A stochastic model for the capacity estimation of non-seismically designed beam-column joints

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Abstract

An analytical model based on nonlinear fracture mechanics was employed to evaluate the shear capacity of a substandard beam-column joint retrofitted by externally applied post-tensioned bars. The strength of the reference specimen was predicted as the capacity of plain concrete under tension with microcracks in the fracture process zone smeared over a band – i.e. crack band theory. Crack minimization effects in the specimens retrofitted by post-tensioned bars were considered by strain energy stored in the bars per unit fracture surface area. Due to the inherent uncertainty in material constitutive laws, the analytical model was evolved to a stochastic level to propose a more advanced model for estimating the capacity. It is found that the experimental results were within the prominent range of Probability Density Functions (i.e. mean ± one standard deviation) of the estimated joint tensile stress.

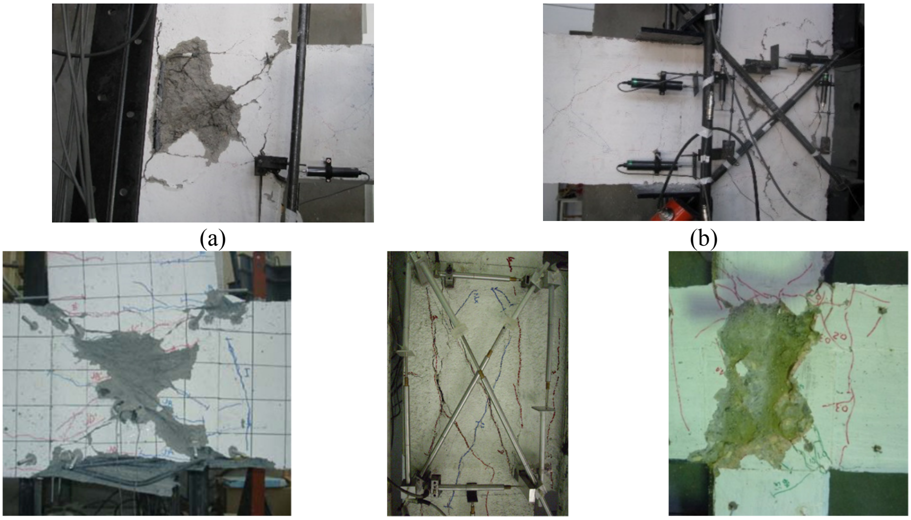
1 Introduction

Non-seismically detailed reinforced concrete structures are vulnerable to high or even moderate seismic actions, which can cause devastating failure of members at local level. The resulting local damage can actuate the global failure mechanism, which brought the requirement to investigate the behaviour of substandard members. Beam-column joint considered as the weakest link in the structural systems must preserve its integrity and have an ability to transfer the seismic forces to the other members under seismic action [1]. Thus, experimental studies on the poorly detailed joints were the subject of many studies [2–6]. Moreover, recent studies demonstrated that the response of joints under multiaxial complex stress mechanism can be reproduced by advanced analytical models [7–9]. However, further progress is still needed as characterizing the behaviour of joints non-conforming to the current seismic codes. Difficulties arise more when the uncertainties in the system are prominent. This study mainly aims to present an analytical model based on nonlinear fracture mechanics. The specimens were selected from four different testing programs. They are selected to be common in terms of failure mode and design principles (i.e. substandard or pre-1970s). The capacity of the specimen was predicted as the strength of plain concrete under tension with microcracks in fracture process zone smeared over a band – i.e. crack band theory [10]. As the effect of uncertainties on the response is more distinct in the models with the local softening approach, the analytical model was evolved to stochastic level. The stochastic model was developed by using Latin Hypercube Sampling (LHS) including statistical correlation among the prominent material parameters. Random parameters of concrete and reinforcement steel were defined in accordance with the material test results and code recommendations. The mean value and prominent range of Probability Density Functions (i.e. mean ± one standard deviation) are thus obtained.

2 Experimental program

The tested specimens are collected from four different testing programs. Beam-column joints have specific deficiencies at joint and global level so that both substandard and pre-1970s design principles can be represented properly. Three of the selected specimens were constructed with plain round bars which can result in bond-slip failure in the overall response. As the estimated analytical model considers concrete fracturing, the slippage of beam longitudinal reinforcing bars should be eliminated. Therefore, the specimens with beam anchor welding, which significantly minimize slip, are selected among the investigated specimens. Not only the analytical model of the as-built specimen was developed but also a specimen retrofitted by diagonally post-tensioned steels rods was also reproduced analytically.

