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A UNIFIED SOLUTION TO PROBLEM OF FLOW OF VISCOPLASTIC FLUIDS PAST A SPHERICAL PARTICLE AND OF FLOW OF VISCOPLASTIC FLUIDS THROUGH FIXED BEDS

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An approach to the calculation of drag and fall velocity of a spherical particle in viscoplastic fluids and to the calculation of pressure drop in flow of viscoplastic fluids through random fixed beds of particles is suggested. It is based on the application of the modified Rabinowitsch–Mooney equations together with corresponding relations for consistency variables.

Introduction

The solution of stabilized laminar flow of generalized Newtonian fluids (GNF) and viscoplatic fluids (VPF) through a tube is described by the Rabinowitsch–Mooney equation in the form

$$\dot{D}w \equiv \frac{2u_{ch}}{l_{ch}} = \frac{4}{\tau_w^3}\int_{\tau_0}^{\tau_w} \tau^2 \dot{D}(\tau)\, d\tau \tag{1}$$

where the dynamic consistency variable τ_w is obtained from the integral form of the momentum balance

$$\tau_w = \frac{\Delta p}{L} l_{ch} \tag{2}$$

and the characteristic velocity of the system u_{ch} is obtained from the integral form of the equation of continuity of flow

$$u_{ch} = \frac{\dot{V}}{S} \tag{3}$$

In Eqs (1) – (3), $\dot{D}_w$ is the kinematic consistency variable, the characteristic linear dimension l_{ch} is the hydraulic radius r_h of the tube, τ_0 is the yield stress, Δp is the pressure drop, L is the length of the tube, $\dot{V}$ is the volumetric flow rate, S is the cross-section of the tube, and $\dot{D}(\tau)$ is the dependence of the shear rate $\dot{D}$ on the shear stress τ, whose course is given generally by the flow curve of non-Newtonian fluid or by the respective flow model. For $\tau_0 = 0$, Eq. (1) assumes the form valid for GNF.

If we write the Rabinowitsch–Mooney equation (1) in the form (4), then the system of Eqs (1) and (4) also defines the relationship necessary for calculation of the effective viscosity of the system, $\mu_{e,w}$

$$\dot{D}_w = \frac{\tau_w}{\mu_{e,w}} \tag{4}$$

For a Newtonian fluid (NF), $\mu_{e,w} = \mu$. Equations (1) – (4) apply only to an incompressible fluid (with $\rho = \text{const.}$, where ρ is the density). Using the quantities ρ, u_{ch}, l_{ch}, and $\mu_{e,w}$, we can express the Reynolds number as

$$Re = \frac{\rho u_{ch} l_{ch}}{\mu_{e,w}} \tag{5}$$

After substituting Eq. (4) in connection with Eq. (1) into Eq. (5) we obtain

$$Re = \frac{2\rho u_{ch}^2}{\tau_w} \tag{6}$$

In the cases of a single particle and fixed beds, we will also need equations of the Rabinowitsch–Mooney type as relationships between the consistency variables, where again the relationships for the dynamic consistency variables are obtained with the help of the integral forms of the momentum balance. These forms for a single particle and fixed beds differ from those valid for the flow of fluid through a tube by the force effect of the solid surface of particles on the fluid being given by not only the frictional resistance but also the shape resistance.

The present paper forms a continuation of our earlier papers dealing with the flow of GNF through random fixed and fluidized beds of particles and with solution of the problem of flow past, or fall of, a single spherical particle in GNF with the use of equations of the Rabinowitsch–Mooney type [1–4].

Theory

The consistency variable τ_w for a flow through a tube represents the frictional resistance referred to its surface area. On introducing this consistency variable (even for external flow past a single spherical particle) the Stokes approach can be expressed by Eqs (7) and (8)

$$\dot{D}_{w,p} \equiv \frac{2u_{ch,p}}{l_{ch,p}} = \frac{\tau_{w,p}}{\mu} \tag{7}$$

$$f_t - f_f - f_s \equiv f_t - f_f(1 + \psi_p) = 0 \tag{8}$$

Here $\dot{D}_{w,p}$ and $\tau_{w,p} \equiv f_f$ are the consistency variables of the problem solved, the characteristic velocity, $u_{ch,p}$, is the maximum velocity, the characteristic linear dimension $l_{ch,p}$ of system is the diameter d of the particle, f_t is the total resistance, f_f is the frictional resistance, f_s is the shape resistance of spherical particle referred to its surface area, and the dimensionless quantity $\psi_p = f_s/f_f$ was called the resistance number. For the solution by Stokes, $\psi_p = 1/2$.

