

The simulation of paintings with different aesthetic variables Temperature and Harmony

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Abstract—The aesthetic characteristics Temperature and Harmony were suggested for measuring of aesthetic impression image composed of simple patterns. These characteristics are influenced by the number of different types of elements contained in the image, and the number of different symmetries in their arrangement. Full details of all steps in these calculations are given in this paper. The primary concern of this research is to investigate the two-dimensional domain of characteristics Temperature and Harmony that depends on the number of different elements. With the use of random generation and evolutionary algorithms, thousands of images will be generated.

I. INTRODUCTION

Australian mathematician Nikos Salingaros together with Professor Allen Klinger suggested a set of aesthetic variables: L (Life — interest rate), C (Complexity — measure of randomness), T (Temperature — diversity of elements), H (Harmony — orderliness), S (Entropy — non-orderliness).

The constraint for this method is the location of each element of the patterns in regular square fields. It is measured by two basic image properties: the number of different types of elements and the number of symmetries in their arrangement [1].

But they did not present the relationship between the number of colors and the set of characteristics. So we will try to deal with this task in our paper.

Klinger and Salingaros stated, that aesthetic impression of image was determined by the variables Temperature and Harmony. They proposed the hypothesis, that by using measurements L and C for patterns in the images, it is possible to determine the overall impression of the image. But we will not discuss the hypotheses they have proposed. This will be the subject of further research.

Other methods for measuring aesthetic variables exist. Some other symmetry measures are suggested in the paper [2]. Creation of paintings with symmetric patterns are investigated in [3]. A brief history of aesthetic measurement is given, for example, in papers [4], [5].

A. Aesthetic variables Temperature and Harmony

Klinger and Salingaros defined two complementary composite measures L and C , where L is Life and C is Complexity. These values are calculated for the image as a whole, and consequently for its sub-elements.

For illustration, we will describe the calculation of these characteristics for a painting which is composed of 64 elements, placed into a regular 8×8 grid. Characteristics are then computed, at first, for an 8×8 square and then for its sub-elements composed of 4×4 and 2×2 squares. For a painting, which is composed of 36 elements placed into a regular 6×6 grid, characteristics are calculated firstly for the 6×6 square, and then also for its sub-elements composed of 3×3 and 2×2 squares. Everything indicates that the value of characteristics L determines how much viewers feel the image "interesting". This could help to quantify the visual connection of two-dimensional patterns with the viewer. Characteristics C determine a rate of randomness of the array.

Images are either immediately understandable or not, on the basis of how they are processed by our mind. It depends not only on the content of the image as a whole, but also on relationships between its substantive elements. Among the operations leading to an immediate understanding of the image, belongs the perception of interrelated (mutually related) elements and the number of its repetitions. The degree of symmetry is determined by comparing the associated elements and their mutual locations in the image. The greatest creations of humans — whether it's buildings, cities, artwork or artefacts — are not simple, nor coincidental, but show a high degree of organized complexity [1].

We can number the elements in the painting to make it easier to refer to specific processes of aesthetic characteristic computation. See Fig. 1. Throughout the paper in the figures, we will show the values of aesthetic characteristics so that the reader can compare the images.

Klinger and Salingaros described characteristics L and C with the help of variables T and H . Variable T (Temperature) expresses the diversity of elements. In our case, it is calculated as the sum of different colours in the image (or in its subblock) minus one.

Variable H (Harmony) measures the correlation of sub-blocks using selected symmetry. We can also refer to H as negative entropy, because the presence of symmetry is linked with the absence of visual fragmentation.

With characteristics T and H formulas for calculation of L and C are:

$$L = TH, \quad (1)$$

$$C = T(H_{max} - H), \quad (2)$$

where H_{max} is maximal H , which can be achieved in a given system.

Entropy S can be calculated as

$$S = H_{max} - H, \quad (3)$$

and so

$$C = TS. \quad (4)$$

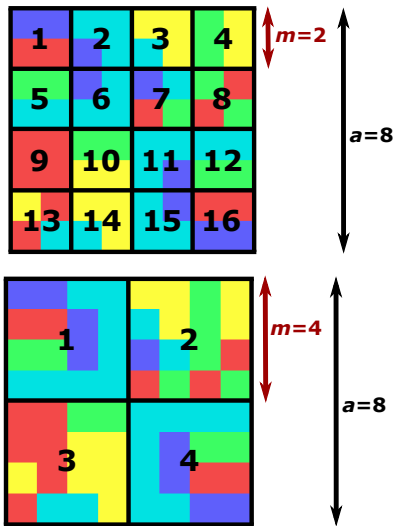
In practice, it is much easier to calculate H (given by the sum symmetry) than S , where it is necessary to determine the degree of disorder. Maximal symmetry H_{max} is a constant for each specific system, and for fixed T , composite measures L and C are different. The following formula applies:

$$C + L = TH_{max}. \quad (5)$$

Why do we measure T , H at first and then we calculate L and C ? Klinger and Salingaros believe that the characteristics T and H , which can be measured from an image, are not directly perceptible to an observer, but there are characteristics L and C , which create an overall impression of the image.

B. Example

Firstly we will compute values T and H , and then by putting them into formulas we will calculate L and C . For calculation T and H , we will have to split the sample image into smaller parts – subblocks (see Fig. 1).



