Comparative Study of Predictive Controllers for Trajectory Tracking of Non-holonomic Mobile Robot

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- **Topic** – non-linear model predictive control of non-holonomic mobile robot
- **Task** – mobile robot trajectory tracking problem
- **Solution** – nonlinear kinematic equation is linearized into two different linear time varying models based on the reference coordinate frame - a successive linear model is derived, considering the world coordinates, by linearizing around the reference points and an error based linear model is derived, considering the local coordinate of mobile robot, by coordinate transformation. Two trajectory tracking NMPCs are designed with these models by minimizing a criteria consisting of state tracking error, control effort and terminal state deviation error.

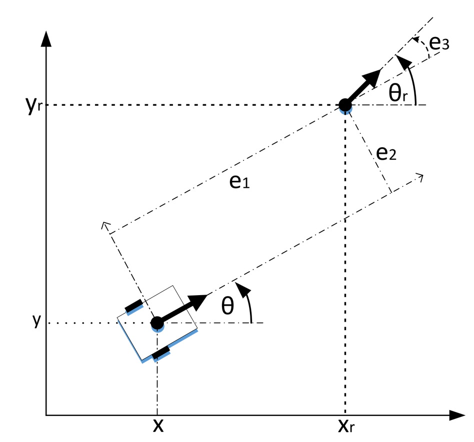
KINEMATIC MODEL OF NON-HOLONOMIC MOBILE ROBOT

$$\dot{\boldsymbol{x}} = \begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \omega\end{bmatrix} \qquad \dot{\boldsymbol{x}}_r = \begin{bmatrix}\dot{x}_r\\ \dot{y}_r\\ \dot{\theta}_r\end{bmatrix} = \begin{bmatrix}\cos\theta_r & 0\\ \sin\theta_r & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_r\\ \omega_r\end{bmatrix}$$

$$v_r(t) = \sqrt{\dot{x}_r(t)^2 + \dot{y}_r(t)^2}$$

$$\theta_r(t) = arctan\left(\dot{y}_r(t), \dot{x}_r(t)\right)$$

$$\omega_r(t) = \dot{\theta}_r(t)$$

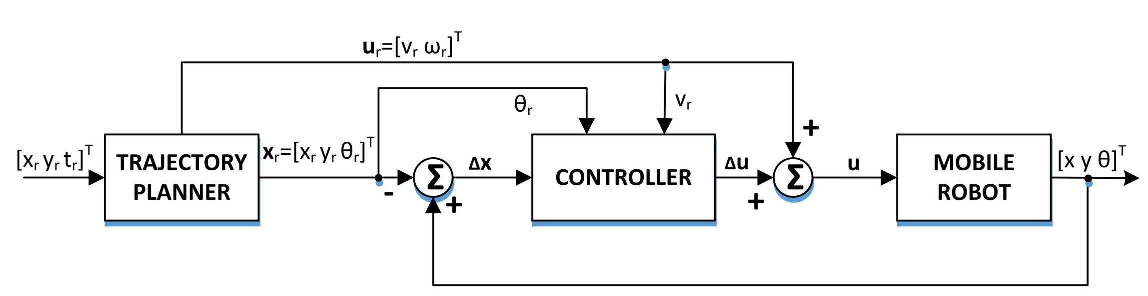


Coordinate system

Successive linear model (M_1)

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}_r, \boldsymbol{u}_r) + \left.\frac{\partial f(\boldsymbol{x}, \boldsymbol{u})}{\partial \boldsymbol{x}}\right|_{\substack{x = x_r\\ u = u_r}} (\boldsymbol{x} - \boldsymbol{x}_r) + \left.\frac{\partial f(\boldsymbol{x}, \boldsymbol{u})}{\partial \boldsymbol{u}}\right|_{\substack{x = x_r\\ u = u_r}} (\boldsymbol{u} - \boldsymbol{u}_r)$$

$$\Delta\dot{\boldsymbol{x}} = \widetilde{\boldsymbol{A}}_S(\boldsymbol{x}_r, \boldsymbol{u}_r).\Delta\boldsymbol{x} + \widetilde{\boldsymbol{B}}_S(\boldsymbol{x}_r, \boldsymbol{u}_r)\Delta\boldsymbol{u}$$