Table 1 summarizes the material properties, dimensions, loading scheme and test setup details of the selected specimens from available literature. More detailed information about the tested specimens i.e. EJ-R&EJ-P-S, JW, T_C3 and C-noSLT can be found in Yurdakul and Avşar [3], Ilki et al. [4], Del Vecchio et al. [5] and Pohoryles [6], respectively. The final damage state of the specimens was visually presented in Fig. 1a-e.



(a) (b) (c) (d) (e)

Fig. 1. Damage state at failure (a) EJ-R [3] (b) EJ-P-S [3] (c) JW [4] (d) T_C3 [5] (e) C-noSLT [6].

Table 1 Detail of the specimens selected from different testing programs.

	Parameter				
Specimen	EJ-R [3]	EJ-P-S [3]	JW [4]	T_C3 [5]	C-noSLT [6]
Concrete Compressive Strength, f_c (MPa)	8.05	9.50	8.00	16.30	29.60
Post-tension	N/A	100kN	N/A	N/A	N/A
Beam Cross-Section (mm)	250 x 500	250 x 500	250 x 500	300 x 500	300 x 450
Column Cross-Section (mm)	250 x 500	250 x 500	250 x 500	300 x 300	300 x 300
Column Axial Load	$0.1A_gf_c$	$0.1A_gf_c$	$0.125A_gf_c$	$0.2A_gf_c$	425kN
Reinforcement	Plain	Plain	Plain	Deformed	Deformed
Test Setup	Loading on the column	Loading on the column	Loading on the beam	Loading on the beam	Loading on the column
Loading Protocol	1 repetition per cycle	1 repetition per cycle	1 repetition per cycle	3 repetitions per cycle	3 repetitions per cycle
Design Principle	Substandard	Substandard	Substandard	Substandard	Pre-1970s
Failure Mode	Joint shear	Joint shear	Joint shear	Joint shear	Joint shear

3 Analytical study

An analytical model based on nonlinear fracture mechanics was employed to evaluate the theoretical tensile capacity of as-built and retrofitted joints by externally applied post-tensioned bars. The capacity of reference specimens was predicted as the strength of plain concrete under tension with microcracks in fracture process zone smeared over a band – i.e. crack band theory [10]. As indicated by Del Vecchio et al. [11], the failure mode of the joint under cyclic loading is characterized by large diagonal corner-to-corner cracks regardless of the dimension and reinforcement details. In addition, in the absence of stirrups at the joint, the principal compressive stress after cracking can be assumed to be inclined at a constant angle, θ, and, in turn, the direction of principal tensile stress is inclined at 90°- θ [11]. In case of the square joint panel, the angle, θ, is 45°. This was also verified by the experiments conducted by Yurdakul and Avşar [3] (Fig. 2). Under the sight of this information, a diagonal crack band can be assumed so the fracture process zone takes place along the diagonal crack. Then, the stress in the crack cohesion for critical strain corresponding to the concrete cracking can be obtained along the band.

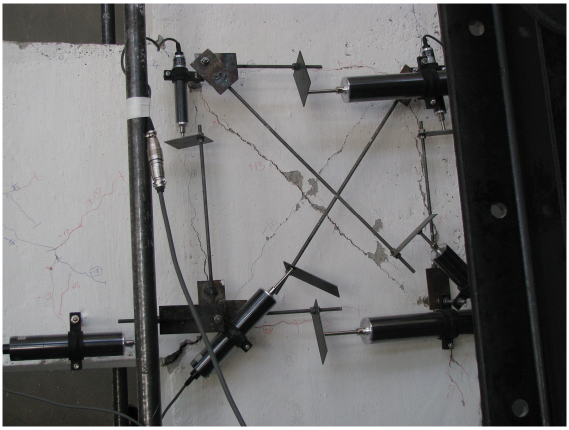


Fig. 2. Corner to corner cracks in X pattern.

