

**SINE WAVE DISTORTION OF THE VOLTAGE  
FOR CONTACT LINE SYSTEM OF 25 KV, 50 HZ AT CZECH  
RAILWAYS**

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**1. Introduction**

The Czech Railways have had in operation electric locomotives equipped with diode traction converters. The diode converter gives a locomotive character of significant non linear load manifested as current generator with poor value of the power factor and with large content of odd harmonics. The harmonic contents of contact current system produce sine wave distortion of the contact line system of 25 kV, 50 Hz.

Specifying of distortion rate is very important from the viewpoint of description of work conditions for:

- A)** watt-hour and var-hour electricity meters applied on the board of electric locomotive,
- B)** traction converter of electric locomotive used in regenerative braking regime.

The same remarks to these viewpoints of:

- A)** CENELEC has prepared the project of prEN.XXXX „**Railway applications - Energy measurement on board trains**“ in May 2004. In the Introduction, the following recommendations are given:

*„Regarding the influence of harmonics, special test procedures had to be incorporated. These tests check the functionality of the meter when the meter is*

exposed to large distortions in the current circuit and the accuracy of the meter with 5th harmonic in the current and voltage circuit. To check the functionality three practical conditions have been specified:

- half-wave rectification (direct current and even harmonics),
- phase-fired control (odd harmonics),
- burst control (sub-harmonics).

To check if the meters accurately measure total energy in the presence of harmonics a test with 5th harmonic in both the current and voltage circuits has been specified. It is assumed that correct measurement of 5th harmonic energy indicates that measurement for other harmonics will be good.“

- B)** Amended text of prEN 50163: October 2003 „**Railway applications - Supply voltages of traction systems**“ in its Annex C, point C.5 „Distortion of the voltage (a.c. and d.c.)“ pays attention on the distortion of contact line system voltage by variation of zero points, which are able to cause undesirable influence the inverter function of the locomotive traction converter:

„The voltage is distorted by traction and auxiliary loads, converter substations and the public distribution system. This results in high and low frequency harmonics which may include, only for a.c., offset (transient) and extra zero crossings. “

## 2. Presumptions and phases of the analysis

- A)** The so-called „amplitude law“ describes the percentage value of current harmonic components for the electric ČD locomotives fitted with the diode traction converter:

$$I_n \cong \frac{100}{n} [\%] \quad n = \frac{f}{50} \text{ (harmonic number, order)} \quad (1)$$

which gives the maximum percentage value of each current component.

- B)** Input impedance of the railway feeding system from the viewpoint of ČD locomotives includes following elements:
- the impedance of the contact line system between the locomotive and the traction substation,
  - the input impedance of the contact line system between locomotive and open end of the contact line system
  - the input impedance of the filter-compensation equipment (FCE) installed in the traction substation for reducing the harmonic components feed into 110 kV line and for improving the electric locomotive power factor of feeding 110 kV line,
  - the substitute reactance of the substation traction transformer,
  - the impedance of feeding line 110 kV is neglected.
- C)** The FCE equipment has two LC branches, one for the 3rd and the other for the 5th harmonic and decompensation branch. Both LC branches are supposed to be tuned

exactly on 150 Hz and 250 Hz. The decompensation branch is supposed to be disconnected, because inductive power of the locomotive is equal to capacitive power of both LC branches.

Under presumptions described above analysis could be divided into 3 phases, with different frequency ranges:

1. **For frequencies 150 Hz and 250 Hz** - the substitute reactance of the substation traction transformer could be neglected, because both LC branches create short circuits for these frequencies.
2. **For frequencies from the 7th up to the proximity of the first resonance frequency for whole traction feeding system** - the substitute reactance of the substation traction transformer is connected in parallel to the FCE. The traction substation has an inductive character composed with:
  - the substitute inductive reactance of the substation traction transformer,
  - the substitute inductive reactance of the LC branches, because the 7th harmonic and the next frequencies are higher than tuned frequencies of both LC branches.
3. **For frequencies near the first resonant frequency of whole traction feeding system** - the feeding system is composed of:
  - contact line system considered as homogeneous electric line,
  - the traction substation with both LC branches and the substitute reactance of the traction transformer,
  - the electric locomotive similar to current generator of harmonic components.

### Phase of analysis No. 1

The distortion of sine wave contact system voltage depends on:

- electrical parameters of contact line system.
- distance of electric locomotive from the traction substation,
- values of the 3rd and 5th locomotive current spectral components.

The analysis is based on the electrical parameters of the contact line system having following values per kilometer:

- inductance  $L \cong 1 \text{ mH} / \text{km}$
- capacitance  $C \cong 15 \text{ nF} / \text{km}$

The characteristic (surge) impedance that is independent on length of contact line system is  $Z_0 = 258,2 \Omega$  and the phase-shift constant for the 50 Hz spectral component is  $\alpha_1 = 1,217 \cdot 10^{-3} \text{ rad} / \text{km}$ . The absolute value of the input impedance of non dissipative contact line with length  $\ell_{TV} [\text{km}]$  is

$$|Z_{TV}| = Z_0 \cdot \operatorname{tg}(\alpha_1 \cdot \ell_{TV} \cdot n) \quad (2)$$

### Example to the phase of analysis No. 1:

Let the situation is described with these values:

- the distance of the electric locomotive from the traction substation  $\ell_{TV} = 25 \text{ km}$
- the 50 Hz component of the locomotive primary current  $I_{LOK,1} = 200 \text{ A}$
- the percentage of the locomotive current harmonic components  $I_{LOK,n} = \frac{100}{n} [\%]$

According to the Ohm's law:

- the 3rd component of the locomotive voltage is 1572 V (5,7 % of nominal voltage),
- the 5th component of the locomotive voltage has the same value.

