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IMPACT OF CORRELATED SAMPLES LOSS ON VoIP QUALITY

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1. Introduction and preliminaries

Quality of the IP telephony service can be measured by two ways. The MOS (mean opinion score) gives the subjective point of view. The technical parameters of network brings the more objective view. Signal-to-noise ratio is one of such technical parameters. It is the ratio of mean power (or energy) of transmitted signal and the noise (difference between transmitted and received signal). The IP telephony service uses the principle of the packet commutation for the speech transmission, for instance in [11]. An analogous speech signal is decomposed into Shannon sequence. The coefficients of the speech signal are segmented into frames. Each frame contains part of the speech lasting 20 ms. The frames create an information part of packets delivered to a receiver's address. During the transmission over IP network, packets may get delayed, or lost. The theory of the impact of packet loss on speech quality has been elaborated in the article [8]. We present the results achieved by modelling short segments of speech as a cyclostationary stochastic proces. The cyclostationary proces is created, if the basis used for discretization consists from regularly shifted functions.

Different shapes of the basis functions lead to different properties of the signal transmission chain. The researcher is searching the best shape of the signal according to required property. The basic property has to be kept: "the regularly shifted functions have to create the basis of the space of all speech signals". We have defined the necessary conditions for regularly shifted functions to create the basis in another papers [6], [7]. Theory of cyclostationary processes can be found in [1], [9]. We have explained some definitions and properties for better understanding of this notion in [8].

2. Reconstruction in case of uncorrelated samples loss

We have shown the case of uncorrelated samples loss in the article [8]. First we used the model applying the Shannon decomposition.

Theorem 1 *Let* $\mathbf{x}(t,\xi)$ *is the stationary, centred,* Ω *-bandlimited stochastic process and let its representation in Shannon basis in case* $T = \frac{\pi}{\Omega}, \quad t \in R$ *is*

$$\mathbf{x}(t,\xi) = \sum_{k=-\infty}^{\infty} \mathbf{x}(kT,\xi) \frac{\sin \Omega(t-kT)}{\Omega(t-kT)} = \sum_{k=-\infty}^{\infty} c_k(\xi)\Phi_k(t)$$

so the basis functions are deterministic and are equal to $\Phi_k(t) = \operatorname{sinc} \Omega(t-kT)$*, coefficients of the process in this basis* $\Phi_k(t)$ *are random variables* $\mathbf{c}_k = \mathbf{x}(kT,\xi)$*. Let the probabilities* $p_{k,}\ \ k \in Z$ *of the coefficients* $\mathbf{c}_k$ *loss, are independent each other and independent of the value of* $\mathbf{c}_k(\xi)$*. Let this probabilities are equal to p* $p_k = p, \quad \forall k \in Z$. *after transition over IP the voice process is reconstructed. Let the reconstructed process* $\mathbf{x}'(t,\xi)$ *is the sum where the lost samples are represented by 0.*

Then the mean power and correlation of noise process $\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi)$ *are respectively*

$$\sigma_{\mathbf{g}}^{\mathbf{2}}(t) = p\sigma_x^{\mathbf{2}}(t) \tag{1}$$

$$R_{\mathbf{g}}(t) = p^2 R_x(t) \tag{2}$$

In the next theorem we used for the discretization the basis created by the function limited in the time, where the lost samples were replaced by zero. The theorem below was proved in [8].

Theorem 2 *Let* $\mathbf{x}(t,\xi)$ *is the stationary, centered,* Ω *-bandlimited stochastic process and let* $\Phi_k(t) = \Phi(t-kT)$ *is the sequence of the functions and let the function* $\Phi(t) = 0$ *for* $|t| > \frac{T}{2}$*. Let the stochastic process can be unique expressed as*

$$\mathbf{x}(t,\xi) = \sum_{k=-\infty}^{\infty} c_k(\xi)\Phi_k(t)$$

where the functions $\Phi_k(t) = \Phi(t-kT)$ *are deterministic, the coefficients of the decomposition of the process in basis* $\Phi_k(t)$ *are random variables* $\mathbf{c}_k(\xi)$*.*

Let the probabilities $p_{k,}\ \ k \in Z$ *of the coefficients* $\mathbf{c}_k(\xi)$ *loss, are independent each other and independent of the value of* $\mathbf{c}_k(\xi)$. *Let this probabilities are equal to p* $p_k = p,\ \ \forall k \in Z$. *Let the reconstructed process* $\mathbf{x}'(t,\xi)$ *is the sum where the lost coefficients are represented by 0.*