$T = 2.792$, $H = 1.042$, $L = 2.908$, $C = 22.217$, colors = 5

Fig. 1. Labels of Elements, and Sample of Structures. Source: own (drawing in JAVA).

We calculate the partial $T(m \times m)$ for the image divided into sixteen subblocks with side 2×2 , into four subblocks with

side 4×4 and then for a whole image (block with side 8×8). Resulting T will be calculated from these partial $T(m \times m)$.

The general formula for calculation of partial $T(m \times m)$ can be written as:

$$T(m \times m) = \frac{1}{n_m} \sum_{i=1}^{n_m} T_i(m \times m), \quad (6)$$

where m is length of the subblock side and n is the number of subblocks in the image. n can be also expressed as:

$$n = a^2/m^2, \quad (7)$$

where a is the length of the picture and m is the length of the subblock side. T takes the value of the sum of different colours in each subblocks reduced by one.

Image divided into blocks 2×2

Particular $T(2 \times 2)$ for the image divided into subblock with sides 2×2 are calculated according to the formula:

$$T(2 \times 2) = \frac{1}{n_2} \sum_{i=1}^{n_2} T_i(2 \times 2). \quad (8)$$

For Fig. 1 variable a is equal to 8 and variable m is equal to 2. We determine n_2 according to the formula (7):

$$n_2 = \frac{a^2}{m^2} = \frac{8^2}{2^2} = 16.$$

After substituting into the formula (8) we get:

$$\begin{aligned} T(2 \times 2) &= \frac{1}{16} \sum_{i=1}^{16} T_i(2 \times 2) = \frac{1}{16} (T_1(2 \times 2) + \dots + \\ &+ T_{16}(2 \times 2)) = \frac{1}{16} (1 + 1 + 1 + 1 + 1 + 1 + 3 + 1 + \\ &0 + 1 + 1 + 1 + 2 + 1 + 1 + 1) = \frac{18}{16} = 1.125. \end{aligned}$$

Image divided into blocks 4×4

Particular $T(4 \times 4)$ for the image divided into subblock with sides 4×4 are computed according formula:

$$T(4 \times 4) = \frac{1}{n_4} \sum_{i=1}^{n_4} T_i(4 \times 4). \quad (9)$$

Firstly, we determine n_4 according to the formula (7):

$$n_4 = \frac{a^2}{m^2} = \frac{8^2}{4^2} = 4.$$

After substituting into the formula (9) we get:

$$\begin{aligned} T(4 \times 4) &= \frac{1}{4} \sum_{i=1}^4 T_i(4 \times 4) = \frac{1}{4} \cdot (T_1(4 \times 4) + \dots + \\ &+ T_4(4 \times 4)) = \frac{1}{4} \cdot (3 + 4 + 3 + 3) = 13/4 = 3.25. \end{aligned}$$

Particular $T(8 \times 8)$ for the block with side 8×8 is computed according formula:

$$T(8 \times 8) = \frac{1}{n_8} \sum_{i=1}^{n_8} T_i(8 \times 8). \quad (10)$$

We determine n_8 according to the formula (7):

$$n_8 = \frac{a^2}{m^2} = \frac{8^2}{8^2} = 1.$$

After substitution into the formula (10) we get:

$$T(8 \times 8) = \frac{1}{1} \sum_{i=1}^1 T_i(8 \times 8) = T_1(8 \times 8) = 4.$$

Then we calculate T by the following formula:

$$T = \frac{1}{|D|} \sum_{m \in D} T(m \times m), \quad (11)$$

where $|D|$ is number of partial $T(m \times m)$, which we have calculated for the image in the previous steps.

In our case we will get:

$$\begin{aligned} T &= \frac{1}{3} (T(2 \times 2) + T(4 \times 4) + T(8 \times 8)) = \\ &= \frac{1}{3} (1.375 + 3.25 + 4) = 2.792. \end{aligned}$$

Calculation of H will be more demanding. Even here in the same way, we divide the image into the subblocks. Variable H measures the presence of symmetry.

We investigate nine kinds of symmetry. Let use labels $h_1 - h_9$, where $h_1 - h_6$ within one subblock represent:

- h_1 — reflective symmetry along the axis x ,
- h_2 — reflective symmetry along the axis y ,
- h_3 — reflective symmetry along the axis of the first and third quadrant,
- h_4 — reflective symmetry along the axis of the second and four quadrant,
- h_5 — rotational symmetry with angle 90°
- h_6 — rotational symmetry with angle 180° .

The other three cases are calculated relative to the other subblocks of the same dimension (in our case it only makes sense to calculate only within 2×2 and 4×4 subblocks):

- h_7 — the identity of the subblock to another subblock of the same size (translational consensus),
- h_8 — translational consensus with another subblock after reflection along the axis x or axis y ,
- h_9 — translational consensus with another subblock after rotation of 90° , -90° or 180° .

Symetries $h_1 - h_9$ may be present or may not be present. Therefore, the value h may only take values 0 (symmetry is not present), or 1 (symmetry is present).