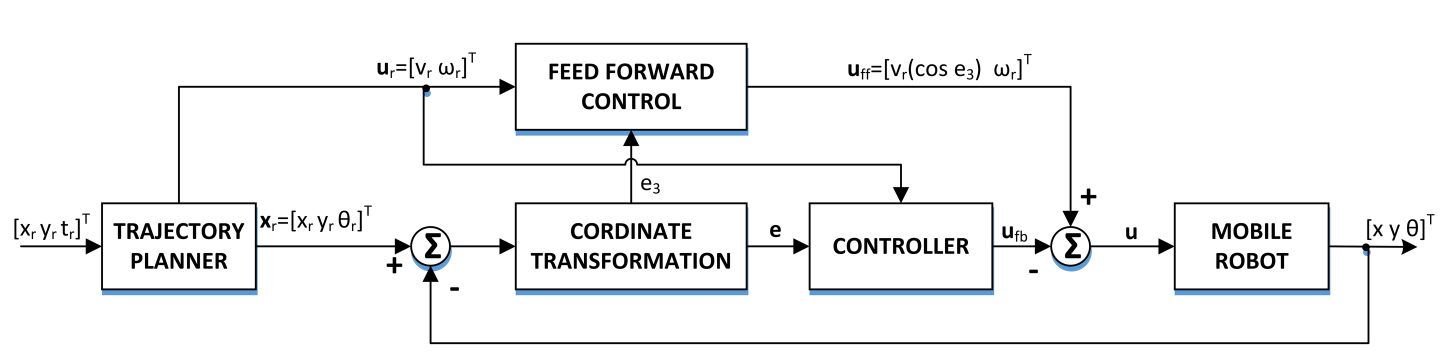


Trajectory tracking kinematic controller with successive linear model

Error based linear model (M_2)

$$\boldsymbol{e} = \begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_r - x\\ y_r - y\\ \theta_r - \theta\end{bmatrix} = \boldsymbol{T_x}(\boldsymbol{x_r} - \boldsymbol{x})$$

$$\dot{\boldsymbol{e}} = \begin{bmatrix}\dot{e}_1\\ \dot{e}_2\\ \dot{e}_3\end{bmatrix} = \begin{bmatrix}e_2\omega - v + v_r\cos e_3\\ -e_1\omega + v_r\sin e_3\\ \omega_r - \omega\end{bmatrix}$$



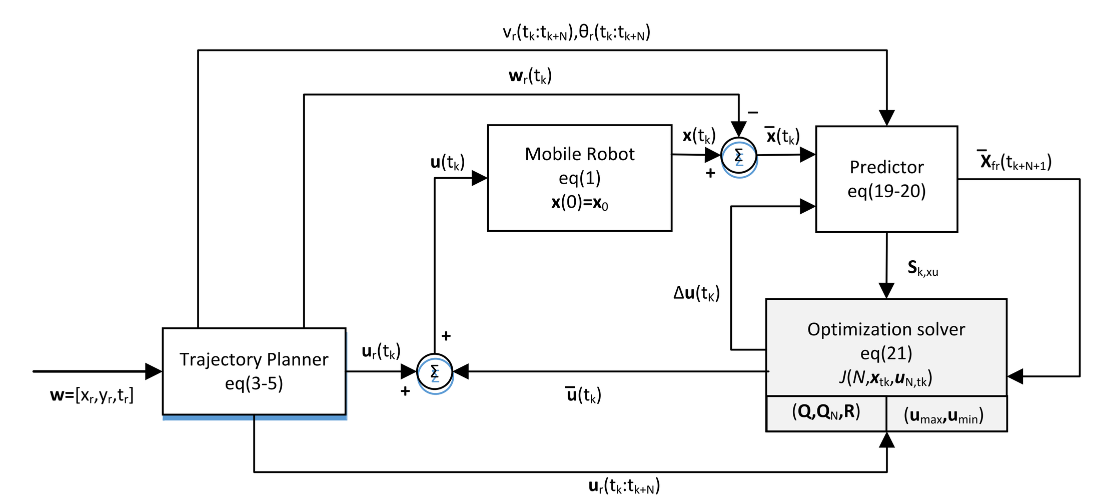
Trajectory tracking kinematic controller with error based linear model

