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NOISE IN THE INDEPENDENT PACKET LOSS OF SPEECH SIGNAL MODELED BY CYCLOSTATIONARY STOCHASTIC PROCESS

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1. Introduction

For the speech transmission, IP telephony service uses the packet commutation principles [6], [7]. This means that after being sampled the speech is segmented into blocks (most frequently parts of speech lasting 50 ms). The blocks create an information part of packets delivered to a receiver address.

During the transmission over IP network, packets get delayed, or lost. Variable delays are eliminated in the jitter buffer (of an IP telephone) [8]. However,

at the moment when the packet samples are to be played the packet may not be delivered in the terminal device. Such an extreme packet delay causes the packet loss again, and the packets delivered later have to be destroyed.

Some preservation mechanisms have been developed to protect the speech quality from being lowered by potential packets losses. The mechanisms include for example reconstructing lost samples from the previous ones, self-correction codes, or a jitter buffer control. So far, no theory of packet loss influence on speech quality has been elaborated. The first work in this area can be seen in [3].

This article presents the results achieved by modeling some parts of speech (vowels) as a cyclostationary process.

2. Preliminaries

Theory of cyclostationary processes can be found in [2], [4]. We give some definitions and properties to better understanding of this notion.

Definition 1.1 A correlation $R(t_1, t_2)$ of the process $\mathbf{x}(t, \xi)$ is the mean value of the product

$$R_{xx}(t_1, t_2) = R(t_1, t_2) = E\{\mathbf{x}(t_1, \xi) \overline{\mathbf{x}}(t_2, \xi)\}$$

Definition 1.2 A stochastic process $\mathbf{x}(t,\xi)$ is called stationary(wide-sense stationary, WSS), if the mean of this process is constant and its correlation depends only on $\tau = t_1 - t_2$

$$E\{\mathbf{x}(t,\xi)\} = \eta$$
 and $E\{\mathbf{x}(t+\tau,\xi)\overline{\mathbf{x}}(t,\xi)\} = R(\tau)$

Theorem 1.1 Let the function $R(\tau)$ is correlation of some stochastic process $\mathbf{x}(t,\xi)$ then:

 $|R(\tau)| \leq R(0)$

Definition 1.3 A stochastic process $\mathbf{x}(t,\xi)$ is called mean square periodic with period T if only if

$$E\{|\mathbf{x}(t+T,\xi)-\mathbf{x}(t,\xi)|^2\}=0$$

for all $t \in R$.

Definition 1.4 The Correlation $R(t_1, t_2)$ is called doubly periodic if only if

 $R(t_1 + mT, t_2 + nT) = R(t_1, t_2)$ for all $t_1, t_2 \in R$.

Theorem 1.2 Stochastic process $\mathbf{x}(t,\xi)$ is called mean square periodic with period T if only if its correlation $R(\tau)$ is doubly periodic.

Definition 1.5 A stochastic process $\mathbf{x}(t,\xi)$ is called cyclostationary process if only if

 $E\{\mathbf{x}(t+T,\xi)\} = E\{\mathbf{x}(t,\xi)\}$ and $R(t_1 + mT, t_2 + mT) = R(t_1, t_2)$ for all $m \in \mathbb{Z}$.

Theorem 1.3 If the stochastic process $\mathbf{x}(t,\xi)$ is stationary, then the process $\mathbf{x}(t,\xi)$ is cyclostationary.

Theorem 1.4 If the stochastic process $\mathbf{x}(t,\xi)$ is mean square periodic, then the process $\mathbf{x}(t,\xi)$ is cyclostationary.