The resulting H value is determined similarly to the T value by calculation of partial $H(m \times m)$. Firstly, the characteristic $H(2 \times 2)$ of the image, that is divided into sixteen subblocks with side 2×2 , is calculated. Then the characteristic $H(4 \times 4)$ of the image, that is divided into four subblocks with side 4×4 , is determined. Finally, the characteristic $H(8 \times 8)$ of the whole image (blog with side 8×8) is computed. The resulting H will be calculated from these partial $H(i \times i)$, $i = 2, 4, 8$.

The general formula for calculation of partial $H(m \times m)$ can be written as:

$$H(m \times m) = \frac{1}{n_m} \sum_{i=1}^{n_m} H_i(m \times m), \quad (12)$$

where m is length of the subblock side and n is the number of subblocks in the image. n_m can also be expressed as $\frac{a^2}{m^2}$, where a is the length of the picture and m the length of the side subblock. $H_i(m \times m)$ is the total sum of symmetries $h_1 - h_9$ measured from the subblock i .

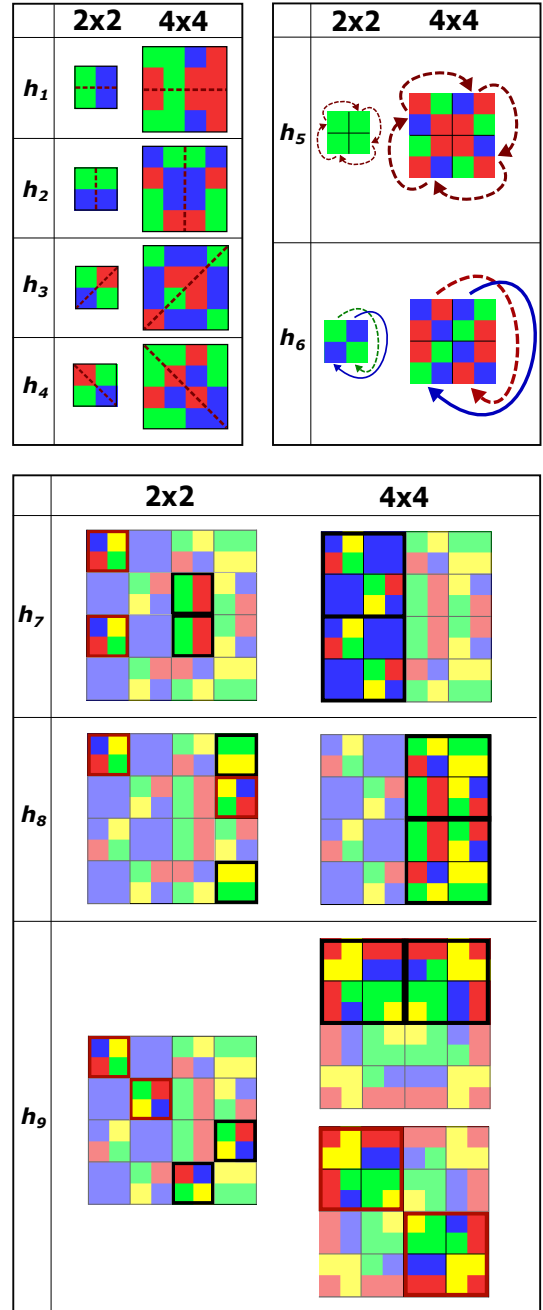


Fig. 2. Symmetries $h_1 - h_9$.

We will determine $H(2 \times 2)$ for the image divided into subblocks with the side 2×2 :

$$H(2 \times 2) = \frac{1}{16} (H_1(2 \times 2) + \dots + H_{16}(2 \times 2)), \quad (13)$$

where

$$\begin{aligned} H_1(2 \times 2) &= h_1(1)(2 \times 2) + \dots + h_9(1)(2 \times 2), \\ H_2(2 \times 2) &= h_1(2)(2 \times 2) + \dots + h_9(2)(2 \times 2), \\ &\dots \end{aligned}$$

$$H_{(16)}(2 \times 2) = h_1(16)(2 \times 2) + \dots + h_9(16)(2 \times 2).$$

For better understanding, first we find $H_1(2 \times 2) - H_{16}(2 \times 2)$:

$$\begin{aligned} H_1(2 \times 2) &= 0 + 1 + 0 + 0 + 0 + 0 + 0 + 1 + 1 = 3, \\ H_2(2 \times 2) &= 0 + 0 + 1 + 0 + 0 + 0 + 0 + 1 + 1 = 3, \\ H_3(2 \times 2) &= 0 + 0 + 1 + 0 + 0 + 0 + 1 + 0 + 0 = 2, \\ H_4(2 \times 2) &= 1 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 1 = 2, \\ H_5(2 \times 2) &= 0 + 1 + 0 + 0 + 0 + 0 + 0 + 1 + 1 = 3, \\ H_6(2 \times 2) &= 0 + 0 + 0 + 1 + 0 + 0 + 0 + 1 + 1 = 3, \\ H_7(2 \times 2) &= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0, \\ H_8(2 \times 2) &= 0 + 0 + 1 + 1 + 0 + 1 + 0 + 0 + 0 = 3, \\ H_9(2 \times 2) &= 1 + 1 + 1 + 1 + 1 + 1 + 0 + 0 + 0 = 6, \\ H_{10}(2 \times 2) &= 0 + 1 + 0 + 0 + 0 + 0 + 0 + 0 + 1 = 2, \\ H_{11}(2 \times 2) &= 0 + 0 + 0 + 1 + 0 + 0 + 0 + 1 + 1 = 3, \\ H_{12}(2 \times 2) &= 0 + 1 + 0 + 0 + 0 + 0 + 0 + 1 + 1 = 3, \\ H_{13}(2 \times 2) &= 0 + 0 + 0 + 1 + 0 + 0 + 0 + 0 + 0 = 1, \\ H_{14}(2 \times 2) &= 0 + 0 + 1 + 0 + 0 + 0 + 1 + 0 + 0 = 2, \\ H_{15}(2 \times 2) &= 0 + 0 + 1 + 0 + 0 + 0 + 0 + 1 + 1 = 3, \\ H_{16}(2 \times 2) &= 0 + 1 + 0 + 0 + 0 + 0 + 0 + 1 + 1 = 3. \end{aligned}$$