The same values correspond for the feeding system of double track contact line system as well, because both FCE's LC branches formed signified short circuits for the 3rd and 5th components.

### Phase of analysis No. 2

This phase of analysis deals with current harmonics from the 7th (350 Hz) up to the proximity of the first resonant frequency of the whole traction feeding system.

Supplementary electric parameters should be newly defined:

- the substitute reactance of the traction transformer  $L_{TT} = 23 \text{ mH}$
- the capacitance of the capacitor group for the 3rd harmonic branch  $C_3 = 6 \text{ }\mu\text{F}$
- the inductance of the 3rd harmonic branch coil  $L_3 = 188 \text{ mH}$
- the capacitance of the capacitor group for the 5th harmonic branch  $C_5 = 2 \text{ }\mu\text{F}$
- the inductance of the 5th harmonic branch coil  $L_5 = 203 \text{ mH}$

The definition of the whole substation substitute impedance forms the first step of this analysis phase. This whole substation is composed of:

- substitute reactance of traction transformer,
- substitute inductance of the 3rd harmonic branch (350 Hz >150 Hz),
- substitute inductance of the 5th harmonic branch (350 Hz >250 Hz).

Common formula for the substitute inductance  $\bar{L}(f)$  of the LC branch in case that the frequency  $f$  is higher then the resonant frequency  $f_{LC,REZ}$  of the LC branch is

$$\bar{L}(f) = L \cdot \left[ 1 - \left( \frac{f_{LC,REZ}}{f} \right)^2 \right] \quad (3)$$

According to this formula, for example, the substitute inductances of the LC branches have following values on the 7th harmonic:

- the 3rd harmonic LC branch  $\bar{L}_{3,7} = 153 \text{ mH}$
- the 5th harmonic LC branch  $\bar{L}_{5,7} = 99,4 \text{ mH}$

These two substitute inductances and the substitute reactance of the traction transformer are connected in parallel. Thus the substitute inductance of the whole substation is  $L_{TNS,7} = 16,65 \text{ mH}$  for the 7th harmonic.

Provided that **the single track-rail** contact line system has from the viewpoint of the locomotive following components ***on the 7th harmonic***:

- the contact line system between the locomotive and the traction substation,
- the traction substation represented with its substitute inductance  $L_{TNS,7}$  formed the shunt impedance of the homogeneous electric line.

The input impedance  $Z_1(n)$  of the finite length of the homogeneous electric line with shunt impedance  $Z_2(n)$  is commonly

$$Z_1(n) = Z_0 \cdot \frac{Z_2(n) + j \cdot Z_0 \cdot \operatorname{tg}(\alpha_1 \cdot n \cdot \ell)}{Z_0 + j \cdot Z_2(n) \cdot \operatorname{tg}(\alpha_1 \cdot n \cdot \ell)} \quad (4)$$

This common formula could be written in analyze of situation

$$Z_1(n) = j \cdot Z_0 \cdot \frac{100 \cdot \pi \cdot n \cdot \bar{L}_{TNS} + Z_0 \cdot \operatorname{tg}(\alpha_1 \cdot n \cdot \ell_{TV})}{Z_0 - 100 \cdot \pi \cdot \bar{L}_{TNS} \cdot \operatorname{tg}(\alpha_1 \cdot n \cdot \ell_{TV})} \quad (5)$$

The result is for example *on the 7th harmonic*  $Z_1(7) = j \cdot 95,37 \ \Omega$

Three supplementary assumptions in the case of **the double track-rail** contact line system are:

- lengths of both contact lines are equal,
- the locomotive is situated at the end of the contact line system,
- the other contact line system is unloaded.

The input impedance of the unloaded contact line system

$$Z_{TV,2}(n) = -j \cdot Z_0 \cdot \operatorname{cotg}(\alpha_1 \cdot n \cdot \ell_{TV})$$

have to be connected in parallel to the substitute impedance of the whole traction substation.