Using the Stokes relationship for splitting the component of shear stress $\tau_{r,0}$ in spherical coordinate system [5], Eq. (8) was obtained [2] for distribution of the quantity τ

$$\tau \equiv \frac{1}{4\pi R^2}\left|\int_{S_s}\vec{\vec{\tau}}\cdot d\vec{S}\right| = \tau_{w,p}\left(\frac{r}{R}\right)^{-2} \tag{9}$$

where R is the radius of spherical particle, $\vec{\vec{\tau}}$ is tensor of shear stress, S_s is spherical surface with a radius $r \geq R$, and $d\vec{S}$ is its elementary part. The integration mentioned eliminates the effect of both variable angles of the spherical coordinate system. The quantity $\tau_{w,p}$ was introduced into the Stokes relationship for the component of shear stress $\tau_{r,0}$ by adopting Eq. (7).

For VPF, Eq. (9) will become

$$\tau - \tau_0 = \tau_{w,p}\left(\frac{r}{R}\right)^{-2} - \tau_0 \tag{10}$$

In analogy to the procedure used in the case of tube (e.g. [6]), the equation of the Rabinowitsch–Mooney type (Eq. (11)) can be derived [2]

$$\dot{D}_{w,p} \equiv \frac{2u_{ch,p}}{d} = \int_{\tau_0}^{\tau_{w,p}}\tau^{-3/2}\dot{D}(\tau)\,d\tau \tag{11}$$

Equation (11) can be written as

$$\dot{D}_{w,p} = \frac{\tau_{w,p}}{\mu_{e,p}} \tag{12}$$

where $\mu_{e,p}$ is the effective viscosity of the system solved. For an NF, $\mu_{e,p} = \mu$.

The Reynolds number for a single particle Re_p can then be defined by Eqs (5) and (6), where the quantities l_{ch}, u_{ch} $\mu_{e,w}$ and τ_w are substituted by the quantities d, $u_{ch,p}$, $\mu_{e,p}$ and $\tau_{w,p}$.

In addition to Eq. (11) we need a relationship for the calculation of the resistance number $\psi_{\mathrm{VPF},p}$ coming from the momentum balance (Eq. (8)) in the form valid for a VPF. We shall express this number as a sum of two terms

$$\psi_{\mathrm{VPF},p} = \psi_p + \psi_{P,p} \tag{13}$$

where the quantity ψ_p expresses the dimensionless influence of viscosity effects and the quantity $\psi_{P,p}$ expresses the dimensionless influence of plastic effects on the value of form resistance and, hence, total resistance of the particle. First of all, we presume that the dependence of quantity ψ_p on the Reynolds number does not depend on the rheological properties of the fluid, so that in the creeping region of flow it is $\psi_p = 1/2$. Thus also the quantity $\psi_{P,p}$ has to be determined with the use of another dimensionless quantity appearing—in addition—in the relationships for VPF.

For $r = R_o > R$ where $\tau = \tau_o < \tau_{w,p}$, Eq. (14) follows from Eq. (10)

$$\frac{R}{R_o} = \left(\frac{\tau_o}{\tau_{w,p}}\right)^{1/2} \tag{14}$$

whereas for $r = R$ it is $\tau = \tau_{w,p}$. On the presumption that the value of quantity $\psi_{P,p}$ is given by the value of R/R_o ratio, the solution of Eqs (13) and (14) yields

$$\psi_{\mathrm{VPF},p} = \psi_p + \left(\frac{\tau_o}{\tau_{w,p}}\right)^{1/2} \equiv \psi_p + \xi_p^{1/2} \tag{15}$$

For a fall of a spherical particle we get, from momentum balance (Eq. (8)) in the form valid for a VPF, the following relation for the $\tau_{w,p} \equiv f_f$ quantity

$$\tau_{w,p} = \frac{(\rho_s - \rho)gd}{6(1 + \psi_{\mathrm{VPF},p})} \tag{16}$$

where ρ_s is the density of the particle and g is the gravitational acceleration. In the case of the fall of the particle, the characteristic velocity $u_{ch,p}$ then corresponds to the fall velocity u_g.

