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ESTIMATION METHODS FOR WEIGHT CRITERIA

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Introduction

It is very problematic to obtain the weight criteria from a decider directly in a numerical form. It is therefore desirable to make the process of weight criteria easier by some tool. These tools can be represented by some methods for estimation of the weight criteria, that is they are represented by some algorithmic processes which, on the basis of subjective information and evaluations, can be used for a construction of the weights.

A computation by means of a Saaty method, follow-up by verification based on Monte-Carlo, will be achieve with computer support, using a modern, visual program language Delphi (in particular: Borland Delphi Enterprise version 7.0).

1. Estimation Methods for Weight criteria

In the following paragraphs we summarize briefly the frequently used methods of weight criterion estimations.

- Method of Order.

The method of order requires only to simply sort the criteria from the most important to the least important. We can classify the most important criterion by a value k (the k is a number of criteria), the second by the number (k–1) and so on to the least

important with the number 1. If a p_i represents the value of the i-th criterion, then it is possible to express the weight criterion estimation by the following relation [3]:

$$v_i = \frac{p_i}{\sum_{i=1}^{k} p_i} \tag{1}$$

- Points method.

The points method presumes that the person who makes decisions (some referee) is capable to evaluate the importance of the criteria at some scale. Such a scale can be the points system from 1 to 10. The more important the criterion is the higher mark it can get. Denoting the i-the criterion by the symbol p_i, we can again use the same relation, as given above, for the weight criterion estimation [3].

Instead of the direct expression of the preferences (represented e.g. by the points method) there can be used more suitable method for the referee. This can be for example the pair comparison. The Fuller's triangle and Saaty's method work on such a principle.

- Fuller's triangle.

By this procedure the referee must deal with the triangle scheme in which the couple of the individual criteria are expressed. It is clear that each pair can be displayed only and exactly once. From each couple one must pick one which is more important than the other one. Such a criterion must be emphasized – e.g. by a circle. It can happen, of course, that the two criteria have the same importance. In such a case the referee must encircle both. If the number of the indications for the i-the criterion is p_i, we can again get for the weight criterion estimation [3]:

$$v_i = \frac{p_i}{\sum_{i=1}^{k} p_i} \tag{2}$$

- Saaty's method.

Saaty's method is a more sophisticated technique of weight criterion estimations. It is also one of the most used procedures in this domain. The application of this method is again based on the comparisons of the pairs, but contrary to the Fuller's triangle method the referee must compare all possible combinations [3].

The Saaty method will be discussed in detail in chapter 3.

2. Monte Carlo Method

The Monte Carlo method is the most frequently used tool in the modeling of the discrete systems [1]. The Monte Carlo method belongs to the stochastic processes, where the inputs are generated randomly according to the defined probability distribution.

It has a wide applicability and the expression "Monte Carlo method" is actually very general. The core is that it is a numeric method which uses stochastic sampling.

For pure mathematical problems it is sometime irreplaceable. It can be used for example for the numerical integration (so-called Monte Carlo integration). It is also suitable for reliable computation of the coefficients for the systems of equations, etc.

It is a flexible tool applied in the solving of many problems where other methods (either analytical or numerical) could not be used. It is also one of the most widely used numerical methods in physics and other domains (it is used e.g. in quantum field theory or condensed matter physics).

For us, the Monte Carlo method is an experimental numerical method, which is convenient especially in the cases where we have many numbers of the continuous factors of risk. More precisely, in our case we deal with the possibility of the “better solutions” in the searching of the weight criterion estimations. Usually we can proceed as follows:

1. We define the functional dependence of the criterion on input.
2. We have to choose risk factors which considerably determine the value of the criterion estimation. This comes out usually from the results of one-parametric or multiparametric sensitiveness analysis.
3. One has to establish the probability of the possible value appearance for the individual risk factors. To the purpose of the modeling of the risk factors at the time, it is possible to define the distribution of the risk factor probability in the time series.
4. It is necessary to generate sets of the random factor values according to the distribution of the probability together with an implementation of the statistical dependence among the individual risk factors.
5. Next we have to substitute the generated data to the functional dependence and perform the calculation of the criterion value.
6. What follows is probably the most important part – a loop construction. The repetition of iteration procedure starts from the point 4. This cycle is terminated – if the sufficient number of iteration is reached. This number can be monitored by the calculation of the interval reliability for the characteristic values (e.g. mean value). These characteristic indicators come from the probability distribution of the criterion values.

The last important point to mention is the problem connected with the construction of the resulting probability distribution of the criterion values and, eventually, the determination of the characteristic indicators for the distribution (as could be for example the mean value, the standard deviation, etc.). It depends on the referee which method is chosen. It is clear that there is no a priori recipe how this can be done the most efficiently.