Then the mean power of the noise process $\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi)$ *can be calculated as*

$$\sigma_{\mathbf{g}}^{\mathbf{2}}(t) = p\sigma_x^{\mathbf{2}}(t)$$

3. Reconstruction in case of correlated samples loss

The VoIP service is based on the packet switching principle. Then the separate samples loss is not usual in IP telephony. Mostly parts of speech lasting 20 ms are lost. The packet have deterministic length but their loss have stochastic occurrence. The good aproximation of this situation can be obtained by Markov on-off model, known as Gilbert model, see [10].

Definition 1 *Gilbert model* $\mathbf{L_k}(\xi)$ *is a Markov chain with two states* 0 *and* 1.

$$\begin{aligned} \mathbf{L_k}(\xi) &= 0 \quad \text{if coefficient} \quad \mathbf{c}_k(\xi) \quad \text{was transmited} \\ &= 1 \quad \text{if coefficient} \quad \mathbf{c}_k(\xi) \quad \text{was not transmited} \end{aligned}$$

A matrix of Gilbert model is

$$\mathbf{P} = \begin{pmatrix} 1-r & r \\ q & 1-q \end{pmatrix} \qquad (3)$$

Figure 1 shows the graph of the transitions of Gilbert model.

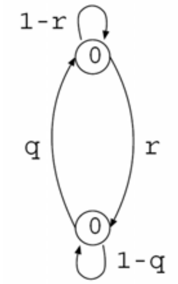

Fig. 1 *Gilbert model*

Theorem 3 *Probability of the loss of the coefficient* $\mathbf{c}_k(\xi)$ *in a stable Gilbert model is* p_k, *for* $\forall k \in Z$, *where*

$$p_k = \pi_1 = \frac{r}{r+q} \tag{4}$$

Proof:

The invariant distribution of the Gilbert model is such distribution π that:

$$\pi = \pi\,\mathbf{P}$$

$$(\pi_0, \pi_1) = (\pi_0, \pi_1)\mathbf{P} = (\pi_0, \pi_1)\begin{pmatrix} 1-r & r \\ q & 1-q \end{pmatrix}$$

By solving

$$\begin{aligned} \pi_0 &= \pi_0(1-r) + \pi_1 q \\ \pi_1 &= \pi_0 r + \pi_1(1-q) \end{aligned}$$

with condition

$$\pi_0 + \pi_1 = 1$$

we get

$$(\pi_0, \pi_1) = \left(\frac{q}{q+r}, \frac{r}{q+r}\right)$$

and then

$$p_k = \pi_1 = \frac{r}{r+q}$$

Theorem 4 *Probability* q_n *of the simultaneous loss of coefficients* $\mathbf{c}_k(\xi)$ *and* $\mathbf{c}_{k+n}(\xi)$ *in stacionary Gilbert model is*

$$q_n = \frac{r}{q+r}\left(\frac{r}{q+r} + (1-r-q)^n \frac{q}{q+r}\right) \tag{5}$$

Proof:

Probability q_n of the simultaneous loss of coefficients $\mathbf{c}_k(\xi)$ and $\mathbf{c}_{k+n}(\xi)$ is

$q_n = P\{loss\ of\ c_{k+n},\ loss\ of\ c_k\} = P\{loss\ of\ c_{k+n}\ /\ loss\ of\ c_k\}\,P\{\ loss\ of\ c_k\} =$

$= P\{loss\ of\ c_{k+n}\ /\ loss\ of\ c_k\}\dfrac{r}{q+r}$.