Theorem 1.5 If the stochastic process $\mathbf{x}(t,\xi)$ is cyclostationary and θ is the random variable uniform in the interval (0,T) and independent of $\mathbf{x}(t,\xi)$ for all $t \in R$. Then the

process $\overline{\mathbf{x}}(t,\xi) = \mathbf{x}(t-\theta,\xi)$ obtained by a random shift of the origin is stationary process with mean $\overline{\eta}$ and correlation $\overline{R}(\tau)$:

$$\overline{\eta} = \frac{1}{T} \int_{0}^{T} \eta(s) ds$$
(1)

$$\overline{R}(\tau) = \frac{1}{T} \int_{0}^{T} R(s + \tau, s) ds$$
(2)

Theorem 1.6 Let $\mathbf{x}(t,\xi)$ is a stochastic process

$$\mathbf{x}(t,\xi) = \sum_{n=-\infty}^{\infty} c_n h(t-nT)$$
(3)

with deterministic h(t), its Fourier transform $H(\omega)$ and stationary sequence of random variables $\mathbf{c}_{\mathbf{n}}$. The mean of the sequence $\mathbf{c}_{\mathbf{n}}$ is $E\{\mathbf{c}_{\mathbf{n}}\} = m_c$ and its correlation is $R_c[m] = E\{c_{n+m}c_n\}$. Then the process $\mathbf{x}(t,\xi)$ is cyclostationary process.

3. Reconstruction of the stochastic signal in the case of independent samples loss

First we use the model applying the Shanon decomposition. Shanon sampling theorem can be found for instance in [1], [5]. The following proposition shows how the original signal changes in case the transmission has been realized as a decomposition into Shanon base and lost values have been replaced by 0.

Definition 2.1 Let $\mathbf{x}(t,\xi)$ is the centered stochastic process. The value of $R(t_1,t_2)$ on $t = t_1 = t_2$:

$$\sigma_x^2(t) = R(t,t) = E\{\mathbf{x}^2(t,\xi)\}$$

is called the average power of the process $\mathbf{x}(t,\xi)$.

Theorem 2.1 Let $\mathbf{x}(t,\xi)$ is the stationary, centered, ω -bandlimited stochastic process

and let its representation in Shanon basis in case $T = \frac{\pi}{\Omega}$, $t \in R$ is

$$\mathbf{x}(t,\xi) = \sum_{k=-\infty}^{\infty} \mathbf{x}(kT,\xi) \frac{\sin\Omega(t-kT)}{\Omega(t-kT)} = \sum_{k=-\infty}^{\infty} c_k(\xi) \Phi_k(t)$$

therefore the basis functions are deterministic and are equal $\Phi_k(t) = \operatorname{sinc} \Omega(t - kT)$, the coefficients of the process in the basis $\Phi_k(t)$ are random variables $\mathbf{c}_k = \mathbf{x}(kT,\xi)$. Let the probabilities $p_{k,k} \in Z$ of the coefficients \mathbf{c}_k loss, are independent each other and independent of the value of $\mathbf{c}_k(\xi)$. Let this probabilities are equal to $p \ p_k = p$, $\forall k \in Z$.

After transition over IP the voice process is reconstructed. Let the reconstructed process $\mathbf{x}'(t,\xi)$ is the sum where the lost samples are represented by 0.

Then the mean power and correlation of noise process $\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi)$ are respectively

$$\sigma_{\mathbf{q}}^{\mathbf{2}}(t) = \rho \sigma_{\mathbf{x}}^{\mathbf{2}}(t) \tag{4}$$

$$R_{g}(t) = \rho^2 R_{\chi}(t) \tag{5}$$

Proof:

Let $L_k(\xi)$ is a random variable such, that

 $\mathbf{L}_{\mathbf{k}}(\xi) = 0 \qquad \text{if coefficient} \qquad \mathbf{c}_{k}(\xi) \qquad \text{was transmit} \\ = 1 \qquad \text{if coefficient} \qquad \mathbf{c}_{k}(\xi) \qquad \text{was not transmit} \end{cases}$

Then

$$P\{L_{k}(\xi) = 0\} = 1 - p_{k} = 1 - p$$
 $P\{L_{k}(\xi) = 1\} = p_{k} = p$

and

$$E\{\mathbf{L}_{\mathbf{k}}\} = 0(1-p) + 1p = p$$

Reconstructed process is

$$\mathbf{x}'(t,\xi) = \sum_{k=-\infty}^{\infty} (1 - \mathbf{L}_{\mathbf{k}}(\xi)) c_k(\xi) \Phi_k(t)$$