The tensile behaviour of concrete in the elastic region was assumed to be uncracked. An exponential function of tension softening was employed in the post-elastic region. The stress in the crack, σ, was calculated according to Hordijk [12], which was presented in Eq. (1).

$$\sigma = f_{ct} \times \left\{ \left[1 + c_1 \left(\frac{w}{w_c} \right)^3 \right] exp \left[-c_2 \left(\frac{w}{w_c} \right) \right] - \frac{w}{w_c} (1 + c_1^3) exp[-c_2] \right\} \tag{1}$$

where w_c is the crack width when the stress releases completely, c_1=3 and c_2=6.93 are the material constants according to Hordijk [12] and w is the crack width. The tensile strength of the concrete (f_{ct}) was selected as $0.56\sqrt{f_c}$ according to ACI 318M-11 [13].

Then, w_c can be obtained by using G_f–w_c relation as proposed by Hordijk [12] (Eq. (2)). Based on the principles of fracture mechanics, the area under Stress-Crack Width curve equals to fracture energy of concrete, G_f, which is the required energy to generate the crack surface per unit area. This value was calculated according to CEB-FIP Model Code [14], which equals to $73 f_{ct}^{0.18}$ in N/m.

$$w_c = 5.14 \frac{G_f}{f_{ct}} \tag{2}$$

The crack width, w, was derived by the crack band theory. As generally accepted, the front of an advancing crack band (microcrack zone), called also the fracture process zone, has a certain characteristic width, h [10]. For plain concrete, it can be considered that crack band width as a material constant that can be determined by experiment and it is expected that h is several-times the maximum aggregate size, which approximately equals to *3g* (g= maximum aggregate size) [10]. Then, the crack width, w, can be derived by Eq. (3) [10].

$$w = \varepsilon_{cr} h \tag{3}$$

ε_{cr} is the strain corresponding to the maximum tensile strength of the concrete (i.e. cracking strain). Wong [15] recommends this value as 0.00008mm/mm, which was also employed in this study.

Under the assumption that smearing of the deformation occurs in the defined band (e.g., diagonal corner to corner crack with a band width of h=3g), Eq. (1) can be used for estimating the stress in the crack, σ, which corresponds to concrete cracking strain. Thus, the principal tensile stress in the joint was estimated for the reference specimen.

Axial load on the specimens should be taken into account since the specimens were tested under the combined effect of lateral and axial load. The effect of axial load considered only as the contribution to the concrete tensile strength. It was thus increased by a coefficient, which considers the state of the stress in the compression strut, proposed by EN 1992-1-1 [16].

It is known from the experimental results that the applied post-tension rods limited the crack propagation. It is assumed that the contribution of axial force (P) in a post-tensioned bar was evaluated by the strain energy stored in the bar per area as shown in Eq. (4). Therefore, the required energy to generate the unit area of crack surface increases. Then, the total energy of the system (G_f^*) was the sum of fracture energy of concrete, G_f, and energy stored in the bars per unit area, U_p, in the retrofitted specimens. In finding the crack width at full stress release, w_c, the sum of energies has been substituted in Eq. (2).

$$U_p = \frac{P \times \Delta l}{2} \times \frac{1}{A_c} \tag{4}$$

The results of the analytical model (i.e. stochastic mean) were presented in Table 4.

4 Stochastic study

One major drawback of the analytical model with the local softening approach is the parameter dependence of the system. This problem is more pronounced in unconfined joints represented as plain concrete. Hillerborg et al. [17] recognized this uncertainty in tensile strain calculations by the development of nonlinear fracture mechanics model for plain concrete [2]. Therefore, the analytical model was combined with a suitable stochastic sampling technique to propose an advanced tool for realistic assessment of the response of the shear critical joint by considering the inherent uncertainties in material constitutive laws.

The prominent material parameters of concrete used in Eq. (1) was firstly defined as a random variable. The randomized values and their distribution were presented in Table 2, which are obtained from Joint Committee on Structural Safety [18] and Pukl et al. [19]. The correlation matrix was presented in Table 3 according to Pukl et al. [19]. Stratified Latin Hypercube Sampling (LHS) including statistical correlation among the prominent material parameters was conducted by FReET software [20] to produce the random samples. The number of simulations was determined in such a way that it was increased until there was no significant change in the computed parameters (e.g. correlation coefficients). The statistical correlation among variables was considered by simulated annealing approach [19,20].

The mean value and prominent range of the distribution (i.e. *mean ± one standard deviation*) were presented in Table 4.

Table 2 Concrete as a random parameter.