TRAJECTORY TRACKING BY NMPC

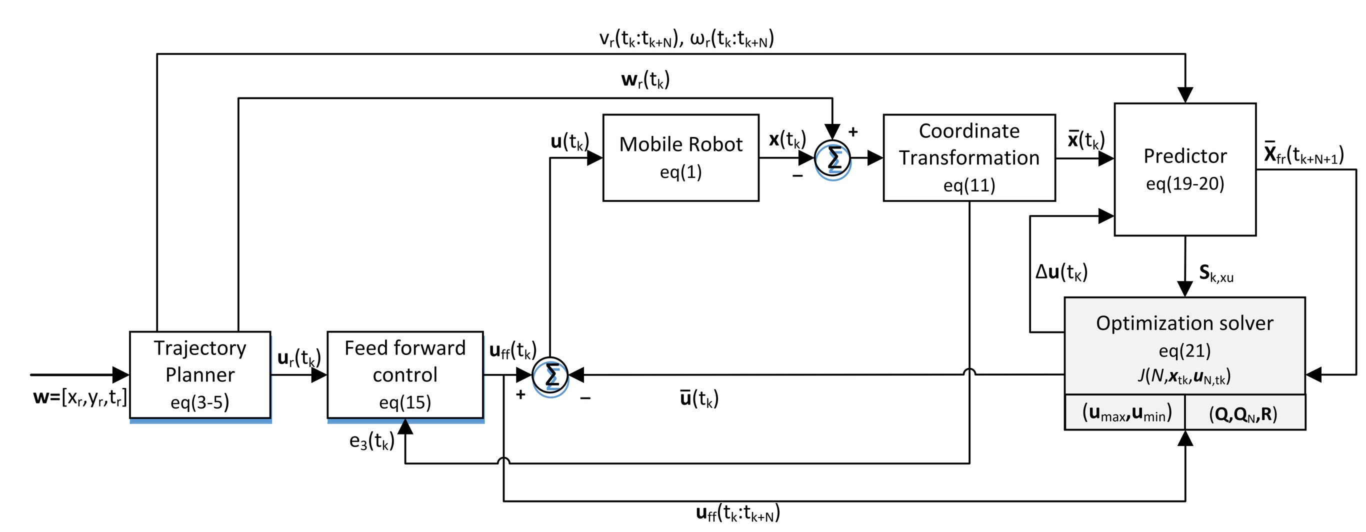
Prediction model $\quad \overline{\boldsymbol{X}}_N = \boldsymbol{S}_{k,xu}\Delta\boldsymbol{u}_N + \boldsymbol{S}_{k,xx}\overline{\boldsymbol{x}}_k + \boldsymbol{S}_{k,xu}\overline{\boldsymbol{u}}_{N,0}$

Cost function

$$J\left(N, \overline{\boldsymbol{x}}_0, \overline{\boldsymbol{u}}_{N,0}\right) = \overline{\boldsymbol{X}}_N{}^T \boldsymbol{Q}\overline{\boldsymbol{X}}_N + \Delta\boldsymbol{u}_N{}^T \boldsymbol{R}\Delta\boldsymbol{u}_N, \quad \Delta\boldsymbol{u}_N = \overline{\boldsymbol{u}}_N - \overline{\boldsymbol{u}}_{N,0}$$



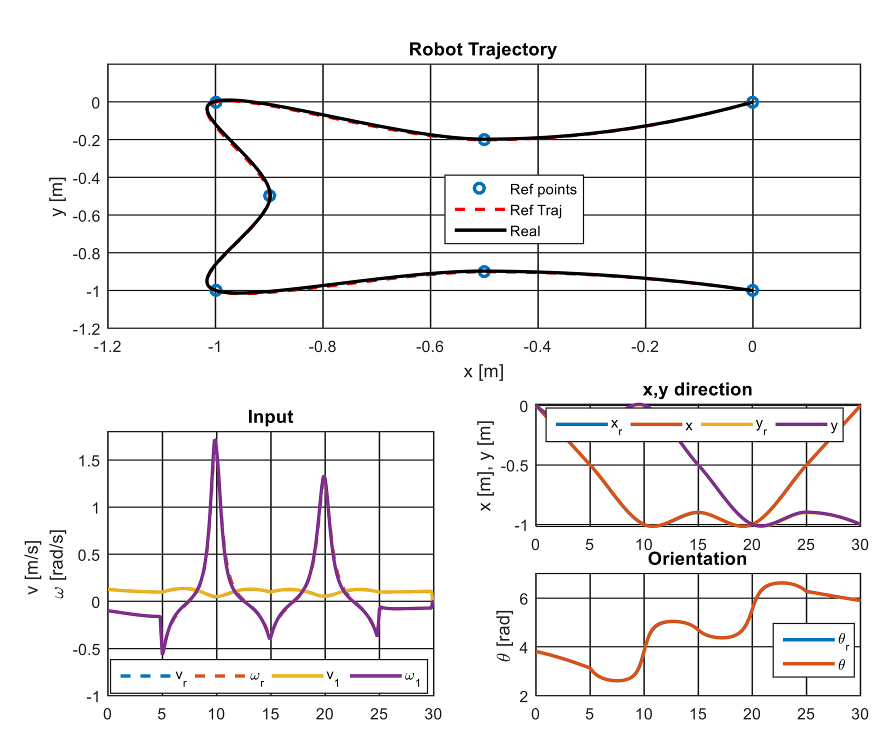
Trajectory tracking $NMPC_1$ with LTV model