The formula for the input impedance  $Z_2(n)$  of the whole feeding system with double track line is from the viewpoint of the locomotive

$$Z_2(n) = j \cdot Z_0 \cdot \frac{100 \cdot \pi \cdot n \cdot \bar{L}_{TNS} \cdot [\cot g(\alpha_1 \cdot n \cdot \ell_{TV}) - \operatorname{tg}(\alpha_1 \cdot n \cdot \ell_{TV})] + Z_0}{Z_0 \cdot \cot g(\alpha_1 \cdot n \cdot \ell_{TV}) - 200 \cdot \pi \cdot n \cdot \bar{L}_{TNS}} \quad (6)$$

For example the result is **on the 7th harmonic**

$$Z_2(7) = j \cdot 96,66 \, \Omega.$$

The difference of both values  $Z_1(7) = j \cdot 95,37 \, \Omega$  and  $Z_2(7) = j \cdot 96,66 \, \Omega$  from the viewpoint of the locomotive could be explained as follows:

- the input impedance of the whole traction substation is

$$Z_{TNS}(7) = 100 \cdot \pi \cdot 7 \cdot L_{TNS,7} = 36,62 \, \Omega$$

- the input impedance of the unloaded contact line system is

$$Z_{TV}(7) = -j \cdot Z_0 \cdot \cot g(\alpha_1 \cdot 7 \cdot \ell_{TV}) = 1194,2 \, \Omega$$

The connection of the unloaded contact line system to the substation has **for the 7th harmonic** component no essential influence upon the input impedance from the viewpoint of the locomotive.

### Example to the phase of analysis No. 2:

In a similar way it is possible to calculate the 7th spectral component of the contact voltage line system on the locomotive pantograph. The value of the 7th locomotive spectral current component is

$$I_{LOK,7} = \frac{200}{7} = 28,57 \, A$$

The input impedances from viewpoint of the locomotive have **for the 7th harmonic** component approximately equally values  $Z_1(7) \cong Z_2(7) \cong 96 \, \Omega$ . The voltage 7th component in the locomotive pantograph is therefore  $2743 \, V \Rightarrow$  cca 10 % of the nominal traction voltage.

### Phase of analysis No. 3

The input impedance of the whole traction feeding system (traction substation, both LC branches, contact line system of single or double track) could be derived on the condition, when the input impedance reaches theoretically unlimited value.

According to this theorem and all input of example's parameters we can make up three values of the resonant frequency:

- the **first** resonance frequency  $f_{REZ,1} = 141,0 \, \text{Hz}$  ( $n_{REZ,1} = 2,82$ ), this frequency is practically independent on the length of contact system,

- the **second** resonance frequency  $f_{REZ,2} = 238,5 \text{ Hz} (n_{REZ,2} = 2,82)$ , this frequency is practically independent on the length of contact system too,
- the **third** resonance frequency  $f_{REZ,3}$  depends on the length of contact system and as follows:

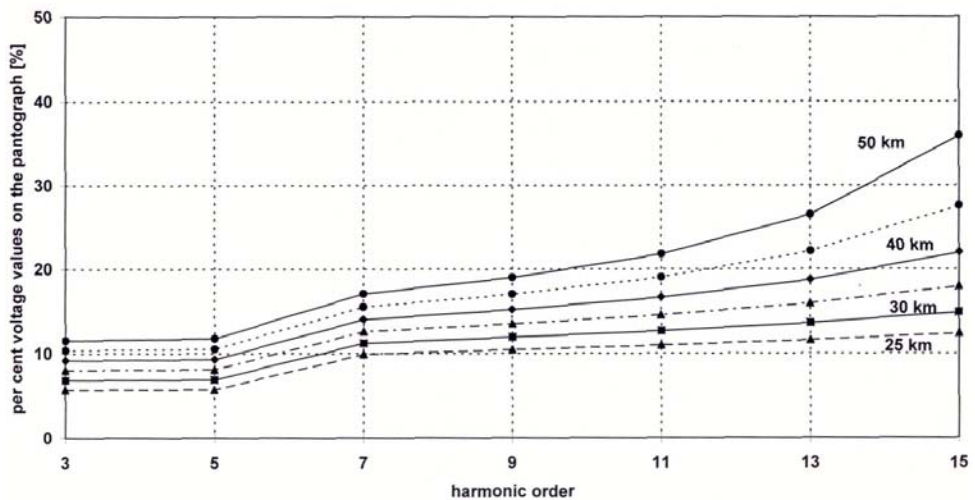
No. of tracks	double track		single track	
Length $\ell_{TV} [km]$	$f_{REZ,3} [Hz]$	$n_{REZ}$	$f_{REZ,3} [Hz]$	$n_{REZ}$
25	1215	24,3	1565	31,3
30	1085	21,7	1385	27,7
35	990	19,8	1240	24,8
40	910	18,2	1130	22,6
45	840	16,8	1035	20,7
50	785	15,7	955	19,1

#### Remarks to this table:

- The 1st and the 2nd resonance frequencies are dependent first of all on parameters of both LC branches and that is why they are independent on the length of contact line system. This values are situated far a field from the odd harmonics (the 3rd and the 5th) produced by the locomotive.
- Further harmonics occur above the 3rd resonance harmonic, because the function „cotangent“ is periodical. For example the 4th resonance harmonic component of double track 25 km is  $f_{REZ,4} = 2580 \text{ Hz} (n_{REZ,4} = 51,6)$ .