Now we can substitute into the formula (13) for calculating $H(2 \times 2)$:

$$\begin{aligned} H(2 \times 2) &= \frac{1}{16} \cdot (H_1(2 \times 2) + \dots + H_{16}(2 \times 2)) \\ &= \frac{1}{16} \cdot (3 + 3 + 2 + 2 + 3 + 3 + 0 + 3 + \\ &\quad 6 + 2 + 3 + 3 + 1 + 2 + 3 + 3) = \frac{1}{16} \cdot 42 = 2.625. \end{aligned}$$

Similarly we calculate $H(4 \times 4)$:

$$H(4 \times 4) = \frac{1}{n_4} \sum_{i=1}^{n_4} H_i(4 \times 4). \quad (14)$$

For image divided into subblocks with side 4×4 we calculate H as follows:

$$H(4 \times 4) = \frac{1}{4} \cdot (H_{(1)}(4 \times 4) + \dots + H_{(4)}(4 \times 4)), \quad (15)$$

where

$$\begin{aligned} H_1(4 \times 4) &= h_1(1)(4 \times 4) + \dots + h_9(1)(4 \times 4), \\ &\dots \\ H_4(4 \times 4) &= h_1(4)(4 \times 4) + \dots + h_9(4)(4 \times 4). \end{aligned}$$

We will calculate $H_1(4 \times 4) - H_4(4 \times 4)$:

$$\begin{aligned} H_1(4 \times 4) &= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 1 = 1, \\ H_4(4 \times 4) &= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0, \end{aligned}$$

$$\begin{aligned} H_3(4 \times 4) &= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0, \\ H_4(4 \times 4) &= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 1 = 1. \end{aligned}$$

Now we can substitute into the formula (14) for calculating $H(4 \times 4)$:

$$\begin{aligned} H(4 \times 4) &= \frac{1}{4} (H_1(4 \times 4) + \dots + H_4(4 \times 4)) \\ &= \frac{1}{4} (1 + 0 + 0 + 1) = \frac{1}{4} \cdot 2 = 0.5. \end{aligned}$$

Now we determine $H(8 \times 8)$:

$$H(8 \times 8) = \frac{1}{n_8} \sum_{i=1}^{n_8} H_i(8 \times 8), \quad (16)$$

after substitution we receive:

$$H(8 \times 8) = \frac{1}{1} \cdot H_1(8 \times 8). \quad (17)$$

Symetries h_7-h_9 will definitely be zero, because there are no other 8×8 blocks, which could be compared with each other. We enumerate $H_1(8 \times 8)$:

$$\begin{aligned} H_1(8 \times 8) &= h_1(1)(8 \times 8) + \dots + h_9(1)(8 \times 8) = \\ &= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0. \end{aligned}$$

After substitution into the formula (17):

$$H(8 \times 8) = \frac{1}{1} \cdot (0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0) = 1 \cdot 0 = 0.$$

Then we calculate resulting H by the following formula:

$$H = \frac{1}{|D|} \sum_{m=1}^{|D|} H(m, m), \quad (18)$$

where $|D|$ is number of particular $H(m, m)$, which we have calculated for the image in the previous steps. Then

$$\begin{aligned} H &= \frac{1}{3} \cdot (H(2 \times 2) + H(4 \times 4) + H(8 \times 8)) \\ &= \frac{1}{3} \cdot (2.625 + 0.5 + 0) = 1.042. \end{aligned}$$

Calculation L a C we already performed by simple substitution into formulas:

$$\begin{aligned} L &= T \cdot H = 2.792 \cdot 1.042 = 2.908, \\ C &= T \cdot (H_{max} - H), \end{aligned}$$

where H_{max} is maximal value of H , which can be achieved in a given system. Maximal symetry H_{max} is constant for every specific system.

Then

$$C = 2.792 \cdot (9 - 1.042) = 22.217.$$

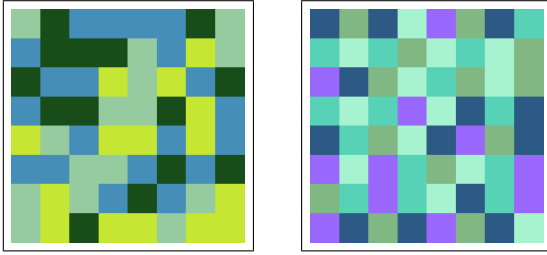
II. DISPUTE ABOUT THE VALUE RANGE OF CHARACTERISTICS L AND C

For a closer examination of the Klinger-Salingaros hypothesis, it is necessary to specify the range of L and C characteristics.

