical particles through these liquids, further experiments will have to be performed and a more suitable viscosity model than power law should be applied as well.

Acknowledgements

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Symbols

a	side of prism square cross-section, m
A_n	projected area of particle perpendicular to direction of motion, m^2
$B(n)$	overall terminal falling velocity correction function defined by Eq. (11)
c	length of prism, m
$c(n)$	coefficient in Eq. (15)
c_D	drag coefficient of particle
d	diameter of sphere or diameter of cylinder, m
d_v	equal volume sphere diameter, m
d_n	diameter of a circle equal to projected area of particle perpendicular to direction of motion, m
D	diameter of test column, m
D_e	effective diameter of the test column with falling particle defined by Eq. (14), m
F_D	magnitude of the drag force acting on a particle, N
F_w	wall effect correction factor defined by Eq. (13)
g	acceleration due to gravity, m s^{-2}
h	height of cylinder, m
K	power law parameter (consistency coefficient), Pa s^n
n	power law flow index
Re_n	Reynolds number for the fall of a particle through a power-law fluid defined by Eq. (5)

u velocity, $m\,s^{-1}$
$u_{t,\infty}$ particle terminal falling velocity in unbounded liquid, $m\,s^{-1}$
$u_{t,o}$ terminal falling velocity of a volume equal sphere in unbounded liquid, $m\,s^{-1}$
$X(n)$ drag coefficient correction function
$Y(n)$ correction function of particle dynamic shape factor
z longest dimension of particle projection into a plane perpendicular to the direction of motion, m

$\dot{\gamma}$ shear rate, s^{-1}
η viscosity of non-Newtonian liquid, Pa s
ρ fluid density, $kg\,m^{-3}$
ρ_s particle density, $kg\,m^{-3}$
φ dynamic shape factor of particle defined by Eq. (7)
ψ particle sphericity

Indexes

h related to horizontal particle orientation
N related to Newtonian fluid
v related to vertical particle orientation

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