Probability $P\{loss\ of\ c_{k+n}\ /\ loss\ of\ c_k\}$ is the element of the matrix $\mathbf{P}^n$ (in the 2nd row and the 2nd column).

$$\mathbf{P}^n = 1^n \mathbf{A} + (k_1)^n \mathbf{A_1} \tag{6}$$

where

$$(\mathbf{I} - z\mathbf{P})^{-1} = \frac{\mathbf{A}}{1-z} + \frac{\mathbf{A}_1}{1-k_1 z}.$$

Left side can be calculated

$$(\mathbf{I} - z\mathbf{P})^{-1} = \begin{pmatrix} 1-z(1-r) & -zr \\ -zq & 1-z(1-q) \end{pmatrix}^{-1} =$$

$$= \frac{1}{(1-z)(1-(1-r-q)z)} \begin{pmatrix} 1-z+qz & zr \\ zq & 1-z+rz \end{pmatrix}$$

Then after splitting into partial fractions and comparing with $\frac{\mathbf{A}}{1-z} + \frac{\mathbf{A}_1}{1-k_1 z}$

$$(\mathbf{I} - z\mathbf{P})^{-1} = \frac{1}{1-z} \begin{pmatrix} \frac{q}{q+r} & \frac{r}{q+r} \\ \frac{q}{q+r} & \frac{r}{q+r} \end{pmatrix} + \frac{1}{1-(1-r-q)z} \begin{pmatrix} \frac{r}{q+r} & \frac{-r}{q+r} \\ \frac{-q}{q+r} & \frac{q}{q+r} \end{pmatrix}$$

we get coefficient $k_1 = 1-r-q$ and matrices

$$\mathbf{A} = \begin{pmatrix} \frac{q}{q+r} & \frac{r}{q+r} \\ \frac{q}{q+r} & \frac{r}{q+r} \end{pmatrix}, \qquad \mathbf{A_1} = \begin{pmatrix} \frac{r}{q+r} & \frac{-r}{q+r} \\ \frac{-q}{q+r} & \frac{q}{q+r} \end{pmatrix}$$

We substitute $\mathbf{A}$, $\mathbf{A_1}$ and $k_1 = 1-r-q$ into equation (6) and we get *n*-th power of matrix $\mathbf{P}$:

$$\mathbf{P}^n = \begin{pmatrix} \frac{q}{q+r} & \frac{r}{q+r} \\ \frac{q}{q+r} & \frac{r}{q+r} \end{pmatrix} + (1-r-q)^n \begin{pmatrix} \frac{r}{q+r} & \frac{-r}{q+r} \\ \frac{-q}{q+r} & \frac{q}{q+r} \end{pmatrix}$$

then

$$p_{22} = \frac{r}{q+r} + (1-r-q)^n \frac{q}{q+r}$$

and

$$q_n = \frac{r}{q+r}\left(\frac{r}{q+r} + (1-r-q)^n \frac{q}{q+r}\right)$$

Next, we will use the proved theorem in the case, when the basis is generated by function $\operatorname{sinc}(t)$. The samples are transmitted in the packets, therefore the sample's losses are correlated. Losing of one sample means losing of all samples in the same packet.

Theorem 5 *Let* $\mathbf{x}(t,\xi)$ *is the centered,* Ω *-bandlimited, stochastic process and let its decomposition into the Shannon basis for* $T=\frac{\pi}{\Omega},\quad t\in R$ *is*

$$\mathbf{x}(t,\xi)=\sum_{k=-\infty}^{\infty}\mathbf{x}(kT,\xi)\frac{\sin\Omega(t-kT)}{\Omega(t-kT)}=\sum_{k=-\infty}^{\infty}c_k(\xi)\Phi_k(t)\,.$$

It means the basis functions are deterministic $\Phi_k(t)=\operatorname{sinc}\Omega(t-kT)$*, coefficients of the process are random variables* $\mathbf{c}_k(\xi)=\mathbf{x}(kT,\xi)$*. Let the probabilities* p_k *of the samples* $\mathbf{c}_k(\xi)$ *loss, are independent of its values and are described by Gilbert model with probabilistic matrix*

$$\mathbf{P}=\begin{pmatrix}1-r & r\\ q & 1-q\end{pmatrix}$$

it means

$$\begin{aligned}&P\{transmission\quad c_{k+1}/\,transmission\quad c_k\}=1-r\\&P\{loss\quad c_{k+1}/\,transmission\quad c_k\}=r\\&P\{transmission\quad c_{k+1}/\,loss\quad c_k\}=q\\&P\{loss\quad c_{k+1}/\,loss\quad c_k\}=1-q\end{aligned}$$

Let the reconstructed process $\mathbf{x}'(t,\xi)$ *is the sum where the lost samples are represented by 0.*