To plot the value range of all L and C characteristics for a maximum four-color images, we would need to generate 4^{64} images, which is about $3,4 \cdot 10^{38}$ combinations. Even for plotting the L and C values for only two-color images, we would need to generate more than $1,8 \cdot 10^{19}$ images. Of course, this is almost impossible.

A. Generating images using a random number generator

During the analysis of the L and C characteristics of randomly generated images, it was found that the L and C values for a large part of the generated images are only displayed in a very limited area of the LC graph. In Fig. 5 is a graph of L and C values of randomly generated images. There were randomly generated 300 million images of two to four colors (100 million images for each number of colors). In the case of the group of generated images, the L and C values were often the same.



$T = 2.6250, H = 0.4167$
 $L = 1.0938, C = 22.5313$
 colors = 4

$T = 3.6667, H = 0.0000$
 $L = 0.0000, C = 33.0000$
 colors = 5

Fig. 3. Images generated using a random number generator.

In Fig. 5, we can see a graph of the L and C characteristics for 300 million randomly generated images, rendered in different colors according to the frequency of matches. The red points in the graph represent separate images, the yellow points represent the consensus of five images, the purple points represent the consensus of fifteen or more images (see the legend of the graph).

In Fig. 4, in detail we can see the matches of the generated images. A graph of relative occurrence frequencies is used to display obtained results.

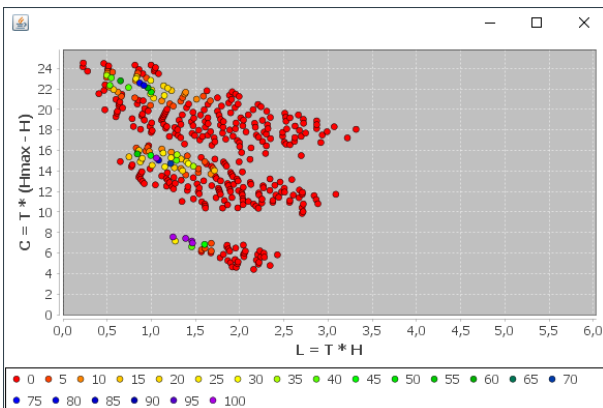


Fig. 4. Detail of match of characteristics L and C of randomly generated images (a graph of relative occurrence frequencies is used to display this).

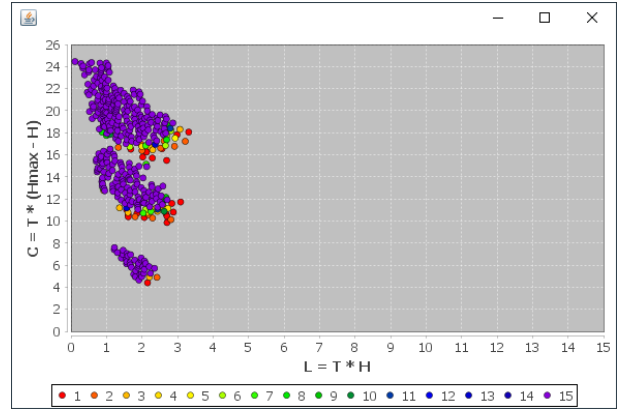


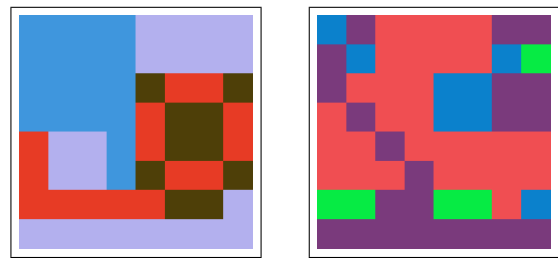
Fig. 5. Graph of characteristics L and C for 300 million randomly generated images (a graph of absolute occurrence frequencies).

Generating images, using only the random number generator, does not appear to be a suitable method to cover the entire field of values of characteristics L and C .

B. Generating images by pattern combining.

However, the range of L and C characteristics is considerably larger. With increasing symmetries, the L characteristic increases in certain circumstances. The C characteristic is most affected by the amount and spacing of each color in the image. However, L and C also have a certain dependence on each other, since both are calculated from the T and H characteristics by the formula (2). Therefore, it will be necessary to deliberately generate certain types of images to expand the value range.

Several ways of generating images have been used. First of all, pattern combining was done. Sets of 2×2 and 4×4 blocks have been created. Each block always contained a certain type of symmetry. The algorithm randomly selected the blocks from the prepared sets and then composed the resulting images.



$T = 1.7292, H = 2.4583$
 $L = 4.2509, C = 11.3116$
 colors = 4

$T = 2.0208, H = 2.0208$
 $L = 4.0838, C = 14.1037$
 colors = 4

Fig. 6. Images created by pattern combining.

Fig. 7 shows the L, C chart for 300 million randomly generated images and 100 million images created by pattern combining. A graph of absolute occurrence frequencies is used to display this.