The stochastic process described noise (error of transmission) is the process

$$\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi) = \sum_{k=-\infty}^{\infty} \mathbf{L}_{\mathbf{k}}(\xi) c_{k}(\xi) \Phi_{\mathbf{k}}(t)$$

The mean power of the noise will be calculated as

$$\sigma_{g}^{2}(t) = E\{g^{2}(t,\xi)\} = E\{\left(\sum_{k=-\infty}^{\infty} \mathbf{L}_{k}(\xi) c_{k}(\xi) \Phi_{k}(t)\right)^{2}\} = E\{\sum_{k=-\infty}^{\infty} \mathbf{L}^{2}_{k}(\xi) c^{2}_{k}(\xi) \Phi^{2}_{k}(t)\} = \sum_{k=-\infty}^{\infty} E\{\mathbf{L}^{2}_{k}(\xi)\} C^{2}_{k}(\xi) C^{2}_{k}(\xi) \Phi^{2}_{k}(\xi)\} \Phi^{2}_{k}(\xi) = \sum_{k=-\infty}^{\infty} E\{\mathbf{L}^{2}_{k}(\xi)\} E\{c^{2}_{k}(\xi)\} \Phi^{2}_{k}(\xi) = E\{c^{2}_{k}(\xi)\} \Phi^{2}_{k}(\xi) C^{2}_{k}(\xi) \Phi^{2}_{k}(\xi)\} \Phi^{2}_{k}(\xi) = E\{c^{2}_{k}(\xi)\} \Phi^{2}_{k}(\xi) C^{2}_{k}(\xi) \Phi^{2}_{k}(\xi) \Phi^{2}_{k}$$

$$= p \sum_{k=-\infty}^{\infty} E\{ c^2_k(\xi) \} \Phi^2_k(t) = p E\{ \sum_{k=-\infty}^{\infty} c^2_k(\xi) \Phi^2_k(t) \} = p \sigma_x^2(t)$$

In the calculation above, we have used the facts below:

The system of the functions $\{\Phi_k(t) = \operatorname{sinc} \Omega(t - kT)\}\$ is orthogonal,

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the random variables $L_k(\xi)$ and $c_k(\xi)$ are independent and

$$E\{\mathbf{L}^{2}_{\mathbf{k}}(\xi)\} = 0^{2}(1-p) + 1^{2}p = p$$

Next we will calculate the correlation of the process $g(t,\xi)$

We will use the fact, that if process $\mathbf{x}(t,\xi)$ is bandlimited, then the processes $\mathbf{x}'(t,\xi)$ and $\mathbf{g}(t,\xi)$ are bandlimited too. The process $\mathbf{g}(t,\xi)$ is called bandlimited, if and only if for its spectral density is $S_g(\omega) = 0$ for $|\omega| > \Omega$. The correlation $R_g(t)$ of the process $\mathbf{g}(t,\xi)$ is the inverse Fourier transform of the function $S_q(\omega)$.

Therefore, the function $R_g(t)$ satisfies the conditions of the Shanon sampling theorem and it can be expressed in the form

$$R_g(t) = \sum_{n=-\infty}^{\infty} R_g(nT,\xi) \frac{\sin\Omega(t-nT)}{\Omega(t-nT)} = \sum_{n=-\infty}^{\infty} R_g(nT,\xi) \operatorname{sinc}\Omega(t-nT)$$
(6)