Then the mean power and correlation of the noise process $\mathbf{g}(t,\xi)=\mathbf{x}(t,\xi)-\mathbf{x}'(t,\xi)$ *are respectively*

$$\sigma_{\mathbf{g}}^2(t)=\frac{r}{q+r}\sigma_x^2(t)\qquad(7)$$

$$R_{\mathbf{g}}(t)=\sum_{n=-\infty}^{\infty}q_nR_{\mathbf{x}}(nT)\operatorname{sinc}\Omega(t-nT)\qquad(8)$$

where q_n *is probability of the current loss of two coefficients with distance equal n*

$$q_n=P\{loss\quad c_{k+n}/\,loss\quad c_k\}=\frac{r}{q+r}\left(1^n\frac{r}{q+r}+(1-r-q)^n\frac{q}{q+r}\right)$$

Proof:

We put the expression (4) into the formula (1). Then we get

$$\sigma_{\mathbf{g}}^2(t) = \frac{r}{q+r}\sigma_x^2(t).$$

The function $R_g(t)$ satisfies the conditions of the Shannon sampling theorem and it can be expressed in the form $R_g(t) = \sum_{n=-\infty}^{\infty} R_g(nT)\frac{\sin\Omega(t-nT)}{\Omega(t-nT)}$ (9)

Calculation of coefficients $R_g(nT)$

$$R_{\mathrm{g}}(nT) = E\{\mathbf{g}(0,\xi)\mathbf{g}(nT,\xi)\} = E\{\mathbf{L_0}(\xi)\mathbf{c_0}(\xi)\operatorname{sinc}(0)\mathbf{L}_n(\xi)\mathbf{c}_n(\xi)\operatorname{sinc}(nT-nT)\} =$$
$$= E\{\mathbf{L_0}(\xi)\mathbf{L}_n(\xi)\}\{\mathbf{c_0}(\xi)\mathbf{c}_n(\xi)\}\operatorname{sinc}^2(0) = q_n R_x(nT)$$

The theorem will be proved after substituing (5) into the last expression.

Theorem 6 *Let* $\mathbf{x}(t,\xi)$ *is the stochastic process and* $\Phi_k(t) = \Phi(t-kT)$ is the sequence of the functions and the function $\Phi(t) = 0$ for $|t| > mT$, where m is arbitrary natural number. Let the stochastic process $\mathbf{x}(t,\xi)$ can be uniquely expressed as

$$\mathbf{x}(t,\xi) = \sum_{k=-\infty}^{\infty} c_k(\xi)\Phi_k(t)$$

where the functions $\Phi_k(t) = \Phi(t-kT)$ are deterministic, and let the coefficients $\mathbf{c}_k(\xi)$ of the decomposition of the process into the basis $\{\Phi_k(t)\}$ create a discrete stationary stochastic process. Let the probabilities p_k of the sampling loss $\mathbf{c}_k(\xi)$ are independent of the values of $\mathbf{c}_k(\xi)$ and are described by Gilbert model with probabilistic matrix

$$\mathbf{P} = \begin{pmatrix} 1-r & r \\ q & 1-q \end{pmatrix}$$

Let reconstructed process $\mathbf{x}(t,\xi)$ is the sum where lost samples are represented by 0.

Then the mean power of the noise process $\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi)$ satisfies the condition:

$$\sigma_{\mathbf{g}}^2(t) \le \sigma_x^2(t)\left(\frac{r}{q+r} + 2\sum_{n=1}^{m} q_n\right).$$

where

$$q_n = \frac{r}{q+r}\left(1^n\frac{r}{q+r} + (1-r-q)^n\frac{q}{q+r}\right).$$

Proof:

The noise stochastic process can be written as

$$\mathbf{g}(t,\xi) = \mathbf{x}(t,\xi) - \mathbf{x}'(t,\xi) = \sum_{k=-\infty}^{\infty} \mathbf{L}_k(\xi)\mathbf{c}_k(\xi)\Phi_k(t)$$

Mean power can be calculated similarly as in theorem 2, only we use formula (5) instead of independence of losses.