Using the integral form of momentum balance and that of equation of continuity of flow we have obtained Eqs (17) and (18) for the consistency variables for the stabilized laminar flow of an incompressible NF through a random fixed bed of particles [7]

$$\tau_{w,b} \equiv \frac{\Delta p}{L} l_{ch,b} = \frac{\Delta p \varepsilon}{L[a_p \omega(1-\varepsilon)(1+\lambda\psi_b)+a_w]} \tag{17}$$

$$\dot{D}_{w,b} \equiv \frac{2u_{ch,b}}{l_{ch,b}} = \frac{2u_{ch}[a_p \omega(1-\varepsilon)(1+\lambda\psi_b)+a_w]}{\varepsilon^2} \tag{18}$$

where $\tau_{w,b}$ is the dynamic consistency variable of the bed, $l_{ch,b}$ is the characteristic linear dimension of the bed, ε is the porosity of the bed, a_p is the specific surface of an individual particle, ω is a correction coefficient for the wetted surface contained in the bed ($\omega = 1$ for a bed of spherical particles where there is point contact between individual particles and where the magnitude of their surface area has been determined with sufficient precision), λ is a shape factor ($\lambda = 1$ for beds of spherical particles), a_w is the specific surface area of the apparatus walls, $\dot{D}_{w,b}$ is the kinematic consistency variable of the bed and $u_{ch,b} = u_{ch}/\varepsilon$ is the mean velocity of a fluid in the voids. The values of correction coefficient for the wetted surface ω and of shape factor λ must be determined experimentally [7].

Since for $\varepsilon = 1$ Eqs (17) and (18) are valid for the tube, where $l_{ch,b} = 1/a_w \equiv r_h$ is the hydraulic radius, we first of all presume that the Eqs (1) and (4) – (6) with consistency variables $\tau_w \equiv \tau_{w,b}$, $\dot{D}_w \equiv \dot{D}_{w,b}$, and effective viscosity $\mu_{e,w} \equiv \mu_{e,b}$ can be used for the calculation of pressure drop in both NF and GNF-fixed bed system ($\tau_o = 0$), too.

For the purposes of treatment of flow of viscoplastic fluid through random fixed bed of particles ($\tau_o > 0$) it is necessary to increase the term $\lambda\psi_b$, appearing in both consistency variables (in analogy to a single particle), by the value $\psi_{P,b}$ expressing the dimensionless influence of plastic effects upon the value of form resistance of particles. For the flow of VPF through a tube it is

$$\frac{R_{w,o}}{R_w} = \frac{\tau_o}{\tau_w} \tag{19}$$

In this equation $R_{w,o} < R_w$ is the distance from the tube axis of radius R_w where the shear stress τ assumes the value of τ_o.

If, in analogy to NF and GNF, we introduce $\tau_w \equiv \tau_{w,b}$ and $\dot{D}_w \equiv \dot{D}_{w,b}$ also for VPF, we get relationship (20) with the use of the same presumption as that used for a single particle

$$\psi_{P,b} \equiv \frac{R_{w,o}}{R_w} = \frac{\tau_o}{\tau_{w,b}} \equiv \xi_b \tag{20}$$

Simultaneously, we presume that the dependences of the quantity ψ_b on the Reynolds number are identical for an NF, GNF and VPF. In the creeping region of flow, the value of quantity ψ_b equals the value valid for a single spherical particle [4] ($\psi_b = 1/2$).

Results and Discussion

For the Robertson–Stiff [8] flow model

$$\dot{D}(\tau) = \left[\frac{\tau}{K}\right]^{1/n} - \dot{D}_o \equiv \left[\frac{\tau}{K}\right]^{1/n} - \left[\frac{\tau_o}{K}\right]^{1/n} \tag{21}$$

where K, n and $\dot{D}_o$ are its parameters, the solution of modified Eq. (1) (in the dimensionless form) yields,

$$\dot{D}_{w,b,d} \equiv \frac{2u_{ch,b}}{l_{ch,b}\dot{D}_o} = \frac{4n\xi_b^{-1/n}(1 - \xi_b^{3+1/n})}{3n+1} - \frac{4}{3}(1 - \xi_b^3) \tag{22}$$

and the solution of Eq. (11) (in the dimensionless form) gives

$$\dot{D}_{w,p,d} \equiv \frac{2u_{ch,p}}{d\dot{D}_o} = \frac{n\xi_p^{-1/n}(1 - \xi_p^{1/n-1/2})}{2-n} + 1 - \xi_p^{-1/2} \tag{23}$$