Random Parameter	Mean Value (μ)	COV [18,19]	Distribution Function [18,19]
f_{ct} [13]	$0.56\sqrt{f_c}$	0.30	Lognormal (2-parameter)
G_f^*	G_f+U_p	0.25	Weibull min (2-parameter)

Table 3 Correlation matrix of the random variables [19].

	f_{ct}	G_f
f_{ct}	1	0.8
G_f	SYM	1

Table 4 Comparison of experimental and predicted joint tensile stress.

Specimen	Principal Tensile Stress				
	Experimental σ_t	Stochastic Mean σ	Stochastic $\sigma \pm$ *one standard deviation*	COV	Distribution Function
EJ-R [3]	$0.66\sqrt{f_c}$	$0.58\sqrt{f_c}$	$0.41\sqrt{f_c}$-$0.75\sqrt{f_c}$	0.29	Beta
EJ-P-S [3]	$0.68\sqrt{f_c}$	$0.61\sqrt{f_c}$	$0.43\sqrt{f_c}$-$0.79\sqrt{f_c}$	0.30	Weibull (3-par)
JW [4]	$0.56\sqrt{f_c}$	$0.60\sqrt{f_c}$	$0.42\sqrt{f_c}$-$0.78\sqrt{f_c}$	0.30	Gamma (3-par)
T_C3 [5]	$0.43\sqrt{f_c}$	$0.56\sqrt{f_c}$	$0.40\sqrt{f_c}$-$0.72\sqrt{f_c}$	0.28	Beta
C-noSLT [6]	$0.39\sqrt{f_c}$	$0.52\sqrt{f_c}$	$0.37\sqrt{f_c}$-$0.67\sqrt{f_c}$	0.28	Beta

When the stochastic model investigated in depth, the trend in the COV values quite similar to each other for all specimens. A close relation between the mean value and experimental results was found for EJ-R by Yurdakul and Avşar [3] and JW by Ilki et al. [4]. For specimens T_C3 by Del Vecchio et al. [5] and C-noSLT by Pohoryles [6], the difference between the stochastic mean and test results is rather high, on the other hand, the prominent range of the stochastic model still covers the experimental results. The difference can be attributed to the position of the fracture process zone. A diagonal fracture process zone with crack band size *3g* is assumed in the model. In the specimen EJ-R and JW, the place of fracture process zone is as assumed due to the existence of welding and the very low concrete compressive strength. As compressive strength and hence the tensile strength increases, the fracture process zone separates all over the joint due to the existence of hairline cracks, which misinterprets the results. When the crack propagation was limited by the retrofit method (i.e. specimen EJ-P-S), the proposed analytical model was rather successful in estimating the joint shear capacity. To summarize, more accurate results can be obtained with a better estimation of the crack band. Moreover, the variability can be overcome by increasing the number of the experiment on each testing program. Nevertheless, the effect of uncertainties in material constitutive laws were minimized by the stochastic model.

5 Conclusion

This study sets out to propose an advanced analytical model for realistic prediction of shear critical beam-column joints. The experimental data were collected from four different testing programs. The capacity of the as-built specimens was considered as a behaviour of plain concrete with fracture process zone smeared over a band. The crack minimization effect in the specimen retrofitted by diagonally placed post-tension bars was considered as a strain energy stored in the rods per unit area. As model with the local softening approach is sensitive to uncertainties in the material constitutive models, the analytical model was evolved to stochastic level. The random samples were developed by using Latin Hypercube Sampling (LHS) including statistical correlation among the prominent material parameters. Random parameters of concrete and reinforcement steel were defined in accordance with the material test results and code recommendations. The constituent outcomes of the stochastic model including mean value, standard deviation and type of probability density function curves are presented.

Based on the results obtained in this study, the following conclusions can be drawn.

- A relatively small difference was found between the mean value of stochastic models (results of the analytical model) and experimental results for as-built specimens with very low concrete strength and welding of beam longitudinal reinforcing bars.
- With the increasing concrete strength, the model was less efficient since the assumed crack band size and location did not match as it was expected.
- The crack minimization of the post-tension bars in the retrofitted specimens were reproduced well with the proposed analytical model.
- Owing to the stochastic model, the prominent range of probability density functions (i.e. *mean ± one standard deviation*) of the estimated joint tensile stress covered the experimental results.
- More experimental data from same testing programs could lead a trustworthy discussion of the results.

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