The coefficients $R_q(nT)$ are

$$R_{g}(nT) = E\{\mathbf{g}(0,\xi)\mathbf{g}(nT,\xi)\} =$$

$$= E\{\sum_{k=-\infty}^{\infty} \mathbf{L}_{\mathbf{k}}(\xi) c_{k}(\xi) \operatorname{sinc} \Omega(0-kT) \sum_{l=-\infty}^{\infty} \mathbf{L}_{l}(\xi) c_{l}(\xi) \operatorname{sinc} \Omega(nT-lT)\} =$$

$$E\{\mathbf{L}_{0}(\xi) c_{0}(\xi) \operatorname{sinc} \Omega(0-0T)\mathbf{L}_{n}(\xi) c_{n}(\xi) \operatorname{sinc} \Omega(nT-nT)\} =$$

$$E\{\mathbf{L}_{0}(\xi)\mathbf{L}_{n}(\xi)\}E\{c_{0}(\xi) c_{n}(\xi)\}\operatorname{sinc}^{2}(\Omega 0) = p^{2}R_{x}(nT)$$

We have used

the orthonormality of the Shanon basis

sinc
$$\Omega(t - kT) = 1$$
 for $t = kT$ $k \in Z$
= 0 for $t = nT$ $n \in Z, n \neq k$
sinc² ($\Omega 0$) = 1²

the independence of the loss of the different samples $L_{k}(\xi)$ and $L_{k+n}(\xi)$

$$E\{\mathbf{L}_0(\xi)\mathbf{L}_n(\xi)\} = E\{\mathbf{L}_0(\xi)\}E\{\mathbf{L}_n(\xi)\} = p^2$$

the stationarity of the sequence $\mathbf{c}_k(\xi)$

$$E\{c_0(\xi) c_n(\xi)\} = R_x(nT)$$

After substituting the coefficients into the equation (6) we get

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$$R_g(t) = \sum_{n=-\infty}^{\infty} p^2 R_x(nT) \operatorname{sinc} \Omega(t - nT) = p^2 R_x(t)$$

Theorem 2.2 Let $\mathbf{x}(t,\xi)$ is the stationary, centered, ω -bandlimited stochastic process and let $\Phi_k(t) = \Phi(t - kT)$ is the sequence of the functions and let the function $\Phi(t) = 0$ for $|t| > \frac{T}{2}$. Let the stochastic process can be unique expressed as

$$\mathbf{x}(t,\xi) = \sum_{k=-\infty}^{\infty} c_k(\xi) \Phi_k(t)$$

where the functions $\Phi_k(t) = \Phi(t - kT)$ are deterministic, the coefficients of the decomposition of the process in basis $\Phi_k(t)$ are random variables $\mathbf{c}_k(\xi)$.

Let the probabilities $p_{k, k} \in Z$ of the coefficients $\mathbf{c}_k(\xi)$ loss, are independent each other and independent of the value of $\mathbf{c}_k(\xi)$. Let this probabilities are equal to $p_k = p, \forall k \in Z$. Let the reconstructed process $\mathbf{x}'(t,\xi)$ is the sum where the lost coefficients are represented by 0.

Then the mean power of the noise process $\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi)$ can be calculated as

$$\sigma_{\mathbf{g}}^{\mathbf{2}}(t) = \rho \sigma_{\mathbf{x}}^{\mathbf{2}}(t)$$

Proof:

similarly as in the theorem 2.1. can be shown that the noise process is

$$\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi) = \sum_{k=-\infty}^{\infty} \mathbf{L}_{\mathbf{k}}(\xi) c_{k}(\xi) \Phi_{\mathbf{k}}(t)$$

where random variable $L_k(\xi)$ denotes

$L_{k}(\xi) = 0$	if coefficient	$\mathbf{c}_k(\xi)$	was transmit
= 1	if coefficient	$\mathbf{c}_k(\xi)$	was not transmit