If in the case of a single spherical particle the fall velocity u_g in wall-unbounded system is known, then the quantity f_t is determined using Eq. (23) and the momentum balance in the dimensionless form

$$\frac{f_t}{\tau_o} - \frac{1}{\xi_p}(1 + \psi_p + \xi_p^{1/2}) = 0 \tag{24}$$

The quantity Re_p is calculated with the help of some form of the dependence $C_D = C_D(Re_p)$ for the fall of spherical particle in NF given in literature (e.g. [9]), using the relationship

$$\psi_p = \frac{C_D Re_p}{16} - 1 \tag{25}$$

We will adopt the form of Reynolds number Re_p of the type given by Eq. (6) in the form

$$Re = \frac{2\rho u_g \xi_p}{\tau_o} \tag{26}$$

If the aim of calculation is to determine the fall velocity of spherical particle, we will adopt the momentum balance in the form

$$\xi_p = \left[\frac{\varphi + [\varphi^2 + 4\varphi(1 + \psi_p)]^{1/2}}{2}\right]^2 \tag{27}$$

where $\varphi = \tau_o/f_t$. The value of fall velocity is then determined using Eqs (23) and (25) – (27) by the method of successive approximations. With respect to the fact that $0 < \xi < 1$ we can choose $\xi_o = 1/2$ in Eqs (23) and (26). However, the method of calculation suggested can probably be used only up to some critical value of the Reynolds number Re_p; after exceeding it, the so-called secondary movement of particle becomes significant and the value has to be determined experimentally.

In similar way we can determine the pressure drop in the flow of Robertson–Stiff [8] fluid through fixed beds. For this purpose, the form of dependence $\psi = \psi(Re_b)$ valid for beds of spherical particles [7] has to be used.

Analogously, it is possible to obtain analytical solutions to the Rabinowitsch–Mooney equations (1) and (11) as well as for the Casson [10] flow model. For Herschel–Bulkley (according to [6]) flow model

$$\dot{D}(\tau) = \left[\frac{\tau - \tau_o}{K'}\right]^{1/n'} \tag{28}$$

Equation (11) can be analytically solved only for certain numerical values of the parameter n' (e.g. 0.6).

For the Bingham flow model (Eq. (21), where $n = 1$ and $K = \mu$), the solution to Eq. (23) with $n = 1$ and Re_p and that to the momentum balance (Eq. (27)) lead to the relationship

$$f_{t,d} = (Bi^{1/2} + 2^{1/2})[2 + \psi_b)Bi^{1/2} + (1 + \psi_p)2^{1/2}] \tag{29}$$

where $f_{t,d} = f_t d/(\mu u_{ch,p})$ is the dimensionless total resistance and $Bi = \tau_o d/(\mu u_{ch,p})$ is the Bingham number. For the creeping region of flow ($\psi_p = 1/2$) and for $Bi \to \infty$, this relationship leads to

$$\frac{f_t d}{3 Bi} \equiv \frac{f_t}{3\tau_o} = \frac{5}{6} \tag{30}$$

This relationship generally follows, independent of the particular flow model of VPF, from the momentum balance (Eq. (24)) for the condition $\tau_{w,p} = \tau_o$ ($\xi_p = 1$). For an NF ($Bi = 0$) and for the creeping region of flow it is $f_{t,d}/3 = 1$.

The comparisons of Eq. (29) in the form valid for the creeping region of flow ($\psi_p = 1/2$), with the numerical solution by Yoshioka et al. [11] (variational principle method, where the arithmetic mean of upper and lower limits of the quantity $f_{t,d}$ was considered), with that by Beris et al. [12] (finite element method), and with that by Blackery and Mitsoulis [13] approximated by the following equation

$$\frac{f_{t,d}}{3} = 1 + 2.93 Bi^{0.83} \tag{31}$$

are presented on Fig. 1.

The experimental results by Ansley and Smith [14], who used tomato sauce (catsup) as the model fluid, are compared with Eq. (29) using relative deviation between experimental and calculated values of $f_{t,d}/3$ quantity in Table I. Here the