3. Saaty method versus Monte-Carlo

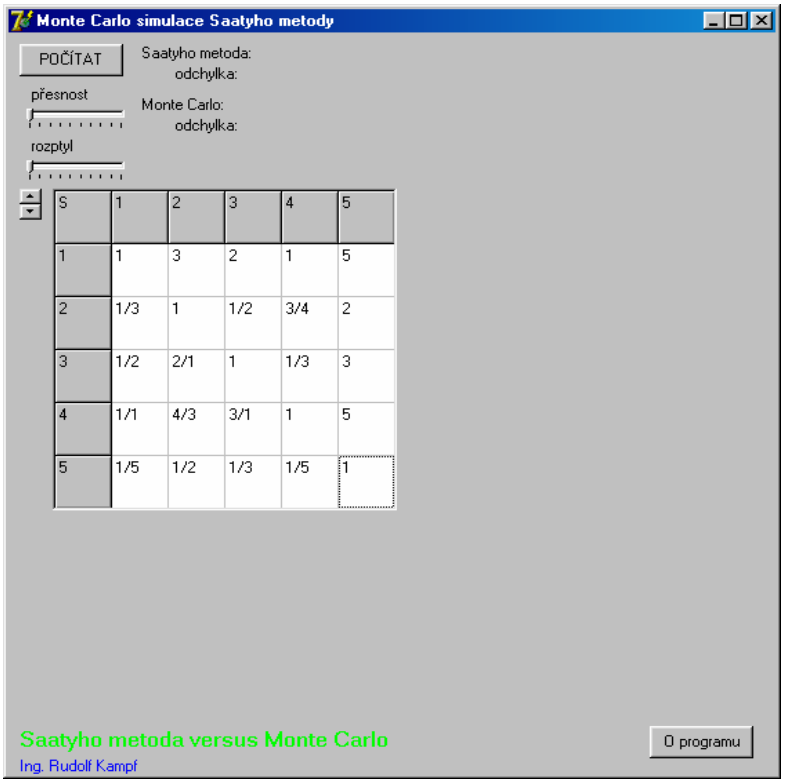
The principle of the multi-criteria decision by Saaty's method consists in the fact that it is not always possible to determinate the weight vector of the criteria and one has to use a method of comparisons, i.e. we compare the weight of individual criterion with each other. The outcome is a numerical determination of the weight of individual criteria.

The Saaty method makes also possible for the decider to express the preferences instead in numerical scale also by a verbal manner [2]. This is a big advantage.

The input is so-called Saaty matrix, where the individual components represent the ratios of the given criteria (relation (3)) [3].

$$s_{ij} \approx \frac{v_i}{v_j}, \qquad i, j = 1,2,\ldots,k. \tag{3}$$

That means there are only "1's" down the main diagonal and the following relation holds sij = 1/sji, i,j = 1, 2, …, k, i.e. all elements are reciprocal value of symmetric elements according to the diagonal. Our program enables only to enter "1's" on the diagonal and the numerical values above diagonal, the values below diagonal are reciprocal (Fig. 1) [4]. The program makes automatically the conversion and writes it down.



S	1	2	3	4	5
1	1	3	2	1	5
2	1/3	1	1/2	3/4	2
3	1/2	2/1	1	1/3	3
4	1/1	4/3	3/1	1	5
5	1/5	1/2	1/3	1/5	1

Fig.1 *Input of values*

The proper computation of global weights from these relative weights is complicated matter. Saaty, for example, proposed the calculation of the eigenvector corresponding to the maximum eigenvalue of our matrix (so-called supermatrix) cf. [3] and (4).

$$Sv = \lambda_{\max} v \tag{4}$$

This vector represents the solution. As was already mentioned, this approach of eigenvector calculation (express by (4)) is not a trivial task and Saaty proposes a simplification (see (5)) [3].

$$v_i' = \left(\prod_{j=1}^{k} s_{ij} \right)^{1/k} \qquad i = 1, 2, \ldots, k$$

$$v_i = \frac{v_i'}{\sum_{i=1}^{k} v_i'} \qquad i = 1, 2, \ldots, k \tag{5}$$

Therefore the calculation of eigenvecor of the given matrix is not accurate and correct though it represents a “good estimation” (Fig 2) [4].