The mean power of the noise process is

$$\sigma_{\mathbf{g}}^{2}(t) = E\{\mathbf{g}^{2}(t,\xi)\} = E\{(\sum_{k=-\infty}^{\infty} \mathbf{L}_{\mathbf{k}}(\xi) \mathbf{c}_{k}(\xi) \Phi_{\mathbf{k}}(t))^{2}\} = E\{\sum_{k=-\infty}^{\infty} \mathbf{L}_{\mathbf{k}}^{2}(\xi) \mathbf{c}_{k}^{2}(\xi) \Phi_{\mathbf{k}}^{2}(t)\} =$$
$$= \sum_{k=-\infty}^{\infty} E\{\mathbf{L}_{\mathbf{k}}^{2}(\xi) \mathbf{c}_{k}^{2}(\xi)\} \Phi_{\mathbf{k}}^{2}(t) = \sum_{k=-\infty}^{\infty} E\{\mathbf{L}_{\mathbf{k}}^{2}(\xi)\} E\{\mathbf{c}_{k}^{2}(\xi)\} \Phi_{\mathbf{k}}^{2}(t) =$$
$$= p \sum_{k=-\infty}^{\infty} E\{\mathbf{c}_{k}^{2}(\xi)\} \Phi_{\mathbf{k}}^{2}(t) = p E\{\sum_{k=-\infty}^{\infty} \mathbf{c}_{k}^{2}(\xi)\} \Phi_{\mathbf{k}}^{2}(t)\} = p \sigma_{\mathbf{x}}^{2}(t)$$

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We have used (like in the theorem 2.1)

$$\Phi_k(t)\Phi_l(t) = 0 \qquad \text{for} \qquad \forall t \in R, \quad k \neq l \quad k, l \in Z$$

and random variables $L_k(\xi)$ and $c_k(\xi)$ are independent and the mean

$$E\{\mathbf{L_k}^2(\xi)\} = 0^2(1-p) + 1^2p = p$$
(7)

Theorem 2.3 Let $\mathbf{x}(t,\xi)$ is the stochastic process and $\Phi_k(t) = \Phi(t - kT)$ is the sequence of the functions and the function $\Phi(t) = 0$ for |t| > mT, where *m* is the arbitrary natural number. Let the stochastic process can be unique expressed as

$$\mathbf{x}(t,\xi) = \sum_{k=-\infty}^{\infty} \mathbf{c}_{k}(\xi) \Phi_{k}(t)$$

where the functions $\Phi_k(t) = \Phi(t - kT)$ are deterministic, and let the coefficients $\mathbf{c}_k(\xi)$ of the decomposition of the process into the basis $\Phi_k(t)$ create a discrete stationary stochastic process. Let the probabilities p_k of the loss of the coefficients $\mathbf{c}_k(\xi)$ are independent each other, they all are independent of the value of $\mathbf{c}_k(\xi)$ and are equal $p_k = p, \quad \forall k \in \mathbb{Z}$. Let the reconstructed process $\mathbf{x}'(t,\xi)$ is the sum where the lost coefficients are represented by 0.

Then the mean power of the noise process $\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi)$ satisfies the condition:

$$\sigma_{\mathbf{g}}^{\mathbf{2}}(t) \leq p \sigma_{x}^{\mathbf{2}}(t) (1 + 2mp)$$

Proof:

The noise process is expressed by

$$\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi) = \sum_{k=-\infty}^{\infty} \mathbf{L}_{\mathbf{k}}(\xi) \, \mathbf{c}_{k}(\xi) \, \Phi_{\mathbf{k}}(t)$$