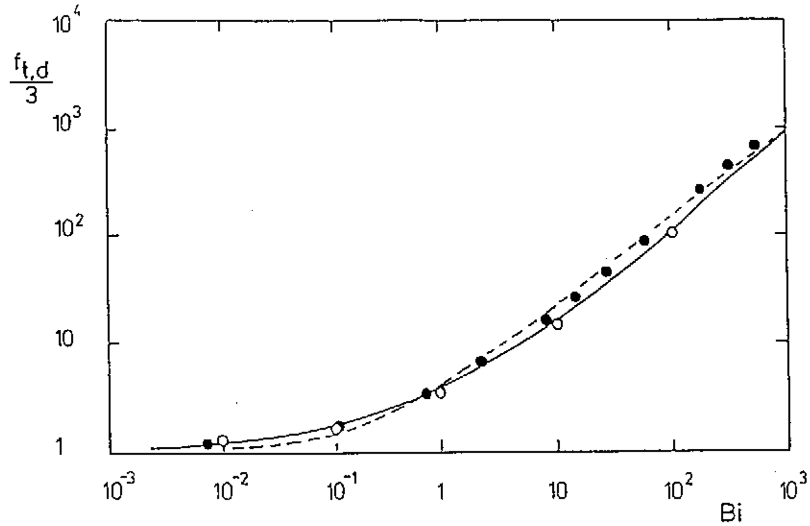

Fig. 1 Comparison of our treatment for Bingham flow model with solutions given in literature in creeping region of flow: ○ – Yoshioka et al. [11], ● – Beris et al. [12], --- Blackery and Mitsoulis [13], —— the solution given in this paper

calculation of Reynolds number Re_p from Eq. (26) adopted the experimental value of fall velocity of particle, and the quantity ξ_p was determined by the method of successive approximations using the momentum balance (Eq. (27)) in which the resistance number ψ_p is given by Eq. (25). The quantity C_D in Eq. (25) was calculated for $Re < 0.2$ using the relationship by Oseen [15]

$$C_D = \frac{24}{Re_p}(1 + \frac{3}{16}Re_p) \tag{32}$$

and for $Re > 0.2$ using the relation by Lapple [16]

$$C_D = \frac{24}{Re_p}(1 + 0.125 Re_p^{0.72}) \tag{33}$$

Table I also gives the comparison with the numerical solution according to Blackery and Mitsoulis [13] (Eq. (31)) in the Reynolds number range where the creeping region of flow ($\psi_p = 1/2$) can be presumed with acceptable accuracy. Table I shows a satisfactory accordance between experiment and the values obtained using the approach suggested in Theoretical. From Table I it follows that

Tab. I Comparison of experimental [14] and calculated values of dimensionless total resistance $f_{t,d}/3$ with application of relative deviation δ_i for tomato sauce

Re_{exp}	Bi_{exp}	$f_{t,d,exp}/3$	δ_i, %, Eq. (29)	δ_i, %, Eq. (31)
0.052	161.9	135.9	-15.9	-32.3
0.067	82.7	82.2	-6.4	-28.8
0.147	120.5	108.8	-12.4	-30.9
0.185	87.2	88.6	-5.0	-26.5
0.270	41.6	49.2	-4.0	-25.1
0.353	39.3	51.0	7.7	-18.6
0.751	31.0	44.5	11.7	
1.65	30.7	40.8	-1.4	
7.09	11.8	21.5	-8.1	
14.1	9.7	20.6	-13.0	

the relative deviations using Eq. (29) are lower than those using numerical solution (Eq. (31)).

Numerical solution to the flow of Casson fluid through a pipe filled with homogeneous porous medium can be found in the literature [17]. In contrast to our solution, it is based on the presumption that Darcy's permeability does not depend on the yield stress τ_0.

However, in practice it is more advantageous to adopt the substantially simpler relations derived on the basis of capillary bed models. The classic models, in which the concept of bed hydraulic radius is used, do not take into account the momentum balance, which respects the fact that in flow through beds of particles the shape resistance exists simultaneously with the friction resistance as in external flow past a single particle. Consequently, the relationship derived from the concept of bed hydraulic radius has to be improved by employing correction factors. Among these models the Blake–Kozeny and Kozeny–Carman models are the most frequently preferred ones [18].

The equation for flow of Herschel–Bulkley fluid through a tube modified using Blake–Kozeny concept was suggested for the flow of VPF through fixed beds of particles by Al-Faris and Pinder [19]. The Kozeny–Carman concept, which is generally used for GNF (e.g. [20]), was not tested by the authors.

The approach suggested starts from the theoretical knowledge that the treatment of the problem at hand needs three equations, viz. the momentum balance, equation of continuity of flow, and rheological equation of state of a fluid. Then the presented equations of the Rabinowitsch–Mooney type, together with the respective relationships for consistency variables, have—in their application—the meaning of integral form of rheological equations of state of GNF and VPF.