$$\sigma_{\mathbf{g}}^{\mathbf{2}}(t) = E\left\{\mathbf{g}^{\mathbf{2}}(t,\xi)\right\} = E\left\{\left(\sum_{k=-\infty}^{\infty} \mathbf{L}_k(\xi)\mathbf{c}_k(\xi)\Phi_k(t)\right)^2\right\} \leq$$

$$\leq \frac{r}{q+r}\sigma_x^{\mathbf{2}}(t) + 2\sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} q_n E\left\{\mathbf{c}_k(\xi)\mathbf{c}_{k+n}(\xi)\right\}\Phi_k(t)\Phi_{k+n}(t) \leq$$

$$\leq \frac{r}{q+r}\sigma_x^{\mathbf{2}}(t) + \sum_{k=-\infty}^{\infty}\sum_{n=1}^{m} q_n E\left\{\mathbf{c}_k(\xi)\mathbf{c}_{k+n}(\xi)\right\}\left(\Phi_k^2(t) + \Phi_{k+n}^2(t)\right) \leq$$

$$\leq \frac{r}{q+r}\sigma_x^{\mathbf{2}}(t) + \sum_{n=1}^{m} q_n 2\sigma_x^{\mathbf{2}}(t) = \sigma_x^{\mathbf{2}}(t)\left(\frac{r}{q+r} + 2\sum_{n=1}^{m} q_n\right).$$

The n-th power of matrix $\mathbf{P}$ can be calculated using the eigenvalues and eigenvectors of matrix $\mathbf{P}$ see [15], where we can find the methods for the theoretical background and practical calculation of those values. For computing we can use MATLAB [4] or some of the open source software mentioned in [5].

4. Conclusions

In multimedia applications of VoIP the quality of speech signal have to be satisfied, see [2], [12]. The problem how to save a quality of voice signal is discussed in [3], [13]. The quality can be measured by using the estimations for noise signal. One of the criterions of quality is the signal to noise ratio. This parameter is calculated as

$$SNR = 10\log\frac{\sigma_x^{\mathbf{2}}(t)}{\sigma_g^{\mathbf{2}}(t)}$$

We have shown in this article, that the decomposition into non-shannon bases can bring better results than Shannon decomposition.

In the future we want to compare results proved in this article with results achieved in case when lost samples are replaced by white noise or by the linear combination of received samples.

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Resumé

VPLYV KORELOVANÝCH STRÁT NA KVALITU VoIP

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Pri prenose reči v službe IP telefónie sa dochádza vplyvom strát paketov k degradácii kvality reči. V článku je ukázaný dolný odhad odstupu rečového signálu od šumu spôsobeného stratami paketov v prípade, že straty paketov sú korelované a úseky reči (typicky samohlásky) predstavujú cyklostacionárny náhodný proces. Korelované straty paketov popisuje Gilbertov model a paket prenáša vzorku rozkladu reči do bázy tvorenej posúvaním generujúcej funkcie bázy. Dosiahnuté výsledky ukazujú na možnosť dosiahnutia lepšej kvality oproti prenosu vzoriek Shannonovej bázy. Článok poukazuje na potrebu hľadania bázy cyklostacionárneho procesu, ktorá by minimalizovala vplyv straty paketov na kvalitu reči v závislosti na charaktere procesu strát.

Summary

IMPACT OF CORRELATED SAMPLES LOSS ON VoIP QUALITY

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Quality of the IP telephony service can be measured as a composition of many parametres. Some of the parametres of quality of IP telephony service can be measured by secondary characteristics of difference between received and transmitted signal. The aim of this paper is to derive estimations for mean power of noise. Using non orthogonal basis the speech signal can be modeled as a cyclostationary random proces. The case of zero sample stuffing is studied. The packet loss is modeled as a correlated samples loss. As a mathematical model of this correlation the Gilbert model is used.

Zusammenfassung

EINFLUSS DER KORRELIERTEN PAKETVERLUSTE AUF DIE QUALITAT DES VoIP DIENSTES

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Bei der Übertragung der Sprache über die IP Telephonie wird das Prinzip des Paketschaltens verwendet. Aufgrund der zufälligen Verspätungen der Pakete während der Übertragung überschreiten einige Pakete die maximale zulässige Verzögerung, die bei der Wiedergabe durch stumme Intervalle ersetzt wird. Die Differenz zwischen dem ursprünglichen und dem wiedergegebenen Signal wird als Rauschen bezeichnet. Im diesen Artikel wird die Wirkung der Paketverluste auf die Charakteristika von Rauschsignal des zweiten Orders beschrieben. Der Schwerpunkt dieser Arbeit wurde auf die Vokalen gelegt, die sich als zyklostationärer zufälliger Prozess gut approximieren lassen.