The mean power of the noise is calculated as

$$\sigma_{\mathbf{g}}^{2}(t) = E\{\mathbf{g}^{2}(t,\xi)\} = E\{(\sum_{k=-\infty}^{\infty} \mathbf{L}_{\mathbf{k}}(\xi) \mathbf{c}_{k}(\xi) \Phi_{\mathbf{k}}(t))^{2}\} = E\{\sum_{k=-\infty}^{\infty} \sum_{n=-m}^{m} \mathbf{L}_{\mathbf{k}}(\xi) \mathbf{L}_{\mathbf{k}+n}(\xi) \mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi) \Phi_{\mathbf{k}}(t) \Phi_{\mathbf{k}+n}(t)\} = \sum_{k=-\infty}^{\infty} E\{\mathbf{L}_{\mathbf{k}}^{2}(\xi)\}E\{\mathbf{c}_{k}^{2}(\xi)\} \Phi_{\mathbf{k}}^{2}(t) + 2\sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} E\{\mathbf{L}_{\mathbf{k}}(\xi)\mathbf{L}_{\mathbf{k}+n}(\xi)\}E\{\mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi)\} \Phi_{\mathbf{k}}(t) \Phi_{\mathbf{k}+n}(t)\} = E\{\mathbf{c}_{k}^{2}(\xi)\}E\{\mathbf{c}_{k}^{2}(\xi)\}\Phi_{\mathbf{k}}^{2}(t) + 2\sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} E\{\mathbf{L}_{\mathbf{k}}(\xi)\mathbf{L}_{\mathbf{k}+n}(\xi)\}E\{\mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi)\}\Phi_{\mathbf{k}}(t)\Phi_{\mathbf{k}+n}(t)\} = E\{\mathbf{c}_{k}^{2}(\xi)\}E\{\mathbf{c}_{k}^{2}(\xi)\}\Phi_{\mathbf{k}}^{2}(t) + 2\sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} E\{\mathbf{L}_{\mathbf{k}}(\xi)\mathbf{L}_{\mathbf{k}+n}(\xi)\}E\{\mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi)\}\Phi_{\mathbf{k}}(t)\Phi_{\mathbf{k}+n}(t)\} = E\{\mathbf{c}_{k}^{2}(\xi)\}E\{\mathbf{c}_{k}^{2}(\xi)\}\Phi_{\mathbf{k}}^{2}(t) + 2\sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} E\{\mathbf{L}_{\mathbf{k}}(\xi)\mathbf{L}_{\mathbf{k}+n}(\xi)\}E\{\mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi)\}\Phi_{\mathbf{k}}(t)\Phi_{\mathbf{k}+n}(t)\}$$

$$= pE\{\sum_{k=-\infty}^{\infty} \mathbf{c}_{k}^{2}(\xi)\} \Phi_{k}^{2}(t)\} + 2p^{2} \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} E\{\mathbf{c}_{k}(\xi) \ \mathbf{c}_{k+n}(\xi)\} \Phi_{k}(t) \Phi_{k+n}(t) \le 0$$

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$$\leq p\sigma_{x}^{2}(t) + p^{2} \sum_{k=-\infty}^{\infty} E\{\mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi)\} \sum_{n=1}^{m} (\Phi_{k}^{2}(t) + \Phi_{k+n}^{2}(t)) \leq$$

$$\leq p\sigma_{x}^{2}(t) + mp^{2} \sum_{k=-\infty}^{\infty} E\{\mathbf{c}_{k}^{2}(\xi)\} \Phi_{k}^{2}(t) + p^{2} \sum_{n=1}^{m} \sum_{k=-\infty}^{\infty} E\{\mathbf{c}_{k+n}^{2}(\xi)\} \Phi_{k+n}^{2}(t) =$$

$$= p\sigma_{x}^{2}(t) + mp^{2}\sigma_{x}^{2}(t) + p^{2} \sum_{n=1}^{m} \sigma_{x}^{2}(t) = p\sigma_{x}^{2}(t)(1 + 2mp)$$

The theorem is proved. In calculating we have used:

For $k, n \in \mathbb{Z}$ and $t \in \mathbb{R}$ such, that $-m \le n \le m$, $-mT \le t \le mT$ is:

$$E\{\mathbf{L}_{\mathbf{k}}(\xi)\mathbf{L}_{\mathbf{k}+n}(\xi)\mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi)\} = E\{\mathbf{L}_{\mathbf{k}}(\xi)\mathbf{L}_{\mathbf{k}+n}(\xi)\}E\{\mathbf{c}_{k}(\xi) \mathbf{c}_{k+n}(\xi)\}$$

because $L_k(\xi)$ and $c_k(\xi)$ are independent.

The mean $E\{\mathbf{L}_{\mathbf{k}}^{2}(\xi)\}$ has been calculated in (7).