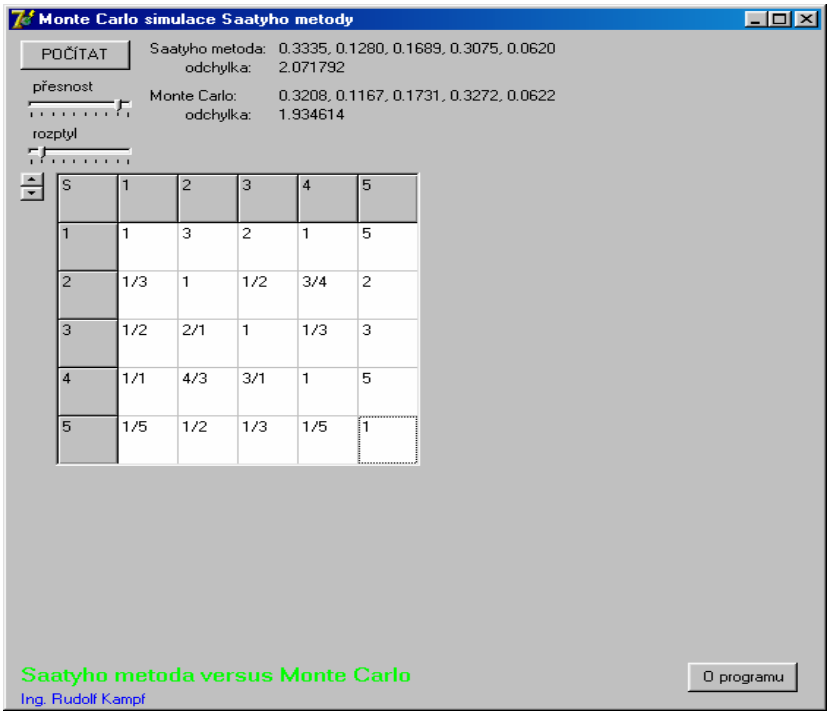


***Fig. 2** Saaty method versus Monte Carlo*

This work presents one additional solution which will offer more accurate results for the estimation of weight criteria. The computation provided by Saaty method will be

verified by Monte-Carlo method [1]. This will represent a mathematical tool to demonstrate the difference between Saaty and Monte-Carlo method in the calculation of weight criteria.

The base still lays in the Saaty method (see (5)). The main idea of Monte-Carlo approach is that in the algorithm we generate a “random fluctuation” around results obtained by the Saaty method, i.e. we need to generate a random number with some variance. Next, the program has to analyze whether this number is better than the Saaty value. For the comparison we used an “optimum”, that is a zero variation. In the case it is not possible to obtain such a consistent result then the result with the smallest variation is chosen. And of course, the smaller it is, the better this result is.

The application makes possible to change the precision by trackbar 

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This exactly means that at the minimum position there are 100 000 iterations, and at the maximum there are one million iterations.

If it is necessary to widen a variance, e.g. in the case when one thinks that is reasonable and required and the values of Saaty method are not accurate we can change the area of random numbers. The application enables us to do it by the following trackbar 

. This is defined as (max-min)/2, where max and min represent maximum and minimum number from Saaty result (5).

The Monte-Carlo method is the valuable aide when the decider did not correctly set the ratios for the individual criteria and therefore one can obtain inconsistent results (6) [4].

$$C.I. = \frac{\lambda_{\max} - k}{k - 1} \tag{6}$$

Conclusion

Saaty method set the value of weight criteria and this is after verified by the experimental numerical method Monte Carlo. This represents a new method which enables to obtain correct and more appropriate value for weight criteria.

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Resumé

METODY ODHADU VAH KRITÉRIÍ

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Získat váhy kritérií od rozhodovatele přímo v numerické podobě je často velmi problematické. Proto je vhodné usnadnit rozhodovateli určení vah kritérií pomocí nějakého nástroje. Tímto nástrojem mohou být metody odhadu vah kritérií. Jedná se o postupy, které na základě subjektivních informací od rozhodovatele konstruují odhady vah.

V uvedém článku je výpočet proveden Saatyho metodou s následnou verifikaci metodou Monte Carlo. Což představuje v podstatě novou metodu umožňující přesnější stanovení hodnot z podílových vah. Výpočet byl proveden s počítačovou podporou, s využitím vizuálního programovacího jazyku Delphi.

Zussamenfassung

DIE METHODEN DER ABSCHÄTZUNG DER KRITERIENGEWICHT

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Gewinnen die Kriteriengewicht direkt vom Entscheidender in der Nummerischenform ist oft sehr problematisch. Darum empfehlen wir des Entscheidenders die Kriteriengewicht durch ein Instrument festsetzen. Solches Instrument können die Methoden für die Abschätzung der Kriteriengewicht sein. Es handelt über die Prozesse, die an der Grundlage der Subjektivinformationen vom Entscheidendern die Abschätzung des Gewichtes konstruiren.

Der Beitrag löst die Berechnung durch die Saatyhmethode mit der folgenden Verifizierung durch die Methode Monte Carlo. Die Lösung vorstellen die neue Methode mit den genaueneren Werte von der Quotegewicht. Die Berechnung war mit den Rechner mit der visuellen Programmierschprache Delphi durchgeführt.

Summary

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It is very problematic to obtain the weight criteria from a decider directly in a numerical form. It is therefore desirable to make the process of weight criteria easier by some tool. These tools can be represented by some methods for estimation of the weight criteria that is they are represented by some algorithmic processes which, on the basis of subjective information and evaluations, can be used for a construction of the weights.

In this article a computation by means of the Saaty method is presented and s followed-up by a verification based on Monte Carlo. This represents a new method for obtaining correct and more appropriate value of weight criteria. The computation was achieved with the computer support, using a modern visual program language Delphi.