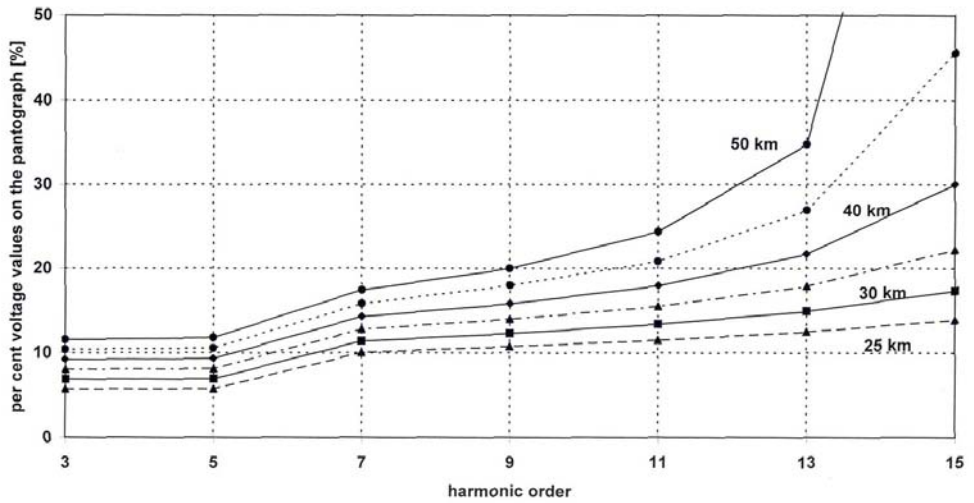
### 3. Final comprehensive results from all three phases of the analysis

- A) The voltage distortion of the contact line system in the proximity of the electric locomotive going at the line end is under supposed traction feeding system configuration differ for the analysed frequency components.
- B) The results of phase analysis No. 1 for the 3rd and 5th harmonic components are under supposed conditions lower than 7th, 9th etc. harmonic components discussed in the phase analysis No. 2.
- C) **Fig. 1** describes the per cent voltage distortion of the harmonic components with harmonic order from 3 up to 15 for lengths of single track line 25 up to 50 km. **Fig. 2** describes the same for a double track line.



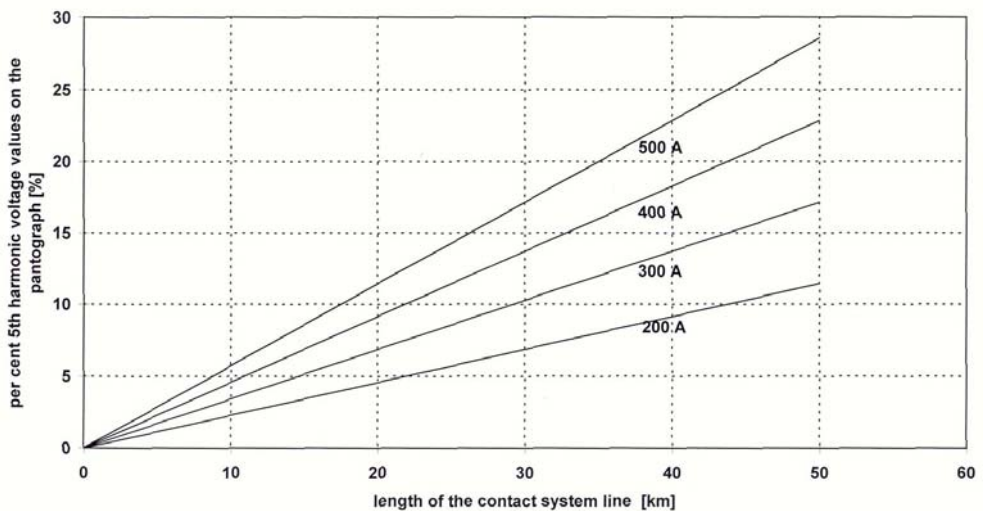
**Fig. 1** Per cent voltage values on the locomotive pantograph located at the end of the single track feeding line





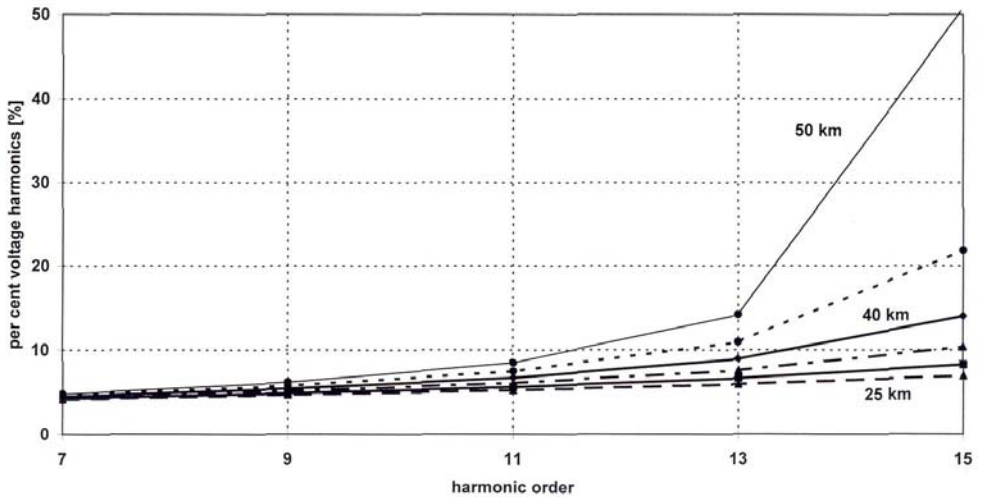
**Fig. 2** Per cent voltage values in the locomotive pantograph located at the end of the double track feeding line

- D) The straight lines connecting the calculated points are valid only as visual aid, not for interpolation.
- E) **Fig. 3** gives an answer to the analysed above point of view **A)** special for the 5th harmonic component percentage distortion for locomotive currents 50 Hz from 200 A up to 500 A.