The most complex form of equation of Rabinowitsch–Mooney type is that for fluidized bed [4]. However, its validity range can not be extended for VPF, since the quantity ξ_p representing the plastic effects, unlike of quantity ψ representing the viscosity effects, does not appear in the relationships for resistance number ψ_{VPF} for a single particle and fixed beds in the same power.

The equations for VPF are the starting points for solving the problem of flow of more sophisticated viscoelastic fluids past a single particle and through a fixed bed of particles with the help of Rabinowitsch–Mooney type equations. Here, of course, the values of quantities $\tau_E/\tau_{w,p,b}$ express the dimensionless influence of elastic effects, and the form of their dependences on suitably defined elasticity number has to be determined experimentally.

Conclusion

An approach has been suggested to the solution both of flow of VPF past a single spherical particle and of flow of VPF through the fixed bed of particles. This approach has an advantage over the procedures used so far in that it is general. It is formulated independently of any concrete flow model of VPF, no empirical correction being needed in the relations for the creeping flow of VPF and systems with spherical particles.

Symbols

a_p specific surface area of an individual particle, m^{-1}
a_w specific surface of walls, m^{-1}
Bi Bingham number
C_D drag coefficient
d diameter of spherical particle, m
D tube and column diameters, m
$\dot{D}$ shear rate, s^{-1}
$\dot{D}_o$ parameter of Robertson–Stiff flow model, s^{-1}
$\dot{D}_w$ consistency variable for flow through tube, s^{-1} (Eq. (1))
$\dot{D}_{w,b}$ consistency variable for flow through fixed bed, s^{-1} (Eq. (18))
$\dot{D}_{w,p}$ consistency variable for fall of single particle, s^{-1} (Eq. (7))
f_f frictional resistance referred to particle surface area, Pa
f_s shape resistance referred to particle surface area, Pa
f_t total resistance referred to particle surface area, Pa
g gravitational acceleration, m s^{-1}

K	parameter of Robertson–Stiff flow model, Pa s^n
K'	parameter of Herschel–Bulkley flow model, Pa s^n
l_{ch}	characteristic linear dimension of the system, m
L	length of tube and bed, m
n	parameter of Robertson–Stiff flow model
n'	parameter of Herschel–Bulkley flow model
Δp	pressure drop, Pa
r	distance from center of spherical coordinate system, m
r_h	hydraulic radius ($= D/4$), m
R	radius of spherical particle, m
R_o	radius of spherical area, m
R_w	radius of tube, m
$R_{w,o}$	distance from tube axis, m
Re	Reynolds number
S	cross-section of tube, m^2
S_s	spherical area, m^2
u_{ch}	characteristic velocity of system for flow through tube, $m\ s^{-1}$ (Eq. (3))
$u_{ch,b}$	characteristic velocity of system for flow through fixed bed, $m\ s^{-1}$
$u_{ch,p}$	characteristic velocity of system for flow past a single particle, $m\ s^{-1}$
u_g	fall velocity, $m\ s^{-1}$
$\dot{V}$	volumetric flow rate, $m^3\ s^{-1}$
δ_i	relative deviation
ε	porosity of bed
λ	shape factor
μ	dynamic viscosity, Pa s
$\mu_{e,b}$	effective viscosity of non-Newtonian fluid—fixed bed system, Pa s
$\mu_{e,p}$	effective viscosity of non-Newtonian fluid—single particle system, Pa s
$\mu_{e,w}$	effective viscosity of non-Newtonian fluid—tube system, Pa s
ξ	ratio of yield stress to dynamic consistency variable
ρ	density of fluid, kg m^{-3}
ρ_s	density of particle, kg m^{-3}
τ	shear stress, Pa
$\vec{\vec{\tau}}$	tensor of shear stress, Pa
τ_o	yield stress, Pa
$\tau_{r,\theta}$	component of shear stress in spherical coordinate system, Pa
τ_w	consistency variable for flow through tube, Pa (Eq. (2))
$\tau_{w,b}$	consistency variable for flow through fixed bed, Pa (Eq. (17))
$\tau_{w,p}$	consistency variable for fall of a single particle, Pa (Eq. (16))
φ	dimensionless yield stress ($= \tau_o/f_t$)
ψ	resistance number
ω	correction factor for wetted surface

Indexes

b	bed
d	dimensionless
exp	experimental
E	elastic
p	particle
P	plastic
VPF	viscoplastic

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