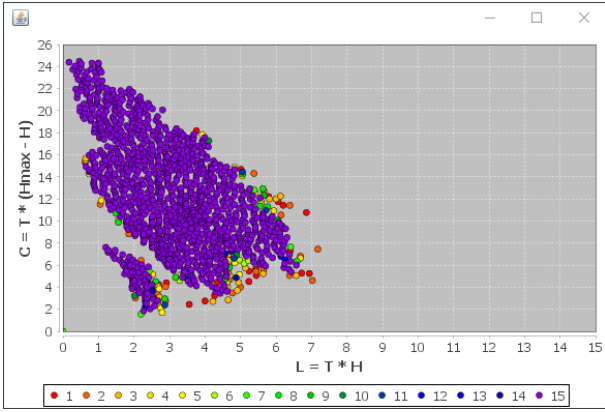


Fig. 7. Graph of L and C values of 300 million randomly generated images and 100 million images created by pattern combining (a graph of absolute occurrence frequencies).

C. Generating images using genetic algorithm

Next, the genetic algorithm was used to generate images.

The principle of the functioning of the genetic algorithm is mentioned in [6]. The genetic algorithm belongs to so-called evolutionary algorithms. It is a stochastic process that tries to find solutions of complex problems for which there is no usable exact algorithm. The principles of evolutionary biology are applied in this case.

Genetic algorithms have several characteristics:

- 1) They work with a whole range of possible solutions to the problem, instead of finding an individual solution,
- 2) they gradually improve the generated solutions by including new solutions; new solutions are obtained by combining (crossing) the original solutions,
- 3) Random changes (mutations) can occur in solutions,
- 4) unsatisfactory solutions are gradually eliminated.

A genetic algorithm in the beginning, has a certain population, where individuals of the population are various solutions to the problem. When crossing, the algorithm gradually generates new solutions to the problem. New individuals are born, who usually have a random portion of genes from one parent and the rest of the genes from the other parent. Each of the newly created individuals is calculated so. Fitness function expresses the quality of the solution represented by this individual. During the crossover, in the chromosome of an individual, a random change of a gene may occur (random mutation). This random mutation may or may not be favorable for further evolution of the species. The process of crossbreeding is constantly repeated, and generations with ever better genetic properties can emerge.

Specification of the used genetic algorithm:

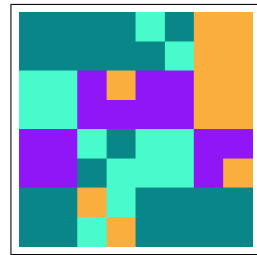
The initial population was composed of a set of 1 000 images. The first third of the population consisted of images generated randomly. The second third of the population contained images created by pattern combining, while the 2×2 and 4×4 subblocks were placed at a random location in the image. The last third of the population contained images created by pattern combining. During the pattern combining

process the subblocks were placed strictly at certain points so as to achieve even greater symmetries.

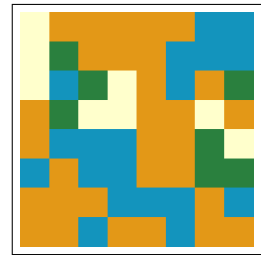
As a fitness function, a function computing the Euclidean distance to the target point $[L, C]$ was selected. The target point was chosen for each generated image randomly.

The population also changed during the calculation. Random mutations were possible. At the multipoint crossover, the two images were divided into three parts, some of which were swapped with one another. During the swap mutation, some points were randomly swapped within one image. The probabilities of occurrence of the mutations varied, and each generated image were different.

Evolutionary calculation was set to finish 2 000 generations. If a stabilization occurred, the calculation would be terminated after 500 iterations that did not improve the maximum fitness value.



$T = 2.1042$, $H = 2.6250$
 $L = 5.5234$, $C = 13.4141$
 colors = 4



$T = 2.1458$, $H = 1.5417$
 $L = 3.3082$, $C = 16.0043$
 colors = 4

Fig. 8. Images obtained with genetic algorithm.

Fig. 9 shows the L, C chart for 300 million randomly generated images, 100 million images created by pattern combining and 13 200 images created by using a genetic algorithm. A graph of absolute occurrence frequencies is used to display this.

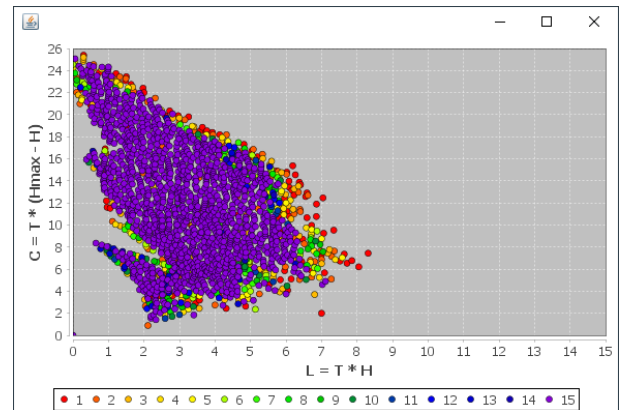


Fig. 9. Graph of L and C values of 300 million randomly generated images, 100 million images created by pattern combining and 13 200 images created by using genetic algorithm (a graph of absolute occurrence frequencies).