**Fig. 3** Per cent 5<sup>th</sup> harmonic values of the voltage on the pantograph going at the end of the feeding line

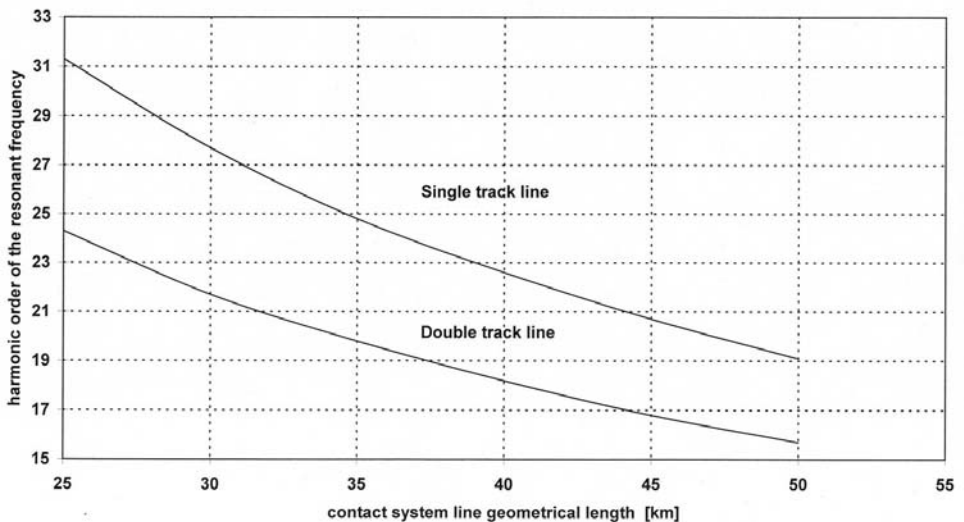
F) **Fig. 4** is based on the results of all three analysis phases for the traction substation output voltage with the utilization of contact line system electric simulation on the frequency from the 7th up to 15th and length of double track line from 25 km up to 50 km.



**Fig. 4** Harmonics of the traction substation output voltage for the double track line and for the locomotive going at his end

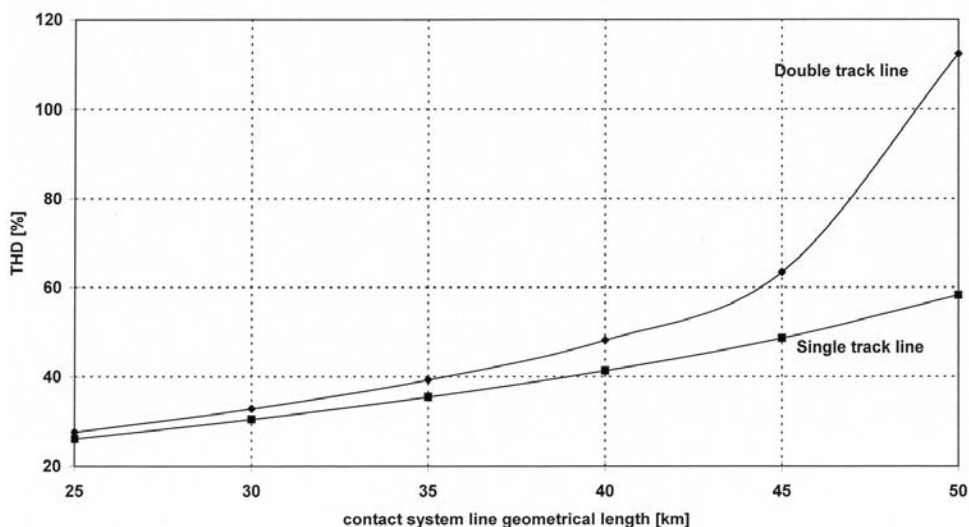
G) The traction substation output voltage 3rd and 5th spectral components are omitted, because they are under supposed conditions short cut with the both LC branches.

H) **Fig. 5** describes single and double track line resonance frequencies of the whole traction feeding system for the length from 25 km up to 50 km.