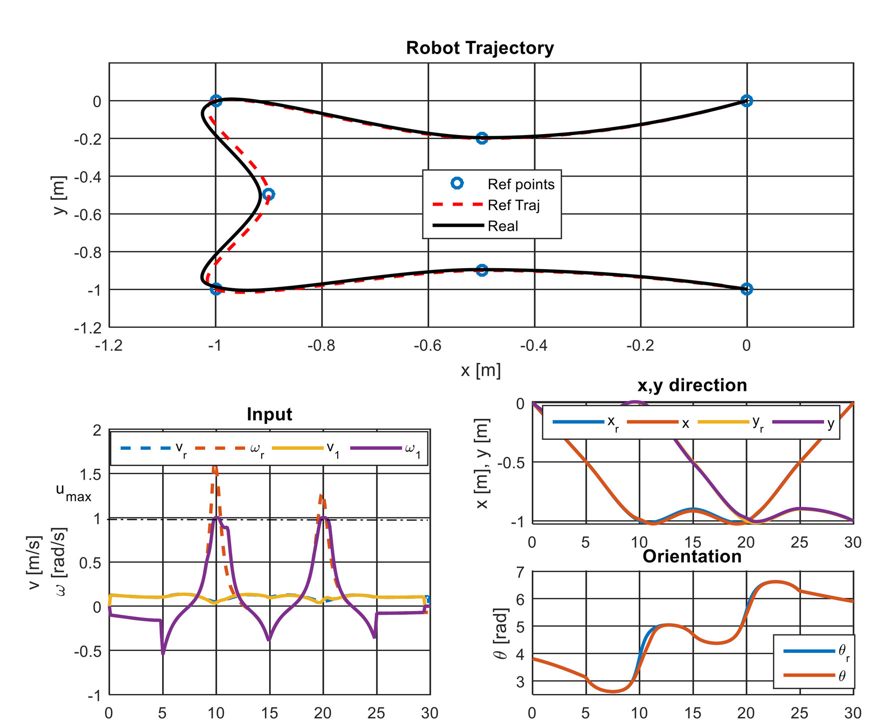
Trajectory tracking $NMPC_2$ with LTV model

Solution $\qquad \min_{\Delta\boldsymbol{u}} J = \Delta\boldsymbol{u}^T M\Delta\boldsymbol{u} + 2\boldsymbol{m}^T\Delta\boldsymbol{u} \qquad \boldsymbol{A_0}\Delta\boldsymbol{u} \leq \boldsymbol{b}_0$

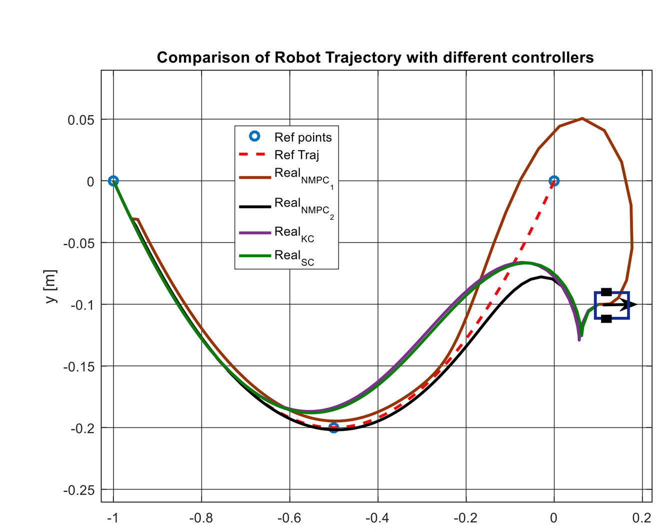
SIMULATION RESULTS



Trajectory tracking with unconstraint $NMPC_1$



Trajectory tracking with constraint $NMPC_2$



Trajectory tracking with different initial conditions