D. Purposeful generating images containing large amounts of symmetry.

A certain number of images especially at the top left and bottom left area of value range were created manually. For

example, the highest values of characteristic C with the minimum L characteristic, were achieved by combining as many colors as possible in each 2×2 and 4×4 subblocks while trying to achieve zero or minimal symmetries. Gradually adding symmetries, the values of the L characteristic increased, and the upper boundary of the value range began to form. This always (verified for two to five color images) forms an angle of 135° with a horizontal axis.

As mentioned above, with increasing symmetries, the L characteristic increases in certain circumstances.

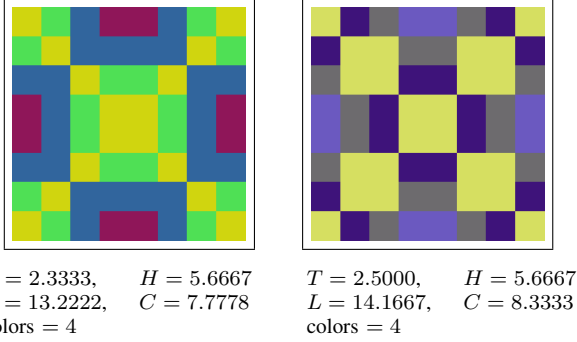


Fig. 10. Images generated to achieve high symmetries.

Therefore, the images with the highest values of L (the images at the boundary of the value range in the right area of the graph) will contain a large number of symmetries. This can be accomplished by focusing on generating images that will also contain h_1 and h_2 symmetry for block 8×8 . If the 8×8 block is positive for the presence of symmetries h_1 and h_2 at the same time, then this 8×8 block must necessarily also have h_6 symmetry, and each of its subblocks 4×4 and 2×2 must be positive for the presence of symmetries h_8 and h_9 . However, if a generated picture would also have in addition to symmetries h_1 and h_2 — symmetries h_3 and h_4 , for this 8×8 block must be necessarily present also symmetries h_5 and h_6 and each of its subblocks 4×4 and 2×2 must be positive for the presence of symmetries h_8 and h_9 too. With the appropriate combination of other symmetries within the individual subblocks 4×4 and 2×2 , we could thus reach the limit values of the L characteristic.

Taking into account equation (1) for calculating characteristics L , it is obvious that the aim is not to find an image with the maximum number of symmetries present. The value of the H characteristic, depending on the number of symmetries, will be maximal for such an image; but the T characteristic, that measures diversity elements, will be zero. This will also cause a zero value of the L characteristic of this image.

Because, the characteristic L is directly proportional to the product of values T and H , to find its maximum, we will need to find the values of characteristics T and H such that their product is the maximum!

To achieve symmetries h_1 and h_2 for block 8×8 in the image, it will be enough to generate only one quadrant of the image, which will then be duplicated, and according to the rules of axial symmetry, successively flipped over into the

other quadrants. With this substantial simplification, we reduce the number of combinations, needed to generate up to four-color images, to 4^{16} (which is approximately 4 billion). Such a number of images can be generated and processed within a few tens of hours. For practical reasons, only images with unique values of L and C were saved when generating.

E. Summary and comparison of the results.

Now we will introduce several generated images (symmetric patterns are chosen in particular) to allow the reader to monitor dependency of aesthetic characteristics and the patterns.

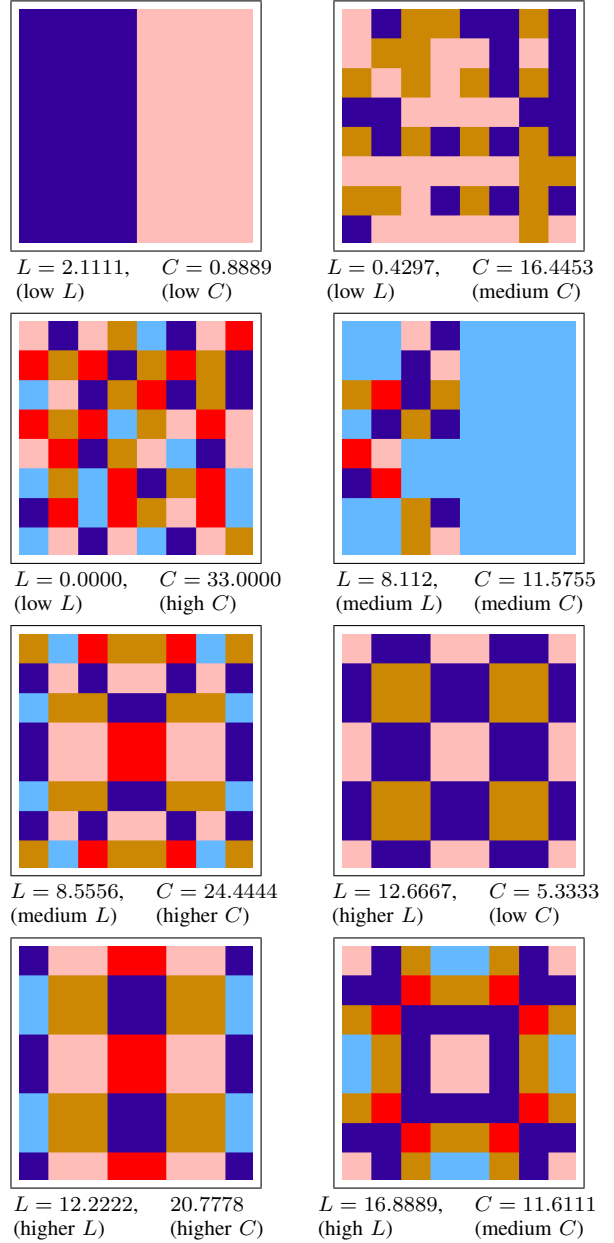


Fig. 11. Images with different values of L and C characteristics.