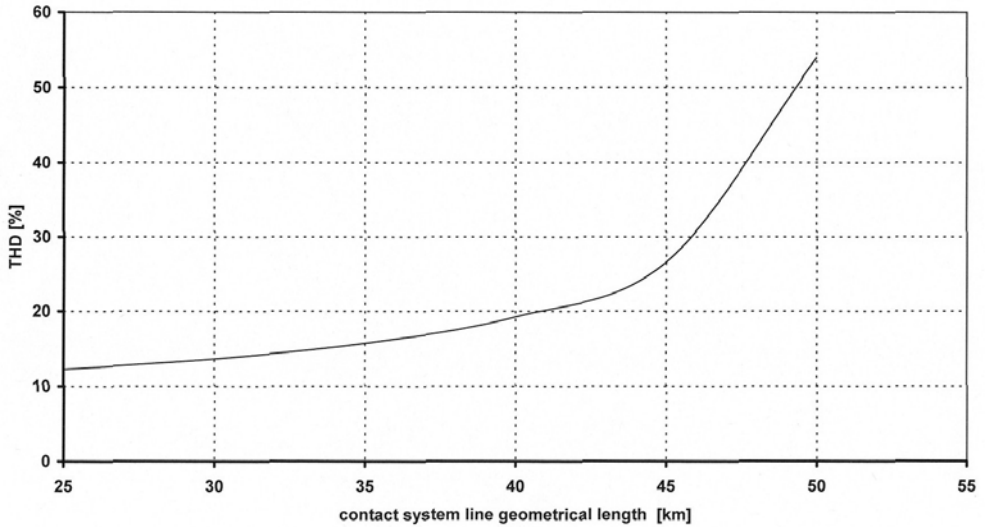
**Fig. 5** Dependence of the whole traction feeding system resonant frequency on the contact system line geometrical length

- I) The voltage distortion of the contact line system for frequencies in proximity to the resonance frequency of the whole traction feeding system is essentially dependent on the rate of traction feeding system damping due to the traction real power consumption. None analysis has been given for these frequencies laying in working railway conditions above the 15th, because their influence on the locomotive traction converter in the mode of regenerative breaking and on the electricity meters is assumed to be negligible.
- J) **Fig. 6** describes THD values of the contact voltage line system for the locomotive at the end of the single and double track feeding system, on frequencies from the 3th up to the 15th and length from 25 km up to 50 km.



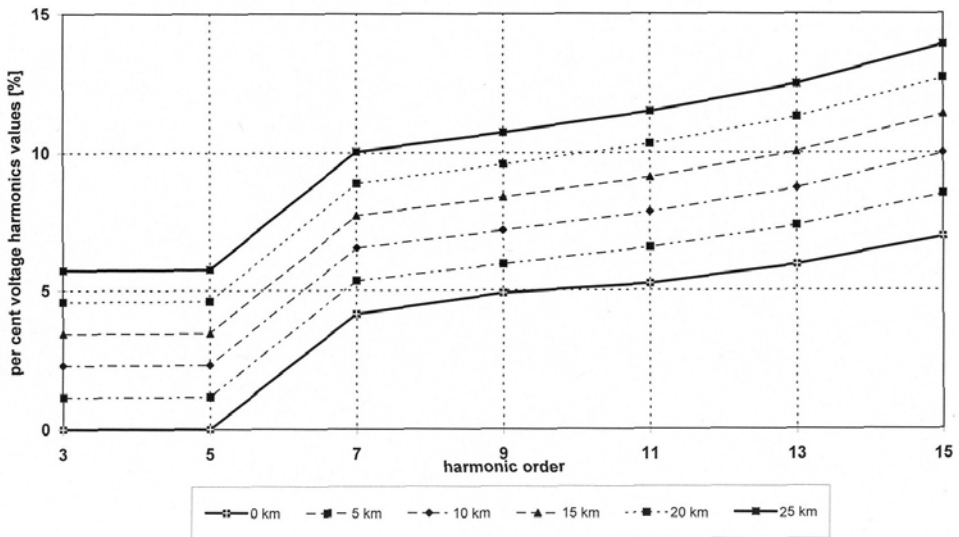
**Fig. 5** THD Voltage values on the pantograph going at the end of contact system line

- K) **Fig. 7** describes THD values of the traction substation voltage output for the double track line.



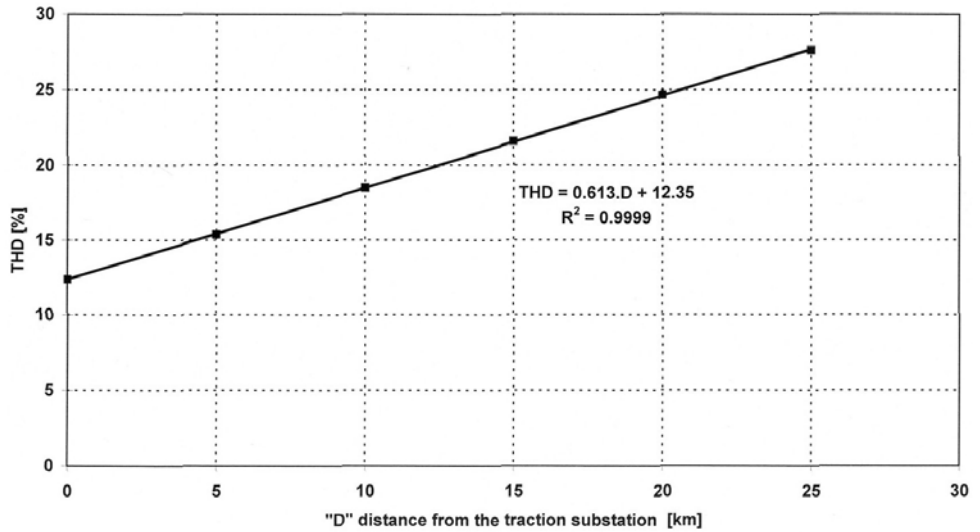
**Fig. 7** THD of traction substation output voltage