The relation

$$E\{\mathbf{L}_{\mathbf{k}}(\xi)\mathbf{L}_{\mathbf{k}+n}(\xi)\} = E\{\mathbf{L}_{\mathbf{k}}(\xi)\}E\{\mathbf{L}_{\mathbf{k}+n}(\xi)\} = \rho^{2}$$

follows from the independence of $L_{k}(\xi)$ and $L_{k+n}(\xi)$. Immediately:

$$\Phi_{k}(t)\Phi_{k+n}(t) = \Phi_{k-n}(t)\Phi_{k}(t)$$

According assumption of the theorem the process $\mathbf{c}_{\mathbf{k}}(\xi)$ is stationary, then

$$E\{\mathbf{c}_{k}(\xi) \ \mathbf{c}_{k+n}(\xi)\} = E\{\mathbf{c}_{k-n}(\xi) \ \mathbf{c}_{k}(\xi)\} \qquad E\{\mathbf{c}_{k}(\xi) \ \mathbf{c}_{k+n}(\xi)\} \le E\{\mathbf{c}_{k}^{2}(\xi)\}$$
$$E\{\mathbf{c}_{k}(\xi) \ \mathbf{c}_{k+n}(\xi)\} \le E\{\mathbf{c}_{k+n}^{2}(\xi)\}$$

The relation $(\Phi_k(t) - \Phi_{k+n}(t)) \ge 0$ is true for all real *t*, then

$$\Phi_{k}^{2}(t) + \Phi_{k+n}^{2}(t) \ge 2\Phi_{k}(t)\Phi_{k+n}(t)$$

The vowels in human speech can be well modeling as a cyclostationary processes.

According the theorem 1.6 we can see that the stochastic processes decomposed into shift basis, like in the theorems 2.1, 2.2 and 2.3, are cyclostationary processes, therefore those decompositions can be used for modeling of the transmission speech over internet.

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Resumé

ŠUM VZNIKAJÚCI PRI NEZÁVISLOM STRÁCANÍ PAKETOV REČI MODELOVANEJ CYKLOSTACIONÁRNYM NÁHODENÝM PROCESOM

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Pri prenose reči v službe IP telefonie sa pre prenos reči využíva princíp prepájania paketov. Z dôvodu náhodného meškania paketov při prenose sieťou, niektoré pakety prekročia povolenú hranicu meškania a sú při prehrávaní nahradené tichom. Rozdiel medzi pôvodným a prehrávaným signálom sa označuje jako šum. V článku popisujeme ako strata vzoriek rečového signálu zmení charakteristiky druhého rádu šumového signálu. Zamerali sme sa na samohlásky, ktoré sa dajú dobre aproximovať ako cyklostacionárny náhodný proces.

Summary

NOISE IN THE INDEPENDENT PACKET LOST OF SPEECH SIGNAL MODELED BY CYCLOSTATIONARY STOCHASTIC PROCESS

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For the speech transmission, IP telephony service uses the packet commutation principle. Some of the samples of speech signal can be lost. The aim of the article is to describe the noise signal by the second order characteristic of the noise as a stochastic process. We choose the vowels which can be well approximated as a cyclostationary stochastic process.

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Zusammenfassung

DAS GERÄUSCH ENTSTEHEND BEI DEM UNABHÄNGIGEN PAKETVERLUST BEI SPRACHE MODELLIERTE DURCH ZYKLOSTATIONÄREN ZUFÄLLIGEN PROZESS

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Das kommt oft in Frage, dass bei IP Telephoniedienste infolge der Verzögerung des Pakets mit den Sprachabtasten einige Abtaste der Sprachsignal verloren wurden. Im Artikel beschreiben wir, wie diese Sprachabtastverlust die Charakteristika des zweiten Rangs des Geräuschsignal wechseln. Der Fokalpunkt steht an die Vokalen, die kann man gut wie zyklostationären zufälligen Prozess approximieren.