In Fig. 12 we will show the position of all the generated images. The L, C chart shows 300 million randomly generated images, 100 million images created by patterns combining,

13 200 images created by using a genetic algorithm, 780 images created manually, and images generated to achieve high symmetries (with h_1 and h_2 symmetry for block 8×8). A graph of absolute occurrence frequencies is displayed below.

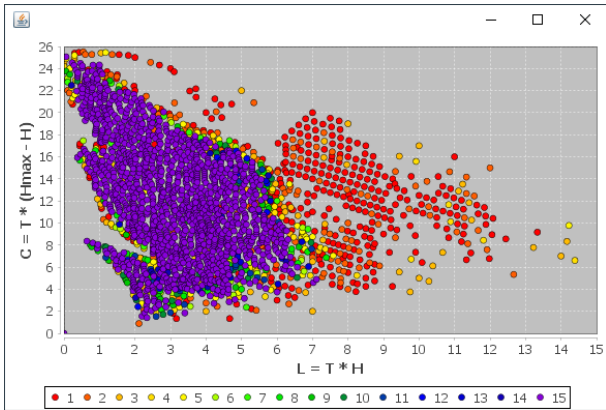


Fig. 12. Graph of L and C values of 300 million randomly generated images, 100 million images created by pattern combining, 13 200 images created by using genetic algorithm and 780 images created manually, and images generated to achieve high symmetries (containing h_1 and h_2 symmetry for block 8×8). A graph of absolute occurrence frequencies is used to display this.

Using these several methods, the L and C function value range was almost found for two, three, four, and five-color images.

Now we wrap the value ranges with convex envelopes (see Fig. 13) and calculate their individual centers of gravity (cf. Tab. I), which we then use for further calculations.

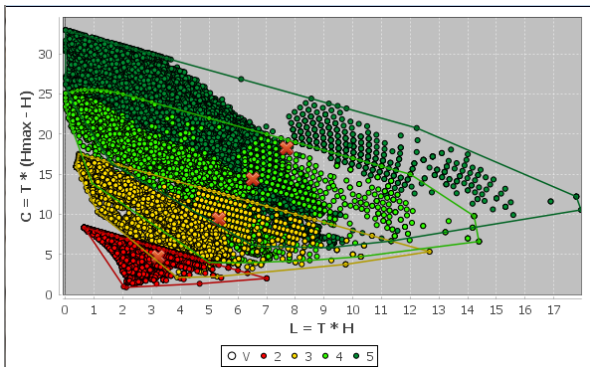


Fig. 13. Values of L and C characteristics for all acquired images.

In Tab. I, we can compare the focus of the range of image group values which depend on the method used to generate them. This can also be compared using the values in Fig. 5, 7, 9, 12. From Tab. I it is obvious that the center of gravity of the RG method has the smallest L coordinates and the largest coordinates of C . Envelopes for image groups generated by the PC and GA methods have a center of gravity with a larger L coordinate than the RG method, but a minor C coordinate. The image group envelope generated by the HS method has the center of gravity with the highest value L .

TABLE I
CENTERS OF GRAVITY OF THE L AND C RANGES.

method	Colors			
	2	3	4	5
random generation	[1.71,6.30]	[1.68,13.77]	[1.45,20.62]	[1.32,26.82]
pattern combining	[2.41,4.22]	[2.86,10.08]	[3.55,14.04]	[3.92,18.77]
genetic generation	[3.23,3.74]	[3.61,9.13]	[3.92,13.99]	[4.73,19.14]
high symmetries	[4.34,3.09]	[5.44,8.33]	[6.64,13.33]	[7.70,17.20]
all	[3.25,3.70]	[5.37,8.36]	[6.55,13.35]	[7.70,17.20]

III. CONCLUSION

In the first instance we tried to prepare an exactly described algorithm for calculation of the aesthetic characteristics of Life and Complexity. We complement the cited literature that did not fulfill this goal. The main aim of this paper was to identify the value range of L and C characteristics. For this purpose, we used several methods of generating images – generating by random algorithm, pattern combining method, generating images using genetic algorithm, and manual image creation with content of specific types of symmetry. We have managed to find a significant part of the value range for two to five color images. We explored the fact, that the value range of L and C characteristics depend on number of colors. Klinger and Salingaros have formulated a hypothesis that claims that with varying aesthetics characteristics, the audience's emotions also change. In order to be able to test this hypothesis on a random sample, we need to know, not only the field of aesthetic characteristics, but also their center of gravity. We found that aesthetic characteristics were conditional on the number of colors. This fact was not previously known in the literature. If the hypothesis of Klinger and Salingaros is accepted, then the emotional dependence on aesthetic characteristics will be described. Based on such knowledge, an image, that produces the greatest possible emotion (e.g. which viewer energises, excites, reassures, etc.) could be traced.

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