- L) The voltage spectral components and THD values on the unloaded track line are in the whole length the same as the traction substation output voltage spectral components.
- M) The voltage spectral components values for 4 points of the loaded line laying between the traction substation and the feeding line end are described on **Fig. 8** for the locomotive approaching the end of the track line and for the line length of 25 km.



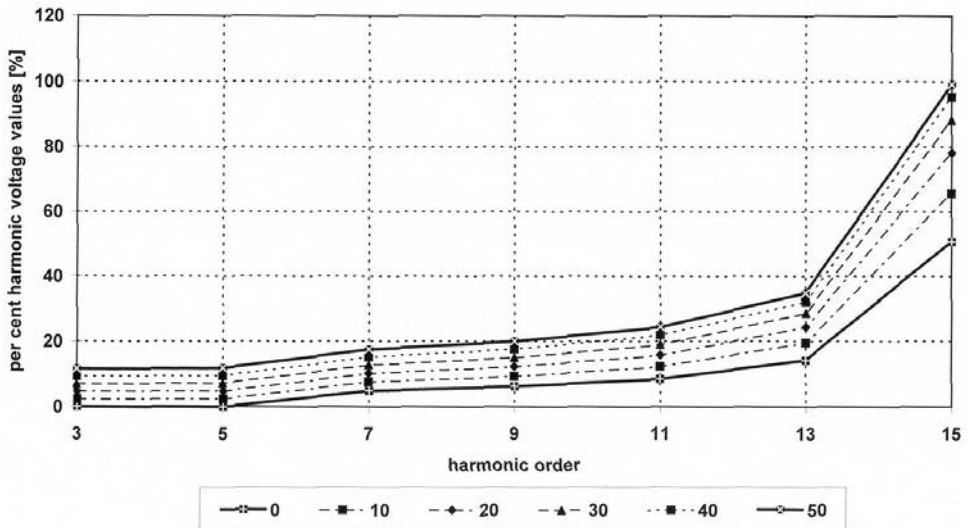
**Fig. 8** Contact system line voltage harmonics in individual points for system length 25 km and the locomotive at the end of this line

N) The THD values of contact voltage line system on various points of the contact line with the length 25 km and the locomotive at the end of the line comprises **Fig. 9**.



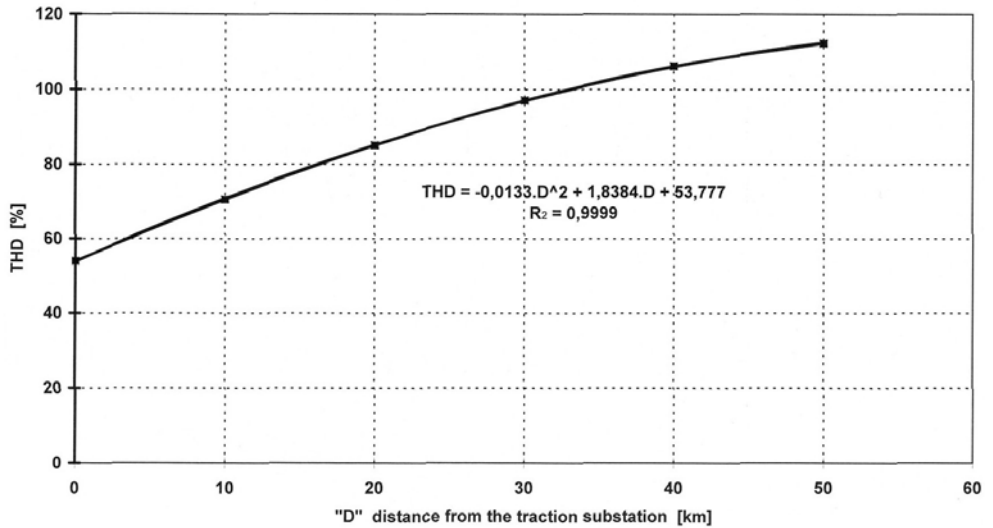
**Fig. 9** THD of the contact system line voltage for the system length 25 km and locomotive at the end of this line

O) **Fig. 10** describes the same case like Fig. 8, but for the line length 50 km.



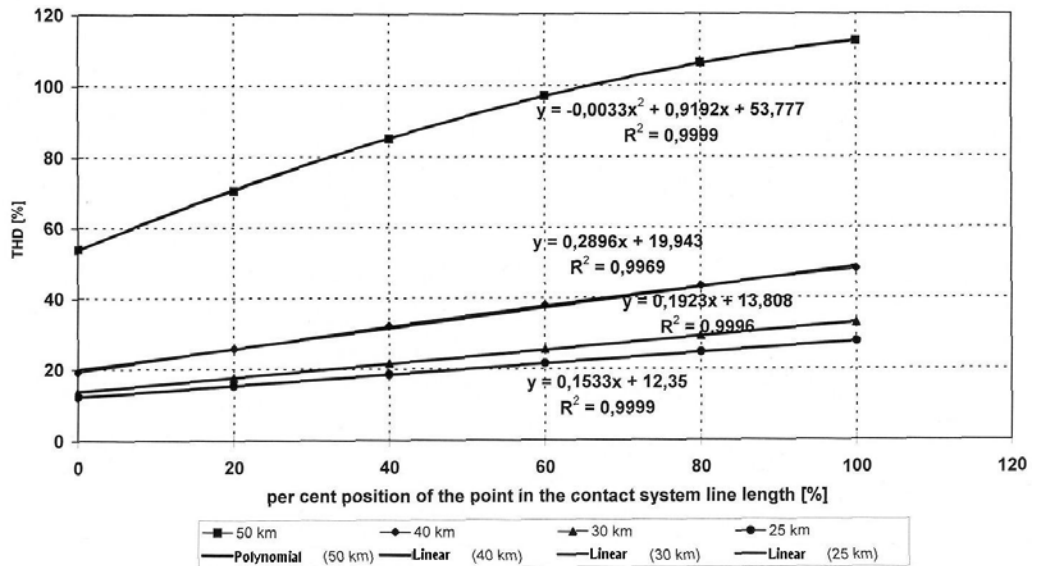
**Fig. 10** Contact system line voltage harmonics in individual points for the system length 50 km and locomotive at the end of this line

P) **Fig. 11** describes the same case like Fig. 9, but for the line length 50 km.



**Fig. 11** THD of contact system line voltage for the system length 50 km and locomotive at the end of this line

Q) **Fig. 12** gives an comprehensive result of the contact voltage line system THD values on various points of contact line (expressed in %) for line length 25 km up to 50 km, the locomotive at the end of the line, with addition of regression curves.



**Fig. 12** THD of the contact system line voltage in individual points of the contact system

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### Literatura

1. SBB/CFF: *Überlagerungsversuche Zürich - Meilen – Rapperswil*. ORE A 122, leden (1977).
2. BURTSCHER H., LEKKAS G. *Labormodell zur Untersuchung der Ausbreitung und Superposition von Oberschwingungen im Bahnnetz Institut für Automatik und Industrielle Elektronik der ETH Zürich*. ORE A 122, (1977).
3. ZWICKY R. *Zwischenbericht über die Untersuchungen zur Superposition von Oberschwingungen im Bahnnetz* (Resultate aufbauend auf Messauswertungen)
4. HLAVA K. *Elektromagnetická kompatibilita (EMC) drážních zařízení*. Skriptum Univerzity Pardubice, (2004).

### Souhrn

#### **ANALÝZA NAPĚŤOVÝCH HARMONICKÝCH V TRAKČNÍM VEDENÍ ČD**

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Studie analyzuje deformace časového průběhu napětí na trakčním vedení při provozu hnacích vozidel s diodovými trakčními měniči. Zjištěné hodnoty harmonických jsou z ohledem na připravované evropské normy i směrnice pro interoperabilitu důležité nejen pro definování podmínek provozu elektroměrů na hnacích vozidlech, ale i pro zajištění bezpečnosti funkce rekuperačního brzdění nových typů hnacích vozidel jednofázové soustavy. Rozbor respektuje použití filtračně kompenzačních zařízení v trakčních napájecích stanicích ČD.

### Summary

#### **SINE WAVE DISTORTION OF THE VOLTAGE FOR CONTACT LINE SYSTEM OF 25 KV, 50 HZ AT CZECH RAILWAYS**

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The paper deals with the wave distortion analysis of the contact system line voltage at the operation of electric motive power with diode traction converters. The ascertained harmonic values are important for the prepared european standards and guidelines about interoperability not only from the point of view of the energy measurements on board trains working conditions, but also from the point of view to insure the regenerative breaking safety of new AC motive power. The analysis respects the filter-compensation equipment installed in ČD traction substations.

## **Zusammenfassung**

### **ANALYSE DER FAHRLEITUNGSPANNUNGOBERWELLEN DER ČD**

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Die Studie analysiert Deformation des Zeitverlaufes der Fahrleitungsspannung beim Betrieb von Triebfahrzeugen mit Diodentraktionsumformer. Die festgestellten Werte sind im Zusammenhang mit den Entwürfen der neuen europäischen Standards und mit den Richtlinien für die Interoperabilität wichtig. Der Belang liegt nicht nur in der Betriebsbedingungen der Elektrizitätszählern am Bord der Triebfahrzeugen, sondern auch für die Betriebssicherheit der Rekuperationsbremsen neuer Triebfahrzeugen des 50 Hz-Traktionssystems. Analyse respektiert die Anwendung von Filter-Kompensations-Einrichtungen in den Bahnunterwerken der ČD.